St Leonard's College

**VCE Unit 4 Specialist Maths**

**EXAMINATION Paper 2**

**Practice Exam 2019**

**Question and Answer Booklet**

**STUDENT NAME:** _____

**TEACHER(S):** Ms S Woolley

**TIME ALLOWED:** **Reading time 15 minutes**

**Writing time 120 minutes**

**INSTRUCTIONS**

**All answers are to be written on the examination paper.**
**Write your answers clearly with relevant working shown.**
**Multiple Choice questions are to be answered by circling the letter corresponding to the chosen answer on the answer sheet provided.**
**A formula sheet is included with this examination paper.**

**Materials permitted**
**One set of bound summary sheets or one bound book**
**A CAS calculator and a scientific calculator**

**STRUCTURE OF BOOKLET / MARKINGSCHEME**

| Exam Section | Number of questions to be answered | Total marks |
|---|---|---|
| A<br>B | 20<br>6<br>Total: | 20<br>60<br>80 |

**Students are not permitted to bring mobile phones and / or any other unauthorized electronic devices into the examination room.**

## SECTION A – Multiple-choice questions

**Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer score 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude $g$ ms$^{-2}$, where $g = 9.8$

**Question 1**

The implied range of $y = \tan^{-1}\left(\frac{x}{a}\right)$ is

**A.** $(-a, a)$
**B.** $(0, a)$
**C.** $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
**D.** $(0, \pi)$
**E.** $R$

**Question 2**

Let $A$, $B$, $C$ and $D$ represent non-zero rational numbers.

The expression $\frac{x^2-2}{(x-1)^2(x^2+2)}$ can be represented in partial fraction form as

**A.** $\frac{A}{x-1}+\frac{B}{x^2+2}$
**B.** $\frac{Ax+B}{(x-1)^2}+\frac{Cx+D}{x^2+2}$
**C.** $\frac{A}{(x-1)^2}+\frac{Bx+D}{x^2+2}$
**D.** $\frac{A}{x-1}+\frac{B}{x-1}+\frac{Cx+D}{x^2+2}$
**E.** $\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{Cx+D}{x^2+2}$

**Question 3**

Let $z = 3\text{cis}\left(\frac{\pi}{5}\right)$.

$\text{Arg}\left(z^6\right)$ is equal to

**A.** $\left(\frac{\pi}{5}\right)^6$

**B.** $-\frac{4\pi}{5}$

**C.** $\frac{4\pi}{5}$

**D.** $\frac{5\pi}{6}$

**E.** $\frac{6\pi}{5}$

**Question 4**

Consider the complex numbers $2z,\ -iz$ and $2z - iz,$ where $z \in C \setminus \{0\}$.
These three complex numbers are plotted in the Argand plane and together with the origin *O*, they form the vertices of a quadrilateral.
The area of this quadrilateral is

**A.** $\frac{1}{|2z|}$

**B.** $|2z|$

**C.** $\left|z^2\right|$

**D.** $2|z|^2$

**E.** $|z| + |2z|$

**Question 5**

The complex number $z_1$ is a solution to the equation $z^n = 1 + \sqrt{3}i$, where *n* is a positive integer.
Which one of the following cannot be true?

**A.** $|z_1| = \sqrt{2}$

**B.** $\text{Arg}(z_1) = 0$

**C.** $|z_1| = 2$

**D.** $\text{Arg}(z_1) = \frac{7\pi}{9}$

**E.** $|z_1| = \sqrt[3]{2}$

**Question 6**

The length of the curve defined by the parametric equations $x = \sqrt{t}$ and $y = e^{2t}$, where $0 \le t \le 3$, is closest to

**A.** 17.0
**B.** 31.2
**C.** 205.7
**D.** 368.2
**E.** 402.7

**Question 7**

Using a suitable substitution, $\int_{-2}^{0} (x+2)\sqrt{1-x}\,dx$ can be written as

**A.** $\int_{1}^{3} (u^{\frac{3}{2}} - 3u^{\frac{1}{2}})\,du$

**B.** $\int_{1}^{3} (3u^{\frac{1}{2}} - u^{\frac{3}{2}})\,du$

**C.** $\int_{1}^{3} (u^{\frac{1}{2}} - u^{\frac{3}{2}})\,du$

**D.** $\int_{0}^{2} (u^{\frac{1}{2}} - u^{\frac{3}{2}})\,du$

**E.** $\int_{0}^{2} (u^{\frac{3}{2}} - 3u^{\frac{1}{2}})\,du$

**Question 8**

Consider the differential equation $\frac{dy}{dx} = 15\sqrt{x}(x+2)$, where $y(1) = y_0 = 26$.
Using Euler's method with a step size of 0.2, an approximation to $y(0.6) = y_2$ is given by

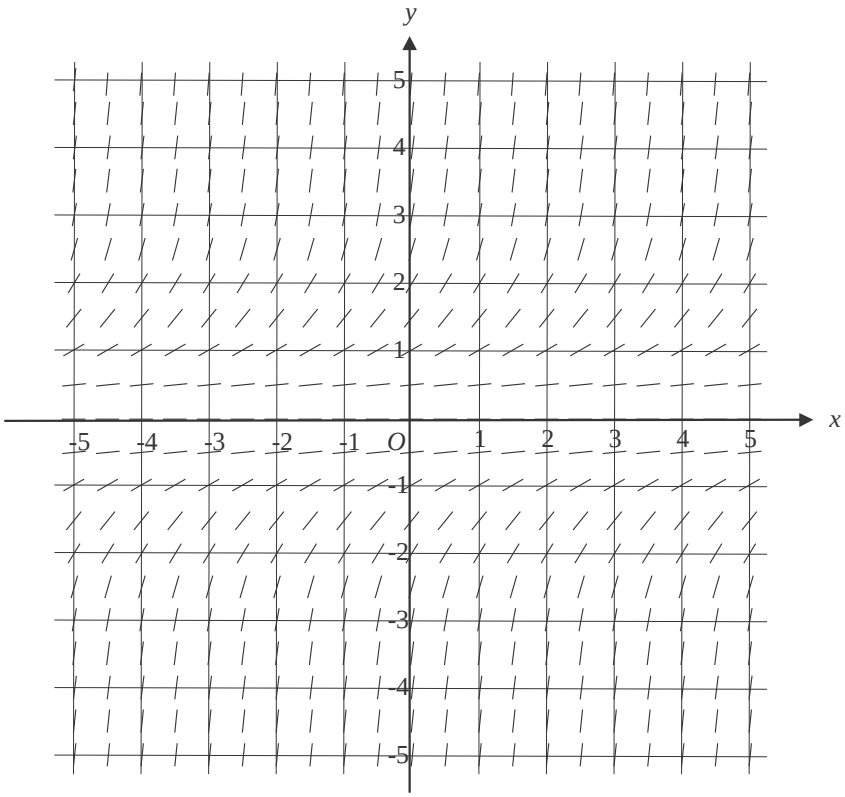
**A.** 9.5
**B.** 10.3
**C.** 17.0
**D.** 35.0
**E.** 42.5

**Question 9**

Consider the function *f* for which $f'(x) < 0$ and $f''(x) < 0$ over its entire domain.
The function $f(x)$ and its gradient function $f'(x)$ would, over this domain, be

**A.** both strictly decreasing.
**B.** both strictly decreasing and the graph of *f* would have a non-stationary point of inflection.
**C.** strictly increasing and strictly decreasing respectively
**D.** both strictly increasing
**E.** both strictly increasing and the graph of *f* would have a non-stationary point of inflection.

**Question 10**

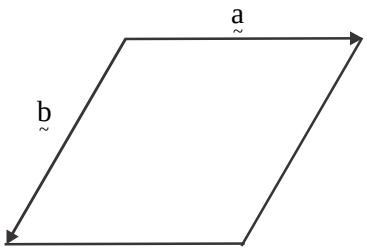



The differential equation that best represents the direction field above is

**A.** $\frac{dy}{dx} = x^2 + y$
**B.** $\frac{dy}{dx} = x^2$
**C.** $\frac{dy}{dx} = y^2$
**D.** $\frac{dy}{dx} = x - y^2$
**E.** $\frac{dy}{dx} = x + y$

**Question 11**

The diagram below shows a rhombus which is spanned by the two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$




Which one of the following statements is **false**?

**A.** $|\underset{\sim}{a}| = |\underset{\sim}{b}|$

**B.** $\underset{\sim}{a} \cdot \underset{\sim}{b} < 0$

**C.** $|\underset{\sim}{a}| = |-\underset{\sim}{b}|$

**D.** $|\underset{\sim}{a}| + |\underset{\sim}{b}| = 0$

**E.** $(\underset{\sim}{a} - \underset{\sim}{b}) \cdot (\underset{\sim}{a} + \underset{\sim}{b}) = 0$

**Question 12**

Let $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = -\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$.

The vector resolute of $\underset{\sim}{a}$ perpendicular to $\underset{\sim}{b}$ is

**A.** $\frac{1}{9}(8\underset{\sim}{i} - 7\underset{\sim}{j} - 11\underset{\sim}{k})$

**B.** $-\frac{5}{3}$

**C.** $\frac{10}{3}(-\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k})$

**D.** $\frac{8}{9}$

**E.** $\frac{10}{9}(-\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k})$

**Question 13**

The position vectors of two speed boats *A* and *B* after time *t* hours are given by

$$\underset{\sim}{r}_A(t) = t\,\underset{\sim}{i} + (4+t)\,\underset{\sim}{j} \quad \text{and} \quad \underset{\sim}{r}_B(t) = (t+2)\,\underset{\sim}{i} + (3-t)\,\underset{\sim}{j}, \qquad t \geq 0$$

where displacement is measured in kilometres from a given reference point on the water. The period of time, in hours, when the two speed boats are within 2.5 km of one another is closest to

**A.** 0.25
**B.** 0.5
**C.** 1.25
**D.** 1.5
**E.** 1.625

**Question 14**

The circular path of a particle moving in the Cartesian plane is given by $\underset{\sim}{r}(t) = 3\sin(t)\,\underset{\sim}{i} + \sqrt{a}\cos(t)\,\underset{\sim}{j}$ where *t* is the time in seconds, $t \geq 0$ and $a \in R^+$.

The direction of motion of the particle when $t = \frac{\pi}{2}$ is

**A.** $-3\underset{\sim}{i}$

**B.** $-3\underset{\sim}{j}$

**C.** $3\underset{\sim}{i}$

**D.** $3\underset{\sim}{j}$

**E.** $9\underset{\sim}{i}$

**Question 15**

The initial velocity of a particle with mass 4 kg is $(3\underset{\sim}{i} - 5\underset{\sim}{j})$ ms$^{-1}$.

This particle has a change of momentum of $(-20\,\underset{\sim}{i} + 44\,\underset{\sim}{j})$ kg ms$^{-1}$.

The velocity of the particle, in ms$^{-1}$, is now

**A.** $-2\,\underset{\sim}{i} + 6\,\underset{\sim}{j}$

**B.** $-2\,\underset{\sim}{i} + 16\,\underset{\sim}{j}$

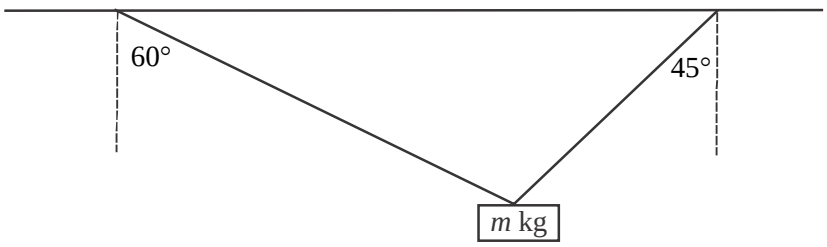
**C.** $-8\,\underset{\sim}{i} + 24\,\underset{\sim}{j}$

**D.** $-17\,\underset{\sim}{i} + 39\,\underset{\sim}{j}$

**E.** $-32\,\underset{\sim}{i} + 96\,\underset{\sim}{j}$

**Question 16**

A mass of *m* kg is suspended from a horizontal bar by two light inextensible strings as shown in the diagram below.




The longer string makes an angle of 60° with the vertical and has tension of $\sqrt{6}$ newtons.
The shorter string makes an angle of 45° with the vertical.
The value of *m* is

**A.** $\dfrac{\sqrt{6}}{g}$

**B.** $\dfrac{3}{g}$

**C.** $\dfrac{\sqrt{3}+3}{\sqrt{2}}$

**D.** $\dfrac{\sqrt{3}\left(\sqrt{2}+1\right)}{2}$

**E.** $\dfrac{\sqrt{2}\left(\sqrt{3}+3\right)}{2g}$

**Question 17**

A mass of 5 kg is acted on by a variable force of *F* newtons and as a result the mass moves in a straight line.
At time *t* seconds, $t \geq 0$, the velocity of the mass, *v* metres per second, and its position, *x* metres from the origin, are given by $v = 1 + 2x$.

The variable force *F* is equal to

**A.** $10$

**B.** $4x + 2$

**C.** $20x + 10$

**D.** $5x^2 + 5x$

**E.** $\dfrac{10}{3}x^3 + 5x^2 + \dfrac{5}{2}x$

**Question 18**

The weights of a population of sea birds are known to be normally distributed with a mean mass of 5.2 kg and a standard deviation of 0.4 kg.
Seven of these birds, chosen at random, are captured and weighed.
The probability that the average weight of these captured birds is less than 5 kg is closest to

**A.** 0.0002
**B.** 0.0041
**C.** 0.0929
**D.** 0.3085
**E.** 0.9072

**Question 19**

The standard deviation of the battery life, in hours, for a particular brand of power tool is 15 hours.
A 95% confidence interval for the mean battery life, in hours, of this brand of power tool is (68.1, 72.3).
The number of power tools in the sample used to calculate this confidence interval is

**A.** 7
**B.** 14
**C.** 49
**D.** 196
**E.** 225

**Question 20**

Annual rainfall in the towns of Alban and Beachtown is normally distributed.
The mean and standard deviation of the annual rainfall, in millimetres, in Alban is 310 and 10 respectively.
For Beachtown, the mean and standard deviation of the annual rainfall, in millimetres, is 640 and 25 respectively.
The annual rainfall in Alban is independent of the annual rainfall in Beachtown.
In a randomly chosen year, the probability that Beachtown will have more than double the annual rainfall of Alban is closest to

**A.** 0.7339
**B.** 0.8241
**C.** 0.8735
**D.** 0.9012
**E.** 0.9088

## SECTION B

**Instructions for Section B**

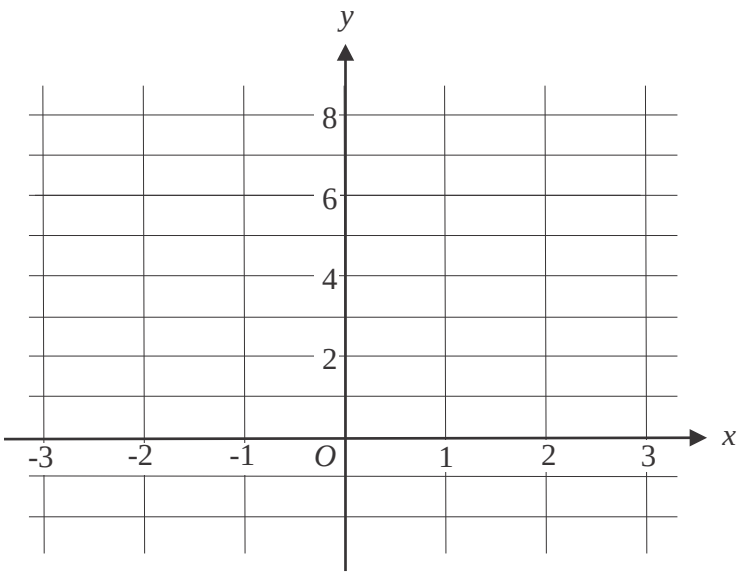
Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude $g$ ms$^{-2}$, where $g = 9.8$

**Question 1** (10 marks)

Consider the function $f$ with rule $f(x) = 5\arccos\left((x-1)^2\right)$.

**a.** Given that $f'(x) = \dfrac{ax-a}{\sqrt{b-(x-b)^c}}$, where $a$, $b$ and $c$ are integers, find the values of $a$, $b$ and $c$. 2 marks

**b.** Sketch the graph of $f$ over its maximal domain on the set of axes below. Label any endpoints and stationary points with their coordinates. 3 marks



**c.** The region enclosed by that part of the graph of $f$ for which $x \in [0,1]$, the line $y = \frac{5\pi}{2}$ and the $y$-axis, is rotated about the $y$-axis to form a solid of revolution.

**i.** Write down a definite integral, in terms of $x$, that gives the length of the curve described above that is used to form the solid of revolution. 1 mark

**ii.** Find the length of this curve, correct to two decimal places. 1 mark

**iii.** Write down a definite integral, in terms of $y$, that gives the volume of the solid of revolution formed. 2 marks

**iv.** Find the volume of the solid formed, in cubic units, correct to two decimal places. 1 mark

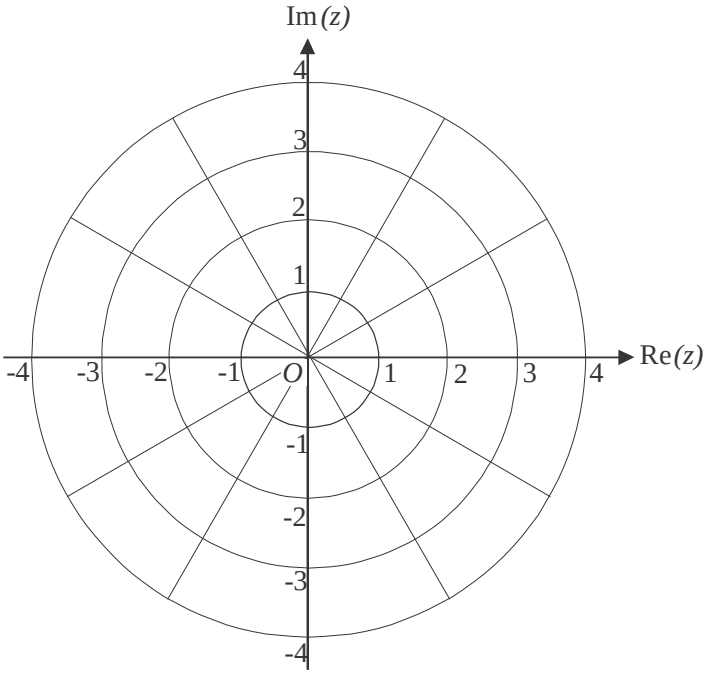
**Question 2** (9 marks)

The relation *S*, in the complex plane, is given by $\left|z-\sqrt{2}-\sqrt{2}i\right|=\sqrt{2}$.

**a.** Show that the Cartesian equation of *S* is $\left(x-\sqrt{2}\right)^2+\left(y-\sqrt{2}\right)^2=2$. 1 mark

**b.** Find, in Cartesian form, the point(s) of intersection of *S* and the graph of the relation $z\bar{z}=2$. 2 marks

**c.** On the Argand diagram below, sketch *S* and the graph of the relation $z\bar{z}=2$. 2 marks



**d.** A ray with equation $\text{Arg}(z) = \alpha\pi, \quad \alpha \in R$, intersects the graph of $S$ **and** the graph of the relation $z\bar{z} = 2$.
Write down the possible values of $\alpha$. 2 marks

**e.** Find the common area enclosed by the graph of $S$ and the graph of the relation $z\bar{z} = 2$. 2 marks

**Question 3** (9 marks)

A tank initially contains 10 kg of sugar dissolved in 100 litres of water.
Water flows into the tank at the rate of 6 litres per minute.
The solution in the tank is kept uniform by stirring and flows out of the tank at the rate of 2 litres per minute.
Let $x$ represent the amount of sugar, in kilograms, present in the tank after $t$ minutes.

**a.** Show that the differential equation relating $x$ and $t$ is $\frac{dx}{dt} = \frac{-x}{50+2t}$. 1 mark

**b.** Hence show that $x = \frac{50}{\sqrt{25+t}}$. 3 marks

A second tank initially contains 10 kg of sugar dissolved in 400 litres of water.
The solution is kept uniform by stirring and flows out of the tank at the rate of 4 litres per minute.
Let $s$ represent the amount of sugar, in kilograms, in this second tank after $t$ minutes.
The differential equation relating $s$ and $t$ is given by $\frac{ds}{dt} = \frac{-s}{100-t}$.

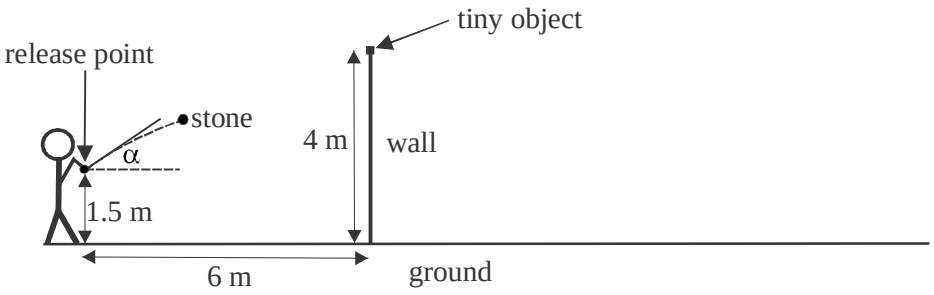
**c.** Verify that $s = 10 - \frac{t}{10}$ satisfies both the differential equation and initial conditions given above. 2 marks

**d.** How many minutes does it take for the tank to be emptied of the sugar solution? 1 mark

**e.** Show that the concentration of sugar in the sugar solution, in kg per litre, is constant throughout the emptying of the tank. 2 marks

**Question 4** (11 marks)

Jack throws a stone which he hopes will hit a tiny object sitting on top of a 4 m high wall. The point at which he releases the stone is 6 m horizontally from the wall and 1.5 m above the horizontal ground.
The angle of projection is $\alpha$ to the horizontal, where $0 < \alpha < 90°$, and the speed of the stone when it is released is 15 $ms^{-1}$. Air resistance can be taken as negligible.




The position vector of the stone at time *t* seconds, $t \geq 0$, relative to the release point is given by $\underset{\sim}{r}(t) = 15t\cos(\alpha)\,\underset{\sim}{i} + (15t\sin(\alpha) - 4.9t^2)\,\underset{\sim}{j}$ where $\underset{\sim}{i}$ is a unit vector in the horizontal direction of motion and $\underset{\sim}{j}$ is a unit vector vertically upwards. Displacement is measured in metres.

**a.** Jack throws the stone with an angle of projection of 60°.

**i.** Find the time, in seconds, when the stone is 6 m horizontally from its release point. 1 mark

**ii.** Hence show that the stone does not hit the tiny object. 1 mark

**iii.** How far vertically above the tiny object does the stone pass? Give your answer in metres correct to two decimal places. 2 marks

**iv.** What is the speed of the stone when it is vertically above the tiny object? Give your answer in $ms^{-1}$ correct to one decimal place. 2 marks

Jack's friend Tom throws an identical stone from the same position where Jack threw his stone.
The speed at which Tom releases his stone is 15 $ms^{-1}$ and he manages to hit the tiny object.

**b.** Find the possible angles of projection, $\alpha$, for Tom's throw.
Give your answer in degrees correct to one decimal place. 3 marks

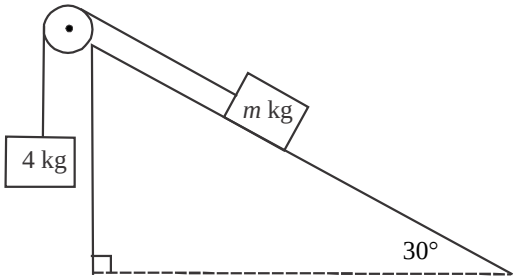
After having skimmed the top of the wall and hit the tiny object, Tom's stone continues on its original path before landing on the ground on the **opposite** side of the wall from where it was thrown.
Its distance from the base of the wall is now 16.4228 m.

**c.** What was the angle of projection of Tom's throw? Give your answer in degrees correct to one decimal place. 2 marks

**Question 5** (11 marks)

A mass of $m$ kg rests on a smooth plane inclined at an angle of 30° to the horizontal. It is connected by a light inextensible string that passes over a smooth pulley to a second mass of 4 kg. The system is in equlibrium.

**a.** Show all the forces acting on each of the masses. 2 marks




**b.** Find the value of $m$. 1 mark

A tank containing a liquid is placed beneath the 4 kg mass so that the mass is touching the surface of the liquid.
The string joining the two masses is cut and the 4 kg mass sinks vertically into the liquid.
The liquid exerts a resistance of $2v$ newtons to the motion, where $v$ ms$^{-1}$ is the velocity of the 4 kg mass $t$ seconds after the string is cut.

**c.** Show that $a = g - \frac{v}{2}$ where $a$ ms$^{-2}$ is the acceleration of the 4 kg mass when its velocity is $v$ ms$^{-1}$. 1 mark

**d.** Find an expression for the time *t*, in seconds since the string was cut, as a function of the velocity *v*, of the 4 kg mass as it sinks.

Give your answer in the form $t = b\log_e\left(\frac{bg}{bg - v}\right)$ where $b \in R$. 2 marks

**e.** Find the limiting (terminal velocity) of the 4 kg mass. 1 mark

**f.** The 4 kg mass reaches the bottom of the tank three seconds after the string is cut. How deep is the liquid in the tank? Give your answer in metres correct to one decimal place. 2 marks

**g.** Find how far the 4 kg mass is below the surface of the liquid when its velocity is $g$ ms$^{-1}$. Give your answer in metres correct to one decimal place. 2 marks

**CONTINUES OVER PAGE**

**Question 6** (10 marks)

A company produces small muffins and large muffins.
The weight of the small muffins is normally distributed with a mean of 108 grams and a standard deviation of two grams.
The weight of a large muffin is twice the weight of a small muffin.

**a.** Find the mean and the standard deviation of the weight of a large muffin. 2 marks

Small muffins are sold to retailers in boxes of 25.

**b.** Find the mean and the standard deviation of the weight of a box of small muffins. (Assume that the packaging weight is negligible). 2 marks

**c.** Find the probability that a randomly selected box containing 25 small muffins will weigh less than 2680 grams. Give your answer correct to three decimal places. 1 mark

The company also produces mud cakes. The weight of these mud cakes is normally distributed with a standard deviation of 20 grams.
The company claims that the mean weight of their mud cakes is 450 grams.
A food regulation authority suspects that the mud cakes are underweight and takes a sample of 30 of them.
The mean weight of the cakes in the sample is 440 grams.
A one-tailed statistical test is carried out to test whether the sample mean weight is significantly less than the weight claimed by the company.

**d.** State two hypotheses that should be used in this statistical test. 1 mark

**e.** Write down an expression for the $p$ value for the test and evaluate it correct to four decimal places. 2 marks

**f.** State whether $H_0$ should be rejected at the 5% level of significance. Give a reason for your answer. 1 mark

**g.** What is the largest value of the sample mean that could provide evidence, at the 5% level of significance, that the mud cakes produced by the company are underweight? Give your answer correct to one decimal place. 1 mark

**END OF QUESTION AND ANSWER BOOK**