Billanook College

July Exam 2017

VCE Specialist Mathematics
Examination 1

Written Examination

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name : _____

Structure of Booklet

Section	Number of Questions	Number of marks
Exam 1	9	38

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
No calculator is permitted in this examination.

Materials supplied:
Question and answer booklet
Formula sheet.

Instructions
Write your name and teacher's name in the space provided above.
Always show your working.
All written responses should be in English

Students are NOT permitted to bring mobile phones and/or any other electronic communications equipment into the examination room.

Question 1 (4 marks)

a. Find all solutions of $z^3 = 8i, z \in C$ in cartesian form. 3 marks

b. Find all solutions of $(z-2i)^3 = 8i, z \in C$ in cartesian form. 1 mark

Question 2 (3 marks)

Find the volume generated when the region bounded by the graph of $y = 2x^2 - 3$, the line $y = 5$ and the y-axis is rotated about the y-axis.

Question 3 (6 marks)

Consider the curve represented by $x^2 - xy + \frac{3}{2}y^2 = 9$.

a. Find the gradient of the curve at any point (x, y). 2 marks

b. Find the equation of the tangent to the curve at the point (3, 0) **and** find the equation of the tangent to the curve at the point $\left(0, \sqrt{6}\right)$.
Write each equation in the form $y = ax + b$. 2 marks

c. Find the acute angle between the tangent to the curve at the point (3, 0) and the tangent to the curve at the point $\left(0, \sqrt{6}\right)$.
Give your answer in the form $k\pi$, where k is a real constant. 2 marks

Question 4 (4 marks)

Chemicals are added to a container so that a particular crystal will grow in the shape of a cube. The side length of the crystal, x millimetres, t days after the chemicals were added to the container, is given by $x = \arctan(t)$.

Find the rate at which the surface area, A square millimetres, of the crystal is growing one day after the chemicals were added. Give your answer in square millimetres per day.

Question 5 (4 marks)

Consider the vectors $\underset{\sim}{a} = 3\underset{\sim}{i} + 5\underset{\sim}{j} - 2\underset{\sim}{k}$, $\underset{\sim}{b} = \underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}$ and $\underset{\sim}{c} = \underset{\sim}{i} + d\underset{\sim}{k}$, where d is a real constant.

a. Find the vector resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$. 2 marks

b. Find the value of d if the vectors are **linearly dependent**. 2 marks

Question 6 (3 marks)

Given that $\cos(x-y)=\frac{3}{5}$ and $\tan(x)\tan(y)=2$, find $\cos(x+y)$.

Question 7 (5 marks)

Solve the differential equation $\sqrt{2-x^2}\,\frac{dy}{dx}=\frac{1}{2-y}$, given that $y(1)=0$. Express y as a function of x.

Question 8 (5 marks)

Let f be a function of a complex variable, defined by the rule $f(z) = z^4 - 4z^3 + 7z^2 - 4z + 6$.

a. Given that $z = i$ is a solution of $f(z) = 0$, write down a quadratic factor of $f(z)$. 2 marks

b. Given that the other quadratic factor of $f(z)$ has the form $z^2 + bz + c$, find all solutions of $z^4 - 4z^3 + 7z^2 - 4z + 6 = 0$ in cartesian form. 3 marks

Question 9 (4 marks)

Evaluate $\int_0^1 \frac{x-5}{x^2-5x+6}\,dx$.