Billanook College

July Exam 2016

VCE Specialist Mathematics
Examination 1

Written Examination

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name : _____

Structure of Booklet

Section	Number of Questions	Number of marks
Exam 1	9	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
No calculator is permitted in this examination.

Materials supplied:
Question and answer booklet
Formula sheet.

Instructions
Write your name and teacher's name in the space provided above.
Always show your working.
All written responses should be in English

*** Please note: questions are not in order ***

Students are NOT permitted to bring mobile phones and/or any other electronic communications equipment into the examination room.

Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagram in this book are not drawn to scale.
Take the acceleration due to gravity to have magnitude $g\ m/s^2$, where $g = 9.8$.

Question 1 (6 marks)

Consider vectors $\underset{\sim}{u} = 2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$, $\underset{\sim}{v} = \left(8 + 3\sqrt{11}\right)\underset{\sim}{i} + 7\underset{\sim}{j}$.

a. Show that the magnitude of $\underset{\sim}{v}$ is $\left|\underset{\sim}{v}\right| = 6 + 4\sqrt{11}$

2 marks

b. Find the angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$.

2 marks

c. Find the resolute vector of $\underset{\sim}{v}$ in the direction of $\underset{\sim}{u}$.

2 marks

TURN OVER

Question 4 (3 marks)

$P(z)$ is a polynomial given by $P(z) = 2z^3 + 5z^2 + 6z + 2$ over the complex field.

a. Show that $z = -\frac{1}{2}$ is a root of $P(z)$.

1 mark

b. Find the other roots of $P(z)$ in the form $rcis(\theta)$.

2 marks

TURN OVER

Question 6 (5 marks)

a. Express $1 + \cos(4x)$ in terms of $\cos(2x)$.

1 mark

b. Use an appropriate substitution u to write the following integral in terms of u only.

$$\int \frac{\sin(2x)}{1 + \cos(4x)} dx$$

2 marks

c. Evaluate the definite integral

$$\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \frac{\sin(2x)}{1 + \cos(4x)} dx$$

2 marks

TURN OVER

Question 8 (6 marks)

a. Write $2x - x^2$ in the form $a - (x - b)^2$.

1 mark

b. Find an antiderivative of

$$\int \frac{1}{x(x-2)}\,dx$$

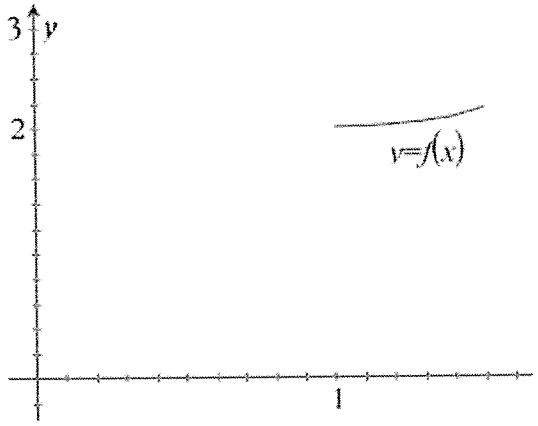
2 marks

Question 8 - continued
TURN OVER

c. A solid revolution is formed by rotating the region enclosed by

$$0 \leq y \leq 1 + \frac{1}{\sqrt{2x-x^2}} \text{ and } 1 \leq x \leq \frac{3}{2}$$

about the x-axis. The graph of $f(x) = 1 + \frac{1}{\sqrt{2x-x^2}}, 1 \leq x \leq \frac{3}{2}$ is shown below.



Find the volume of the solid revolution.

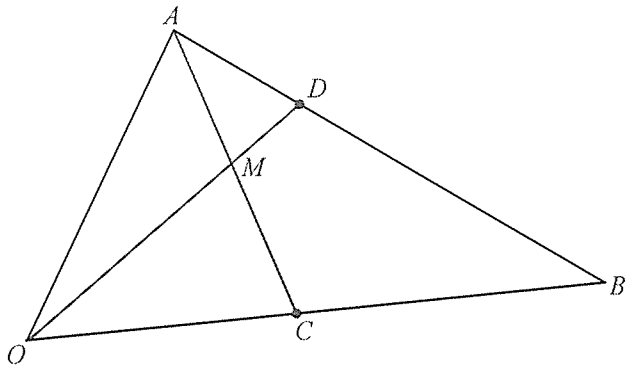
3 marks

END OF QUESTION AND ANSWER BOOK

Question 2 (5 marks)

OAB is a triangle where C is the midpoint of $\overline{OB}$ and $3\overline{AD} = \overline{AB}$.

$\overline{OD}$ and $\overline{AC}$ intersect at point M.



Let $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$, $\overrightarrow{CM} = \alpha\overrightarrow{CA}$ and $\overrightarrow{OM} = \beta\overrightarrow{OD}$.

a. Express $\overrightarrow{CM}$ in terms of

i. $\underset{\sim}{a}, \underset{\sim}{b}$ and α. 1 mark

ii. $\underset{\sim}{a}, \underset{\sim}{b}$ and β 2 marks

b. Hence find the values of α and β. 2 marks

Question 6 (4 marks)

Find the gradient of the tangent to the curve defined by $2xy - \arctan\left(\frac{x}{2}\right) + y^2 = 5 - \frac{\pi}{4}$ at the point $(2,1)$.

Question 8 (5 marks)

A particle moves in a straight line with acceleration a m/s^2, velocity v m/s and position x m, relative to a fixed point on the line. The relationship between the position and the velocity of the particle at time t seconds, $t \geq 0$, is given by $v = \sqrt{2x+4}$. The particle is initially at the fixed point.

a. Show that the acceleration of the particle is constant. 2 marks

b. Find the value of x when $t = 3$. 3 marks

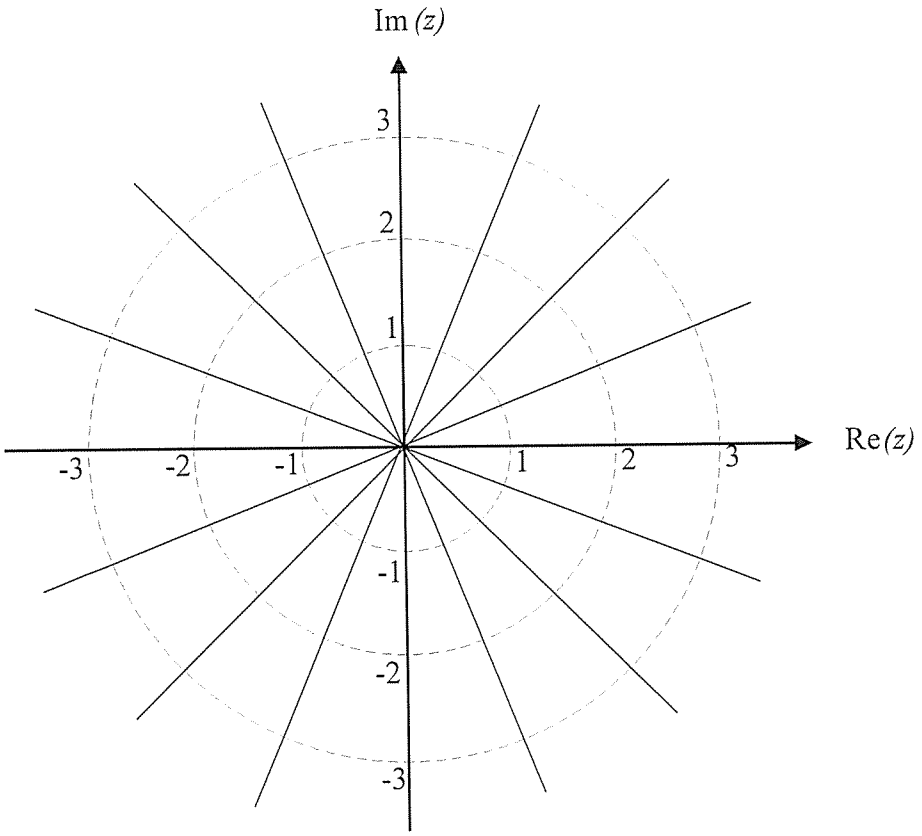
Question 1 (4 marks)

a. Find all the solutions to the equation $z^4 + 2z^2 - 3 = 0,\ z \in C$.

3 marks

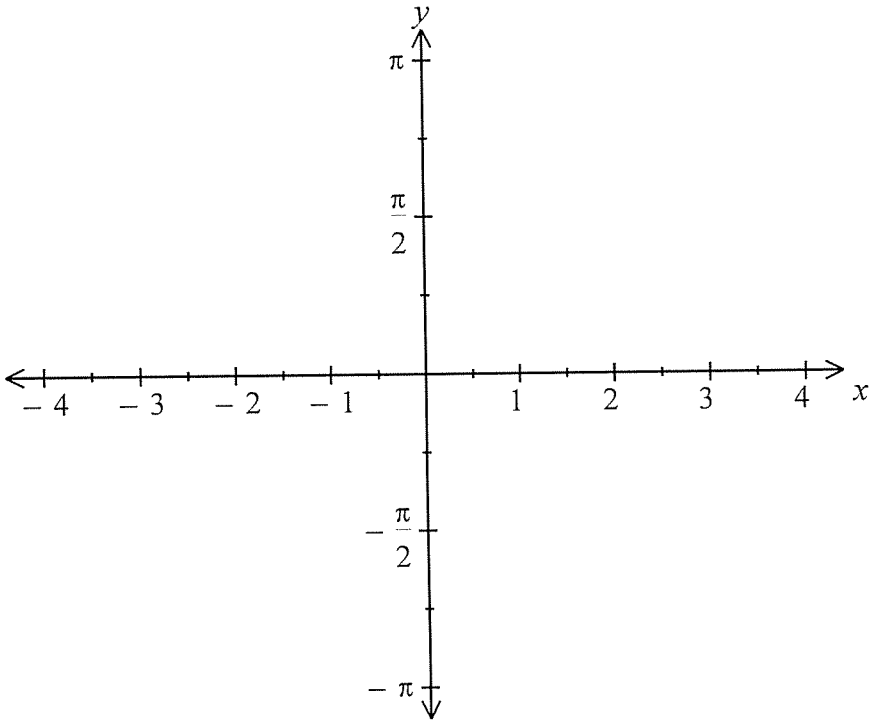
b. Plot the roots found in part a. on the Argand diagram below.

1 mark



Question 8

a. On the axes below sketch the graph of the curve $y = 2\arcsin\left(\frac{x}{3}\right)$. State the coordinates of the point of inflection and label the coordinates of all endpoints.



2 marks