SPECIALIST MATHEMATICS

Written examination 2

TSSM™
Creating VCE Success

2023 Trial Examination

SOLUTIONS

SECTION A

Question 1

B

Explanation:

$sinx + \cos x = 0$
$\tan x = -1$

$\text{x} = \frac{3\pi}{4} + n\pi, n \in Z$

Question 2

E

Explanation:

$\text{y} = e^{|x|}$ is not differentiable at the point $(0,1)$ due to the function being non-smooth at this point

The gradient of $\text{y} = e^{|x|}$ approaches -1 from the left and $+1$ from the right as x approaches 0.

Question 3

C

Explanation:

$\sec\theta = -4$

$\cos\theta = -\dfrac{1}{4}$

Using Pythagoras' theorem:

$\sin\theta = \frac{\sqrt{15}}{4}$ (second quadrant) or $-\frac{\sqrt{15}}{4}$ (third quadrant).

$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \dfrac{\sqrt{15}}{4} \times \left(-\dfrac{1}{4}\right) = -\dfrac{\sqrt{15}}{8}$ or $2 \times \left(-\dfrac{\sqrt{15}}{4}\right) \times \left(-\dfrac{1}{4}\right) = \dfrac{\sqrt{15}}{8}$

$\operatorname{cosec}\theta = \frac{8}{\sqrt{15}}$ or $-\frac{8}{\sqrt{15}}$

Question 4

B

Explanation:

For $y = cos^{-1}\left(\sqrt{x-2}\right)$ to be defined, $-1 \le \sqrt{x-2} \le 1$

Check the graph of $y = cos^{-1}\left(\sqrt{x-2}\right)$ on your CAS.

This will occur when $x \in [2,3]$

This will result in $y \in [1, \frac{\pi}{2}]$

Question 5

E

Explanation:

$g: [8, \infty) \to R$ $where$ $g(x) = 12 \sin^{-1}(\frac{4}{x})$

Let $y = 12 \sin^{-1}(\frac{4}{x})$

The inverse is $x = 12 \sin^{-1}(\frac{4}{y})$

$\frac{4}{y} = sin\left(\frac{x}{12}\right)$

$y = 4\left(sin\left(\frac{x}{12}\right)\right)^{-1}$

Question 6

D

Explanation:

$P(2i) = (2i)^3 + (2 - i)(2i)^2 + n(1 - i)(2i) + 4 = 0$

$-8i - 4(2 - i) + 2ni(1 - i) + 4 = 0$

$-8i - 8 + 4i + 2ni + 2n + 4 = 0$

$-4i - 4 + 2ni + 2n = 0$

$n = -2$

$z^3 + (2 - i)z^2 - 2(1 - i)z + 4 = 0$

$(z - 2i)(z^2 + iz + 2z + 2i) = 0$

$(z - 2i)(z + 2)(z + i) = 0$

$z = 2i, \quad z = -i, \quad z = -2$

Question 7

D

Explanation:

Which statement relating to $\boldsymbol{a} = 2\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = -\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ is untrue?

A. The magnitude of $\boldsymbol{a}$ is 3 units.

$|\boldsymbol{a}| = \sqrt{4+4+1} = \sqrt{9} = 3$

B. The dot product of the two vectors is -9.

$\boldsymbol{a}.\boldsymbol{b} = (2\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}).(-\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}) = -2 - 6 - 1 = -9$

C. The angle between the two vectors is closest to $155°$.

$\boldsymbol{a}.\boldsymbol{b} = -9 = 3 \times \sqrt{11} \times \cos\theta, \theta \approx 155°$

D. The cross product of the two vectors is $-\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}$.

$\boldsymbol{a} \times \boldsymbol{b} = -\boldsymbol{i} + \boldsymbol{j} + 4\boldsymbol{k}$ **FALSE**

E. The two vectors are neither parallel nor perpendicular.

$\boldsymbol{a} \neq k \times \boldsymbol{b}, \qquad \boldsymbol{a}.\boldsymbol{b} = -9 \neq 0$

Question 8

E

Explanation:

$\boldsymbol{m}.\boldsymbol{n} = u - 2 + 6 = 3 \times \sqrt{u^2 + 1 + 9} \times \cos 60°$

$u + 4 = 3 \times \sqrt{u^2 + 10} \times \frac{1}{2}$

$2u + 8 = 3\sqrt{u^2 + 10}$

Question 9

B

Explanation:

$P = (1,1,0), Q = (2,1,-1), R = (1,2,-1)$ is:

$\overrightarrow{PQ} = \boldsymbol{i} + 0\boldsymbol{j} - \boldsymbol{k}, \qquad \overrightarrow{PR} = 0\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$

$\overrightarrow{PQ} \times \overrightarrow{PR} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$

Select any point in plane, say P

$1(x-1) + 1(y-1) + 1(z-0) = 0$

$x + y + z = 2$

Question 10

B

Explanation:

$$\frac{dF}{dt} = 1.04F - 10000$$

$$t = \frac{1}{1.04}\int \frac{1.04}{1.04F - 10000}\,dt$$

$$t = \frac{1}{1.04}log_e(1.04F - 10000) + c$$

$$0 = \frac{1}{1.04}log_e(1.04 \times 200000 - 10000) + c$$

$$c = -\frac{1}{1.04}log_e(198000)$$

$$t = \frac{1}{1.04}log_e\left(\frac{1.04F - 10000}{198000}\right)$$

$$F = \frac{1}{1.04}(198000e^{1.04t} + 10000)$$

Question 11

C

Explanation:

Proof by contradiction.

1. Assume the original statement is true. (In this case.)
2. That assumption leads to a mathematical inconsistency.
3. The original assumption must be false.

Question 12

E

Explanation:

Statement **B** is not the converse of statement A. The converse of statement **A** is:
Given θ is the internal angle at Q in the right-angled triangle PQR, then $\sin\theta = \frac{3}{5}$.

Question 13

D

Explanation:

$f: [0,2] \to R$ where $f(x) = x^3$

$$S = \int_0^2 2\pi y\sqrt{(1 + \left(f'(x)\right)^2)}$$

$$= \int_0^2 2\pi(2x^3)\sqrt{(1 + \left((6x^2)\right)^2)}$$

Use CAS

$\approx 806\ U^2$

Question 14

B

Explanation:

$$\int_0^{\frac{\pi}{3}} \frac{e^{\tan x}}{cos^2 x} dx$$

Let $u = \tan x$

$\frac{du}{dx} = sec^2 x$

$$= \int_0^{\sqrt{3}} (e^{\tan x} \, sec^2 x \,) \, dx$$

$$= \int_0^{\sqrt{3}} \left(e^u \, \frac{du}{dx}\right) \, dx$$

$$= \int_0^{\sqrt{3}} (e^u \,) \, du$$

Question 15

D

Explanation:

$a = 2 - x$

$$\frac{d}{dx}\left(\frac{1}{2} v^2\right) = 2 - x$$

$$\frac{1}{2} v^2 = \int (2 - x) \, dx$$

$v^2 = -x^2 + 4x + c$

When $x = 0, v = 4$

$16 = -0^2 + 4(0) + c$

$c = 16$

$v^2 = -x^2 + 4x + 16$

$v^2 = -4^2 + 4(4) + 16$

$v^2 = 16$

$v = 4\ ms^{-1}$
(Reject $v = -4$ since particle's initial velocity is positive and it has turned around.)

Question 16

B

Explanation:

$E(A) = 10, Var(A) = 3, E(B) = 12, Var(B) = 4, C = 2A + B$

$E(C) = E(2A + B) = 20 + 12 = 32$

$Var(C) = Var(2A + B) = 4 \times Var(A) + Var(B)$

$= 4 \times 3 + 4 = 16$

$\Pr(C < 30) = \Pr(z < \dfrac{30 - 32}{\sqrt{16}})$

$= \Pr(z < -0.5) \approx 0.309$

Question 17

C

Explanation:

$$\Pr\left(\frac{-0.8}{\frac{\sigma}{\sqrt{30}}} < z < \frac{0.8}{\frac{\sigma}{\sqrt{30}}}\right) = 0.99$$

$$\Pr\left(z < \frac{0.8}{\frac{\sigma}{\sqrt{30}}}\right) = 0.995$$

$$\frac{0.8}{\frac{\sigma}{\sqrt{30}}} \approx 2.57584$$

$$\sigma \approx 1.701$$

Question 18

A

Explanation:

$$\boldsymbol{v}(t) = 6\sqrt{t}\,\boldsymbol{i} - \left(\frac{1}{t+1}\right)\boldsymbol{j}$$

$$\boldsymbol{a}(t) = \frac{3}{\sqrt{t}}\boldsymbol{i} + \left(\frac{1}{(t+1)^2}\right)\boldsymbol{j}$$

$$\boldsymbol{x}(t) = 4t^{\frac{3}{2}}\boldsymbol{i} - log_e(t+1)\boldsymbol{j} + \boldsymbol{c}$$

$$\boldsymbol{x}(t) = 4t^{\frac{3}{2}}\boldsymbol{i} - log_e(t+1)\boldsymbol{j} + 2\boldsymbol{j}$$

$$|\boldsymbol{a}(t)| = \sqrt{\left(\frac{3}{\sqrt{t}}\right)^2 + \left(\frac{1}{(t+1)^2}\right)^{\mathbf{2}}} = 2$$

Solve on CAS. $t \approx 2.255\ s$

$$\boldsymbol{x}(2.255) = 4(2.255)^{\frac{3}{2}}\boldsymbol{i} - log_e(2.255+1)\boldsymbol{j} + 2\boldsymbol{j} \approx 13.55\ \boldsymbol{i} + 0.82\ \boldsymbol{j}$$

Question 19

D

Explanation:

$h:\left[\frac{\pi}{2}, a\right] \to R$ where $h(x) = \cos^{-1} x$

$$V(y) = \pi \int_{\frac{\pi}{2}}^{a} (\cos y)^2 dy = 2$$

Solve on CAS.

$a \approx 3$

Question 20

A

Explanation:

$\boldsymbol{a}(t) = -9.8\boldsymbol{j}$

$\boldsymbol{v}(t) = 30\cos 50°\, \boldsymbol{i} + (30\sin 50° - 9.8t)\boldsymbol{j}$

$\boldsymbol{x}(t) = (30\cos 50° t)\boldsymbol{i} + (30\sin 50°\, t - 4.9t^2 + 3.5)\boldsymbol{j}$

$30\sin 50°\, t - 4.9t^2 + 3.5 = 0$

Solve on CAS.

$t \approx 4.8377\ s$

$\boldsymbol{x}(4.8377) = (30\cos 50° \times 4.8377)\boldsymbol{i} + 0\boldsymbol{j}$

$\approx 93.3\boldsymbol{i} + 0\boldsymbol{j}$

SECTION B

Question 1 (10 marks)

a. (1 mark)

Answer:

$z_2 = 2 - i$

1A

b. (2 marks)

Answer:

$(z - 2 - i)(z - 2 + i)$

$= z^2 - 2z + iz - 2z - 2i + 4 - iz + 2i + 1$

1W

$= z^2 - 4z + 5$

$a = 1, b = -4, c = 5$

1A

c. (2 marks)

Answer:

$Q(z) = z^2 - 4i - 3 = 0$

$Q(z) = (z - 2 - i)(z - m - ni) = 0$

1W

$(z - 2 - i)(z - m - ni) = z^2 - 4i - 3$

Equating coefficients:

1A

$= z^2 - mz - niz - 2z + 2m + 2ni - iz + mi - n = z^2 - 4i - 3$

$m = -2, n = -1$

1A

d. (2 marks)

i.

Answer:

$z^4 = p + qi, \qquad p, q \in R$

$(2+i)^4 = p + qi = -7 + 24i$

$p = -7, q = 24$

1A

The four roots lie symmetrically in a circle.

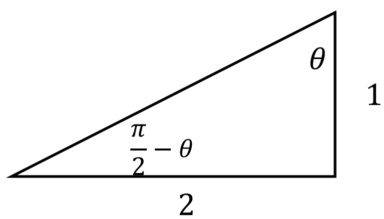
$z_1 = 2 + i$
$z_4 = (2+i)i = -1 + 2i$
$z_5 = (-1+2i)i = -2 - i$
$z_6 = (-2-i)i = 1 - 2i$

1A

ii. (3 marks)

Answer:

Consider the first root $x_1 = 2 + i$
$\theta = \tan^{-1} 2$
$\tan\theta = 2$



Relevant angle in first quadrant is $\frac{\pi}{2} - \theta$

1W

$x_1 = 2 + i = \sqrt{5}\, cis\left(\frac{\pi}{2} - \theta\right)$

1W

$x_4 = (2+i)i = -1 + 2i = \sqrt{5}\, cis(\pi - \theta)$

$x_5 = (-1+2i)i = -2 - i = \sqrt{5}\, cis\left(\frac{3\pi}{2} - \theta\right)$

$x_6 = (-2-i)i = 1 - 2i = \sqrt{5}\, cis(2\pi - \theta) = \sqrt{5}\, cis(-\theta)$

1A

Question 2 (9 marks)

a. (3 marks)

Answer:

$\boldsymbol{a}(t) = 2\cos t\ \boldsymbol{i} + \sin t\ \boldsymbol{j}$

$\boldsymbol{v}(t) = 2\sin t\ \boldsymbol{i} - \cos t\ \boldsymbol{j} + \boldsymbol{c}$

$\boldsymbol{v}(0) = 2\sin 0\ \boldsymbol{i} - \cos 0\ \boldsymbol{j} + \boldsymbol{c} = -1\boldsymbol{j}$

$\boldsymbol{c} = 0\boldsymbol{j}$

1W

$\boldsymbol{v}(t) = 2\sin t\ \boldsymbol{i} - \cos t\ \boldsymbol{j}$

$\boldsymbol{r}(t) = -2\cos t\ \boldsymbol{i} - \sin t\ \boldsymbol{j} + \boldsymbol{c}$

$\boldsymbol{r}(0) = -2\cos 0\ \boldsymbol{i} - 0\ \boldsymbol{j} + \boldsymbol{c} = 2\boldsymbol{i}$

$\boldsymbol{c} = 4\boldsymbol{i}$

$\boldsymbol{r}(t) = (4 - 2\cos t\,)\boldsymbol{i} - \sin t\ \boldsymbol{j}$

1W

$x = 4 - 2\cos t\,, y = -\sin t$

$cos^2 t + \sin^2 t = 1$

$$\left(\frac{4-x}{2}\right)^2 + (-y)^2 = 1$$

$$\frac{(x-4)^2}{4} + y^2 = 1$$

1A

b. (3 marks)

Answer:

$\boldsymbol{v}(t) = 2\sin t\ \boldsymbol{i} - \cos t\ \boldsymbol{j}$

$|\boldsymbol{v}(t)| = \sqrt{(2\sin t)^2 + (\cos t)^2}$

$= \sqrt{1 + 3\,sin^2 t}$

1W

$|\boldsymbol{v}(t)|_{\boldsymbol{max.}} = \sqrt{1+3} = 2\ ms^{-1}$, $t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$ so, $t = \frac{(2n+1)\pi}{2}, n \in Z$

$|\boldsymbol{v}(t)|_{\boldsymbol{min.}} = \sqrt{1+0} = 1\ ms^{-1}$, $t = 0, \pi, 2\pi, \ldots$ so, $t = n\pi, n \in Z$

1W, 1A

c. (3 marks)

Answer:

$\boldsymbol{a}(t).\boldsymbol{v}(t) = (2\cos t\ \boldsymbol{i} + \sin t\ \boldsymbol{j}).(2\sin t\ \boldsymbol{i} - \cos t\ \boldsymbol{j}) = 0$

$4\cos t \sin t - \cos t \sin t = 0$

1W

$3\cos t \sin t = 0$

$\cos t = 0$ or $\sin t = 0$

1W

$t = 0, \frac{\pi}{2}, \pi, \ldots\ t = \frac{n\pi}{2}, n \in Z$

1A

Question 3 (13 marks)

a. (1 mark)

Answer:
$450 \pm 1.96 \times 8$
$434\ mm$ to $466\ mm$

1A

b. (3 marks)

Answer:

$\bar{x} = 450, \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{30}}$

1W

$450 \pm 2.576 \times \frac{8}{\sqrt{30}}$

1W

$446.2\ mm$ to $453.8\ mm$

1A

c. (2 marks)

Answer:

$$\Pr(\bar{y} < 446) = \Pr\left(z < \frac{446 - 450}{\frac{8}{\sqrt{25}}}\right) \approx 0.006$$

1W, 1A

d. (3 marks)

Answer:

i.

$H_0 \qquad \mu = 450mm$
$H_1 \qquad \mu < 450mm$

1A

ii.

$$z = \frac{447 - 450}{\frac{8}{\sqrt{20}}} \approx -1.6771$$

$\Pr(z < \ -1.6771) \approx 0.047$
Since $0.047 < 0.05$ Reject H_0

1W, 1A

e. (3 marks)

Answer:

$E(1.2X) = 1.2 \times 450 = 540mm$
$Var(1.2X) = 1.2^2 \times 64 = 92.16mm$
$SD(1.2X) = 9.6mm$

1A

$$z = \frac{550 - 540}{9.6} \approx 1.0417$$

$\Pr(z > \ 1.047) \approx 0.14878$
Use CAS
Binomial PD with $n = 10, x = 2, p = 0.14878$
The chance that exactly 2 pinbolts are longer than $550mm$ is 0.2746

1W, 1A

Question 4 (12 marks)

a. (2 marks)

Answer:
$a = 9.8, u = 0, t = 2.5$

1W

$s = \frac{1}{2}at^2 + ut$
$s = \frac{1}{2}(-9.8)(2.5)^2 + 0$
$s = -30.625\ m$

The particle falls $30.625\ m$

1A

b. (5 marks)

Answer:

$a = kt - 9.8$

$v = \int (kt - 9.8)dt$

1W

$v = \frac{1}{2}kt^2 - 9.8t + 0$

$v = \frac{1}{2}kt^2 - 9.8t$

1W

$x = \int \left(\frac{1}{2}kt^2 - 9.8t\right) dt$

$x = \frac{1}{6}kt^3 - 4.9t^2 + 0$

$x = \frac{1}{6}kt^3 - 4.9t^2$

1W

$-30.625 = \frac{1}{6}k(2.6)^3 - 4.9(2.6)^2$

Solve on CAS.

$k \approx 0.853$

1A

$v(2.6) = \frac{1}{2} \times 0.853 \times (2.6)^2 - 9.8 \times 2.6 \approx -22.60\ ms^{-1}$

1A

c. (5 marks)

Answer:

$a = -0.5v - 9.8$

$t = \int \left(\frac{1}{-0.5v - 9.8}\right) dt$

1W

Solve on CAS.

$t = -2(ln|v + 19.6| + 1.6094) + c$

$v = 0, t = 0$

$0 = -2(ln|0 + 19.6| + 1.6094) + c$

$c \approx 9.16986$

$t = -2(ln|v + 19.6| + 1.6094) + 9.16986$

1W

$v = e^{((\frac{t-9.16986}{-2} - 1.6094)} - 19.6$

$v = e^{((-0.5t+2.9755)} - 19.6$

$x = \int \left(e^{((-0.5t+2.9755)} - 19.6\right) dt$

$x = -2e^{((-0.5t+2.9755)} - 19.6t + c$

$x = 0, t = 0$

$0 = -2e^{(2.9755)} + c$

$c = 39.2$

$x = -2e^{((-0.5t+2.9755)} - 19.6t + 39.2$

1W

$-30.625 = -2e^{((-0.5t+2.9755)} - 19.6t + 39.2$

Solve on CAS. $t \approx 3.1481\ s$

1W

$v(3.1481) = e^{((-0.5\times3.1481+2.9755)} - 19.6 \approx 15.54\ ms^{-1}$

1A

Question 5 (16 marks)

a. (1 mark)

Answer:

$f: A \to R \ where \ f(x) = \dfrac{x+1}{\sqrt{4-x^2}}$

$4 - x^2 > 0$

$x \in (-2,2)$

1A

b. (2 marks)

Answer:

$x -$ intercept

$\dfrac{x+1}{\sqrt{4-x^2}} = 0$

$x = -1$

$(-1,0)$

1A

$y -$ intercept

$f(0) = \frac{0+1}{\sqrt{4-0^2}} = \frac{1}{2}$ $(0, \frac{1}{2})$

1A

c. (2 marks)

Answer:

$$f(x) = \frac{x+1}{\sqrt{4-x^2}}$$
$$f'(x) = \frac{x+4}{\sqrt{(4-x^2)^3}}$$

1W

For $x \in (-2,2)$, $x + 4 > 0$ and $\sqrt{(4-x^2)^3} > 0$

So, $\frac{x+4}{\sqrt{(4-x^2)^3}} > 0$

1A

d. (2 marks)

Answer:

This will occur at the inflection point.
Use CAS.

$$f(x) = \frac{x+1}{\sqrt{4-x^2}}$$

$$f'(x) = \frac{x+4}{\sqrt{(4-x^2)^3}}$$

$$f''(x) = \frac{-2(x^2+6x+2)}{(x^2-4)\sqrt{(4-x^2)^3}} = 0$$
$$x^2 + 6x + 2 = 0$$

1W

$$\left(x + 3 - \sqrt{7}\,\right)\left(x + 3 + \sqrt{7}\right) = 0$$

Reject $-3 - \sqrt{7}$

$$x = -3 + \sqrt{7} \approx -0.3542$$

$$ff(-0.3542) \approx 0.33$$

1A

e. (3 marks)
Answer:

$$\int_{-1}^{0}\left(\frac{x+1}{\sqrt{4-x^2}}\right)dx$$

$$=\int_{-1}^{0}\left(\frac{x}{\sqrt{4-x^2}}\right)dx+\int_{-1}^{0}\left(\frac{1}{\sqrt{4-x^2}}\right)dx$$

Let $u=4-x^2$ $\quad\quad \frac{du}{dx}=-2x=\int_{3}^{4}\left(\frac{-\frac{1}{2}\times\frac{du}{dx}}{\sqrt{u}}\right)dx+\int_{-1}^{0}\left(\frac{1}{\sqrt{4-x^2}}\right)dx$

$$=-\frac{1}{2}\int_{3}^{4}\left(u^{-\frac{1}{2}}\right)du+\int_{-1}^{0}\left(\frac{1}{\sqrt{4-x^2}}\right)dx$$

1W

$$=\left[-u^{\frac{1}{2}}\right]_{3}^{4}+\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_{-1}^{0}$$

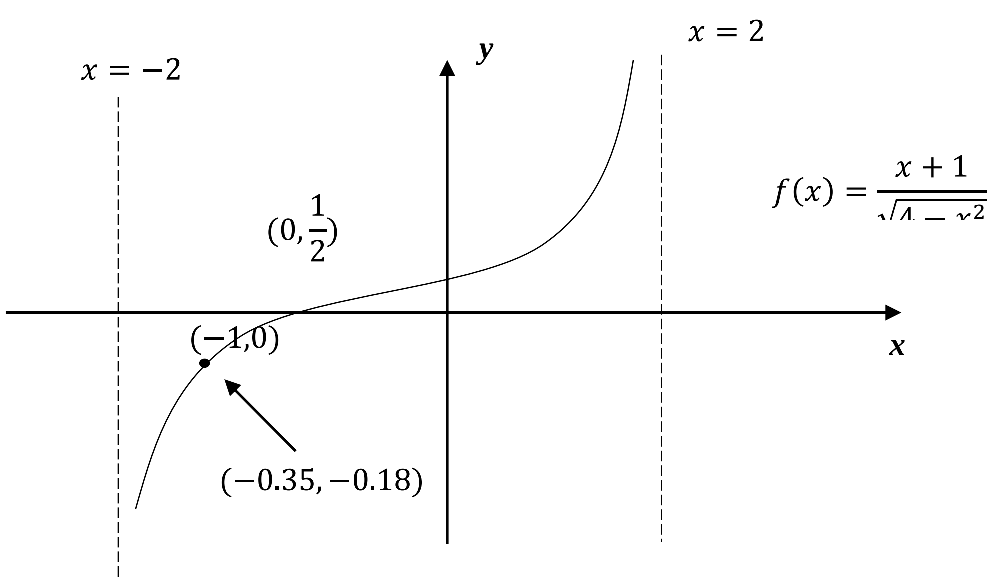
1W

$$=-2+\sqrt{3}+\sin^{-1}(0)-\sin^{-1}\left(-\frac{1}{2}\right)$$

$$=-2+\sqrt{3}+\frac{\pi}{6}$$ **1A**

f. (3 marks)

Answer:



Shape **1A**, Intercepts **1A**, Asymptotes **1A**,

g. (1 mark)

Answer:

$$\frac{5}{4-x^2}=\frac{5}{(2-x)(2+x)}=\frac{5}{4(2-x)}+\frac{5}{4(2+x)}$$

1A

h. (2 marks)

Answer:

$$V_x=\pi\int_{-1}^{0}\left(\frac{x+1}{\sqrt{4-x^2}}\right)^2\,dx$$

$$=\pi\int_{-1}^{0}\frac{(x+1)^2}{4-x^2}\,dx$$

$$=\pi\int_{-1}^{0}\frac{x^2+2x+1}{4-x^2}\,dx$$

$$=\pi\int_{-1}^{0}\left(\frac{2x+5}{4-x^2}-1\right)\,dx$$

$$=\pi\int_{-1}^{0}\left(\frac{2x}{4-x^2}+\frac{5}{4(2-x)}+\frac{5}{4(2+x)}-1\right)\,dx$$

1W

$$=\pi\left[-log_e(4-x^2)+\frac{5}{4}log_e\left(\frac{2+x}{2-x}\right)-x\right]\begin{matrix}0\\-1\end{matrix}$$

$$=\pi\left(-log_e(4)+\frac{5}{4}log_e\left(\frac{2}{2}\right)-0-(-log_e(3)+\frac{5}{4}log_e\left(\frac{1}{3}\right)-(-1))\right)$$

$$=\pi\left(\,log_e\left(\frac{3}{4}\right)+\frac{5}{4}log_e(3)+1\right)$$

1A

1 + 2 + 2 + 2 + 3 + 3 + 1 + 2 = 16 marks