

2023 Trial Examination

STUDENT
NUMBER

--	--	--	--	--	--	--	--	--

Letter

--

SPECIALIST MATHEMATICS

Unit 1 Written Examination 2

Reading time: 15 minutes

Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
A	20	20	20
B	5	5	60
Total			80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 16 pages.

Instructions

- Print your student number in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

THIS PAGE IS BLANK

SECTION A – Multiple Choice Questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

What would be a counterexample to the statement, “all prime numbers are odd”?

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5

Question 2

Which notation indicates that statements A and B are equivalent?

- A. $A' \Rightarrow B'$
- B. $A \Rightarrow B$
- C. $A \Leftarrow B$
- D. $A \Leftrightarrow B$
- E. $B' \Rightarrow A'$

Question 3

367 people attend a conference. The assertion that at least two people at the conference must share the same birth date is an example of:

- A. Equivalence
- B. The pigeon-hole principle
- C. Mathematical induction
- D. Conditionality
- E. Contrapositive

SECTION A – continued
TURN OVER

Question 4

Let $X = \{1, 2, 3, 4, 5\}$, $Y = \{2, 5, 6\}$, $Z = \{1, 7\}$

$Y \cap Z =$

- A. \emptyset
- B. $\{1, 2, 5, 6, 7\}$
- C. 0
- D. $\{2, 5\}$
- E. ∞

Question 5

The number of edges on a regular graph with 12 vertices each of degree 5 is:

- A. 15
- B. 20
- C. 25
- D. 30
- E. 60

Question 6

How many vertices does a complete graph with 45 edges have?

- A. 12
- B. 10
- C. 9
- D. 8
- E. 6

Question 7

Consider the binary number made up of n 1's and no zeroes. In base 10, this number equals:

- A. $2^n + 1$
- B. 2^n
- C. $2^n - 1$
- D. $10^n + 1$
- E. $10^n - 1$

Question 8

In Boolean Algebra, the statement $\neg(M \wedge N) =$

- A. $M \vee N$
- B. $\neg M \wedge \neg N$
- C. $M \wedge N$
- D. $\neg M \vee N$
- E. $\neg M \vee \neg N$

Question 9

The prime decomposition of 200 is of the form:

- A. $a^3 \times b^2$ where a and b are different prime numbers and $b < a$
- B. $a^3 \times b^2$ where a and b are different prime numbers and $b > a$
- C. $a^2 \times b^2$ where a and b are different prime numbers and $b > a$
- D. $a^3 \times b^3$ where a and b are different prime numbers and $b > a$
- E. $a^2 \times b^4$ where a and b are different prime numbers and $b > a$

Question 10

$20 + 10 + 5 + 2.5 + \dots =$

- A. 39
- B. 39.9
- C. 39.99
- D. 40
- E. 50

Question 11

A fish population in a large lake can be modelled by the information:

P_n = population in the n th year

Each year the population increases by 4%

Each year 5000 fish either die or are caught by anglers

Initially, the lake has 200000 fish.

The related recurrence relation is:

- A. $P_{n+1} = 0.04P_n - 5000, P_0 = 200000$
- B. $P_{n+1} = 1.04P_n + 5000, P_0 = 200000$
- C. $P_{n+1} = 0.04P_n + 5000, P_0 = 200000$
- D. $P_{n+1} = 1.04P_n - 5000, P_0 = 200000$
- E. $P_n = 1.04P_{n+1} - 5000, P_0 = 200000$

SECTION A – continued
TURN OVER

Question 12

The first 4 terms of the recurrence relation: $t_{n+1} = 3t_n + 1, t_1 = 4$ are:

- A. 4, 13, 40, 121
- B. 4, 15, 48, 147
- C. 13, 40, 120, 361
- D. 15, 48, 147, 444
- E. 4, 13, 42, 129

Question 13

What is the minimum number of terms (n) required so that:
 $7 + 11 + 15 + 19 + \dots + t_n > 10^5$?

- A. 222
- B. 223
- C. 224
- D. 225
- E. 226

Question 14

The number $\frac{a}{a-\sqrt{b}}$ will be positive if:

- A. $a < 0, b \geq 0$
- B. $a = 0, b > 0$
- C. $a > 0$
- D. $a < b$
- E. $a^2 = b$

Question 15

The sum of the degrees of all the vertices in a particular connected graph is 32. The number of edges in the graph:

- A. cannot be determined without further information.
- B. is less than 10
- C. is greater than 20
- D. is 16
- E. is between 8 and 16

Question 16

The number of ways in which a team of 6 can be formed with at least 3 females from a group of 5 females and 4 males is:

- A. 48
- B. 56
- C. 60
- D. 70
- E. 74

Question 17

How many different ways can the letters of the word: PADDINGTON be arranged?

- A. 3628800
- B. 907200
- C. 453600
- D. 226800
- E. 302400

Question 18

For the matrix: $A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix}$, which statement is true?

- A. A^2 is defined.
- B. A has 2 columns
- C. A has a defined inverse matrix.
- D. The element in first row, second column does not exist.
- E. A can be subtracted from a 2×3 matrix.

Question 19

Which statement relating to the matrix: $B = \begin{bmatrix} -1 & 1 & 2 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$ is true?

A. $2B = \begin{bmatrix} -2 & 1 & 2 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$

B. $B + 1 = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 2 & 0 \\ 2 & 4 & 1 \end{bmatrix}$

C. $B^2 = -\begin{bmatrix} -3 & -6 & 3 \\ 1 & 2 & 1 \\ 1 & -4 & 1 \end{bmatrix}$

D. $B - B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- E. $C + B$ is always defined, where C is a non-zero matrix.

SECTION A – continued
TURN OVER

Question 20

The equations of two parallel, non-identical straight lines are written in the matrix form:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$$

Which statement is not true?

A. $ax + by = e$

B. $cx + dy = f$

C. $ad = bc$

D. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}$

E. Matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is singular.

END OF SECTION A

SECTION B – Extended Response Questions

Instructions for Section B

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (9 marks)

Consider the set of numbers $M = 3^x, x \in N$

a. Write down the first 4 elements of the set M .

2 marks

b. Consider two elements from the set M . Let these elements be M_1 and M_2 . Let y be defined by $y = \frac{M_1}{M_2}$. What set of numbers does y belong to?

1 mark

c. Prove by **mathematical induction** that $3^x, x \in N$ is an odd number.

4 marks

SECTION B – Question 1 - continued

TURN OVER

d. Hence prove that by that $9^x, x \in N$ is also an odd number.

2 marks

Question 2 (8 marks)

Four couples (8 people altogether) are to be seated .

a. In how many different ways can they seat themselves in a straight line?

1 mark

b. In how many different ways can they seat themselves in a straight line with each couple next to each other?

2 marks

c. In how many different ways can they seat themselves around a circular table?

1 mark

SECTION B – Question 2 - continued

- d. In how many different ways can they seat themselves around a circular table with each couple next to each other?

2 marks

Suppose that Raj, Beth, Garry and Fran are four of the eight people being seated.

- e. In how many different ways can the eight people seat themselves in a straight line with Raj next to Beth, but Garry not next to Fran?

2 marks

Question 3 (9 marks)

Daisy invests \$50000 with a yearly return of 5%. At the end of each year, she adds a further \$3000. The first order recurrence relation is of the form:

$$A_{n+1} = aA_n + b, A_0 = c, n \geq 0$$

- a. Identify the values a, b, c .

3 marks

SECTION B – Question 3 - continued
TURN OVER

- b. Find the general formula (not a recurrence relation) that gives the value of the investment after n years, hence find the value of the investment, correct to the nearest dollar, after 20 years.

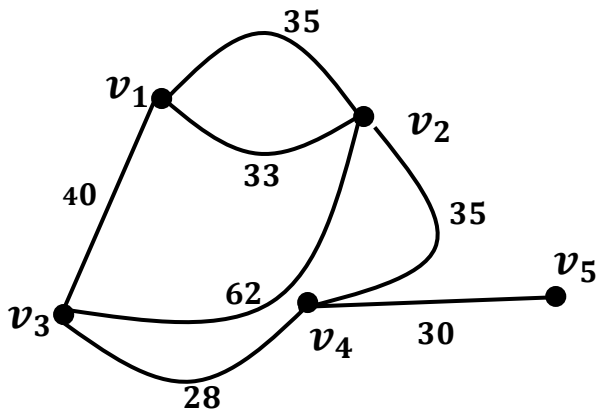
3 marks

- c. Suppose instead, that after 3 years, (when the investment is valued at A_3), Daisy is offered a return of only 3%. Daisy leaves the money invested without adding any extra money at the end of each year. Write a rule that gives the value of the investment x years after the start of the original investment (V_x).

3 marks

Question 4 (10 marks)

Consider the following connected graph:



- a. Show that Euler’s formula for planar, connected graphs holds: $v + f - e = 2$

1 mark

- b. Each vertex represents a small town and each edge has a weight that represents the time in minutes required to clear the road (edge) of snow. One snow plough is available. Find the minimum spanning tree, draw it and explain its significance.

2 marks

- c. Complete the adjacency matrix for the original graph, A .

$$A = \begin{bmatrix} 0 & 2 & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

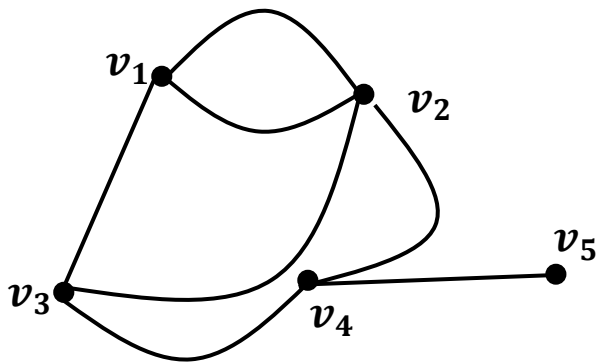
2 marks

SECTION B – Question 4 - continued
TURN OVER

d. Find A^2 and interpret its meaning.

2 marks

e. Add two edges so that an Euler circuit exists. How many different ways can this be done? Illustrate with drawings. (The graph must remain two dimensional and must remain planar.)



3 marks
SECTION B – continued

Question 5 (7 marks)

In a group of 20 students, 14 study mathematics (M), 10 study science (S) and 2 study neither.

- a. Use the inclusion-exclusion principle (or otherwise) to determine the number of students that study both mathematics and science.

2 marks

Of the group of 20 students, 9 study business management (B).

It is also known that 5 students study both mathematics and business management, 2 students study all 3 subjects and one student studies none of the 3 subjects.

- b. Use the inclusion-exclusion principle (or otherwise) to determine the number of students that study both business management and science.

2 marks

A committee of 5 is chosen randomly from the group of 20 students.

- c. How many committees are possible?

1 mark

- d. How many committees have no science students and at least 3 business management students?

2 marks

Question 6 (8 marks)

Consider the matrix: $C = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

- a. Find the inverse matrix of C .

2 marks

- b. Given $AC - BC = D$, use matrix algebra to write matrix B in terms of the other matrices.

2 marks

- c. What must be true about the dimensions of each of the other matrices A , C and D .

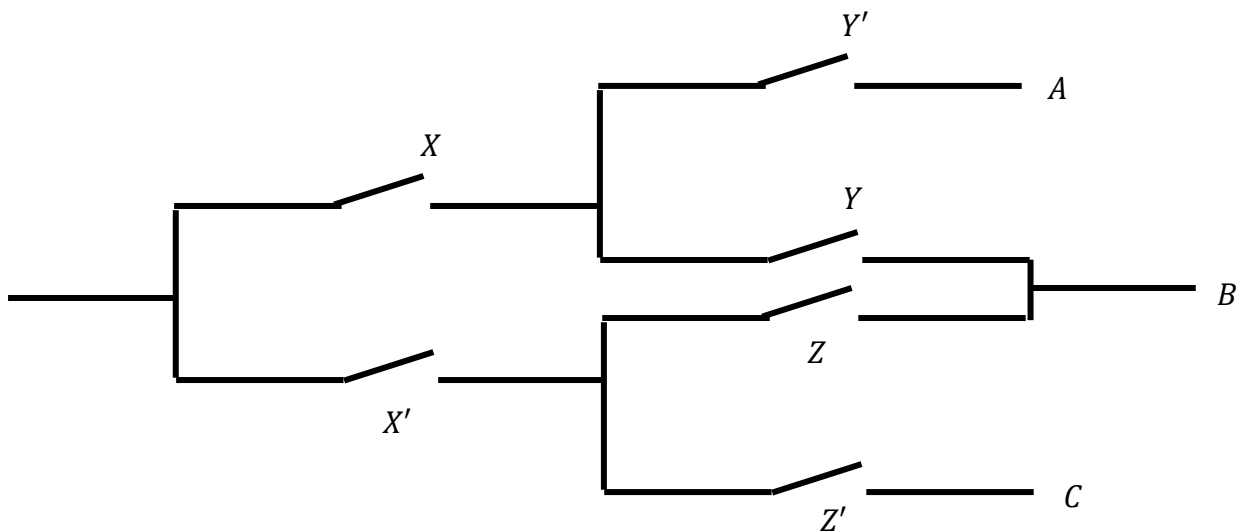
1 mark

- d. Given $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 0 & 3 \end{bmatrix}$, $D = 3A$ find matrix B .

3 marks

Question 7 (6 marks)

Consider the following switching circuit:



- a. Use Boolean notation to describe each of the outputs A , B and C .

3 marks

- b. Complete the following truth table hence simplify where possible any of the outputs A, B or C hence comment on the outcome of $A \vee B \vee C$.

X	Y	Z	A	B	C
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

3 marks

Question 7 (3 marks)

The following PSEUDOCODE illustrates the set up required to find the maximum of two numbers.

```

input x, y
If x > y then
print x
else
print y
end if

```

2023 SPECIALIST MATHEMATICS EXAM 2

Write PSEUDOCODE with similar notation that finds the sum of the first 6 terms of a geometric series: $a + ar + ar^2 + \dots + t_6$

3 marks

END OF QUESTION AND ANSWER BOOK