Student Name:



SPECIALIST MATHEMATICS 2023

Unit 4
Key Topic Test 3 – Differential Equations
Technology Free

Recommended writing time*: 45 minutes Total number of marks available: 30 marks

QUESTION BOOK

© TSSM 2023 Page 1 of 7

^{*} The recommended writing time is a guide to the time students should take to complete this test. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this test: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the room for this test: blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this test.

Materials supplied

• Question and answer book of 7 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this test.

© TSSM 2023 Page 2 of 7

Question 1 (7 marks)

a. $\frac{dy}{dx} = \frac{x-1}{2y}$, given that $y(0) = \frac{1}{2}$

3 marks

b.
$$\frac{dy}{dx} = (1 - 2y)(1 - y)$$
, given that $y(0) = 2$

4 marks

2023 SPECIALIST MATHEMATICS KEY TOPIC TEST

Question 2 (3 marks)				
Show that $y = (1 + 2x)e^{-x}$ is a solution to the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$.				

3 marks

© TSSM 2023 Page 4 of 7

Question 3 (7 marks)

Liquid is pouring into a large container and is leaking out of a hole in the base. The differential equation below shows the relationship between the height, h in cm, of the container and time, t in seconds.

$$\frac{dh}{dt} = 0.6 - 0.04\sqrt{h}$$

a.	Show that the time taken to fill the container from empty to a height of 100cm is given by $t = \int_0^{100} \frac{25}{15 - \sqrt{h}} dh$.
	30 15-√h
	3 marks
b.	Hence, find the exact time taken to fill the container to a height of 100cm.

4 marks

Page 5 of 7

© TSSM 2023

Question 4 (8 marks)

The population of bacteria, P(t), in a Petri dish satisfies the logistic differential equation

$$\frac{dP}{dt} = 2P\left(4 - \frac{P}{6000}\right)$$

where t is measured in hours and the initial population is 5000 bacteria.

a.	Find the maximum number of bacteria predicted by this model.	
b.	Find the number of bacteria when the population is growing at its fastest rate.	1 marl
c.	Solve the differential equation to find P as a function of t .	2 marks

5 marks

© TSSM 2023 Page 6 of 7

2023 SPECIALIST MATHEMATICS KEY TOPIC TEST

Question 5 (3 marks)
Use Euler's method to find y_2 given that $\frac{dy}{dx} = \frac{1}{x}$, given that $y(1) = \frac{1}{4}$.
Use a step size of 0.1.
Question 6 (2 marks)
Find the general solution of the differential equation $\frac{dx}{dt} = -\frac{x}{40}$.

END OF KEY TOPIC TEST

© TSSM 2023 Page 7 of 7