

Student Name: _____



SPECIALIST MATHEMATICS 2023

Unit 4

Key Topic Test 3 – Differential Equations

Technology Free

Recommended writing time*: 45 minutes

Total number of marks available: 30 marks

QUESTION BOOK

* The recommended writing time is a guide to the time students should take to complete this test. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this test: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the room for this test: blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this test.

Materials supplied

- Question and answer book of 7 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this test.

Question 1 (7 marks)

Solve the following differential equations and express y as a function of x .

a. $\frac{dy}{dx} = \frac{x-1}{2y}$, given that $y(0) = \frac{1}{2}$

3 marks

b. $\frac{dy}{dx} = (1 - 2y)(1 - y)$, given that $y(0) = 2$

4 marks

Question 4 (8 marks)

The population of bacteria, $P(t)$, in a Petri dish satisfies the logistic differential equation

$$\frac{dP}{dt} = 2P \left(4 - \frac{P}{6000} \right)$$

where t is measured in hours and the initial population is 5000 bacteria.

- a. Find the maximum number of bacteria predicted by this model.

1 mark

- b. Find the number of bacteria when the population is growing at its fastest rate.

2 marks

- c. Solve the differential equation to find P as a function of t .

5 marks

