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SPECIALIST MATHEMATICS 2023

Unit 4
Key Topic Test 2- Antidifferentiation applications
Technology Active

Recommended writing time*: 45 minutes
Total number of marks available: 30 marks

SOLUTIONS

SECTION A: Multiple-choice questions (1 mark each)

Question 1
Answer: **D**

Explanation:
$Area = \frac{1}{2}\left(y(-1) + y\left(-\frac{1}{2}\right)\right) \times \frac{1}{2} + \frac{1}{2}\left(y\left(-\frac{1}{2}\right) + y(0)\right) \times \frac{1}{2} + \frac{1}{2}\left(y(0) + y\left(\frac{1}{2}\right)\right) \times \frac{1}{2}$
$Area = \frac{1}{4}\left(\sqrt{3} + 2\sqrt{2} + 2\right)$

Question 2
Answer: **A**

Explanation:

$For\ concave\ up -\ \ f''(x) > 0$ (sketch on CAS)

Question 3
Answer: **B**

Explanation:

$\int_0^3 \left(\frac{1}{3}\right)^x dx = \frac{1}{2}\left(\left(\frac{1}{3}\right)^0 + \left(\frac{1}{3}\right)^1\right) + \frac{1}{2}\left(\left(\frac{1}{3}\right)^1 + \left(\frac{1}{3}\right)^2\right) + \frac{1}{2}\left(\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3\right) = 0.963$

Question 4
Answer: **E**

Explanation:

$-2\int_{\frac{3\pi}{4}}^{\pi} \sin(2x)\ dx = 2\int_{\pi}^{\frac{3\pi}{4}} \sin(2x)\ dx$

Question 5
Answer: **A**

Explanation:

$V_x = \pi \int_0^{\frac{\pi}{2}} \cos^6(x)\ dx = \frac{5\pi}{32}$
$V_y = \pi \int_0^1 \cos^{-1}\left(\sqrt[6]{y}\right)\ dy = \frac{5\pi}{32}$

Question 6
Answer: **B**

Explanation:

$x = \sin^2(t)\ \ and\ \ y = \cos^2(t)$
$\frac{dx}{dt} = 2\sin(t)\cos(t)\ \ and\ \frac{dy}{dt} = -2\cos(t)\sin(t)$
$Length\ of\ the\ curve = \int_0^{\frac{\pi}{2}} \sqrt{2\sin^2(2t)}\, dt$

Question 7
Answer: **D**

Explanation:

$A = 2\pi \int_0^8 2x^{\frac{1}{3}} \sqrt{1 + \left(\frac{2}{3}x^{-\frac{2}{3}}\right)^2}\ dx = 163.31$

Question 8
Answer: **E**

Explanation:

$A = 2\pi \int_0^{27} x^{\frac{1}{3}} \sqrt{1 + \left(\frac{1}{3}x^{-\frac{2}{3}}\right)^2}\ dx = 384.05$

SECTION B:

Question 1

a. $\frac{dV}{dt} = 0.048e^{0.4t}$

$V = 0.12\ e^{0.4t} + c$

$t = 0,\ V = 0 \rightarrow V = 0.12\ e^{0.4t} - 0.12$

2 marks

b. $0.12e^{0.4t} = V + 0.12$

$e^{0.4t} = \frac{V+0.12}{0.12}$

$0.4t = \ln\left(\frac{V+0.12}{0.12}\right)$

$t = \frac{1}{0.4}\ln\left(\frac{V+0.12}{0.12}\right)$

$t = \frac{5}{2}\ln\left(\frac{100V+12}{12}\right)$

$t = \frac{5}{2}\ln\left(\frac{25V+3}{3}\right)$

3 marks

c. $V = 0.12\ e^{0.4\times 7\ln(6)} - 0.12$

$V = 0.12e^{\ln(6)^{2.8}} - 0.12$

$V = 0.12 \times 6^{2.8} - 0.12$

$Max\ volume = 18\ L$

2 marks

d. $Volume = 2\pi \int_1^2 \sqrt{x}\sqrt{1 + \left(\frac{1}{2\sqrt{x}}\right)^2}\ dx$

$Volume = 2\pi \int_1^2 \sqrt{x}\sqrt{1 + \frac{1}{4x}}dx$

$Volume = 2\pi \int_1^2 \sqrt{x}\frac{\sqrt{4x+1}}{2\sqrt{x}}\ dx$

$Volume = \pi \int_1^2 \sqrt{4x+1}\,dx$

$Volume = \frac{\pi}{6}\left[(4x+1)^{\frac{3}{2}}\right]_1^2$

$Volume = \frac{\pi}{6}\left(27 - 5\sqrt{5}\right)$

4 marks

Question 2

a. $x(t) = 3\cos(2t) \;\; and \;\; y(t) = 3\sin(2t)$

$\frac{dx}{dt} = -6\sin(2t)\,,\; \frac{dy}{dt} = 6\cos(2t)$

$Length = \int_0^{\frac{\pi}{4}} \sqrt{36\sin^2(2t) + 36\cos^2(2t)}\; dt = \int_0^{\frac{\pi}{4}} 6\, dt = \frac{3\pi}{2}$

3 marks

b. $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$x^2 + y^2 = 9$

1 mark

c. $Area = \int_{2\sqrt{2}}^{3} \sqrt{(9 - y^2)}\, dy$

$Area = 0.12\ sq\ units$

2 marks

d. $Volume = \pi \int_0^1 (9 - x^2)\, dx$

$Volume = \pi \left[9x - \frac{x^3}{3}\right] \frac{1}{0} = \frac{26\pi}{3}\ cubic\ units$

2 marks

e. $Area = 2\pi \int_0^1 \sqrt{9 - x^2} \sqrt{1 + \left(-\frac{x}{\sqrt{9-x^2}}\right)^2}\, dx$

$Area = 2\pi \int_0^1 \sqrt{9 - x^2} \sqrt{\frac{9}{9-x^2}}\, dx$

$Area = 2\pi \int_0^1 3\, dx$

$Area = 6\pi$

3 marks

END OF KEY TOPIC TEST SOLUTIONS