

Student Name: _____



SPECIALIST MATHEMATICS 2023

Unit 4

Key Topic Test 1 – Antidifferentiation applications Technology Free

Recommended writing time*: 45 minutes

Total number of marks available: 30 marks

QUESTION BOOK

* The recommended writing time is a guide to the time students should take to complete this test. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this test: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the room for this test: blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this test.

Materials supplied

- Question and answer book of 7 pages.

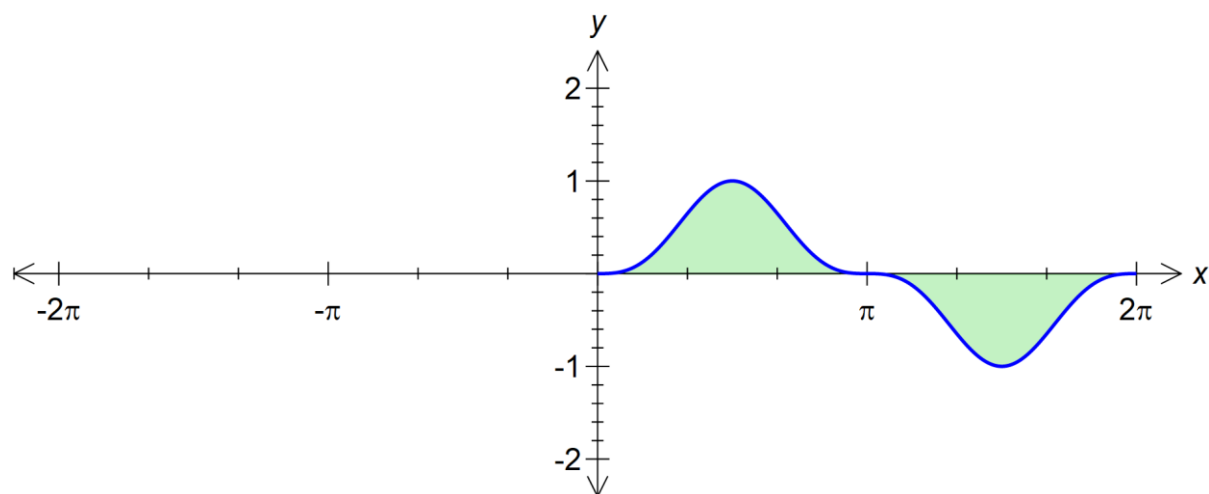
Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this test.

Question 1 (6 marks)

The graph of $y = \sin^3(x)$ over the interval $[0, 2\pi]$ is shown below.



- a. Write down an integral that will calculate the area bounded by the graph of $y = \sin^3(x)$ and the x-axis between $x = 0$ and $x = 2\pi$.

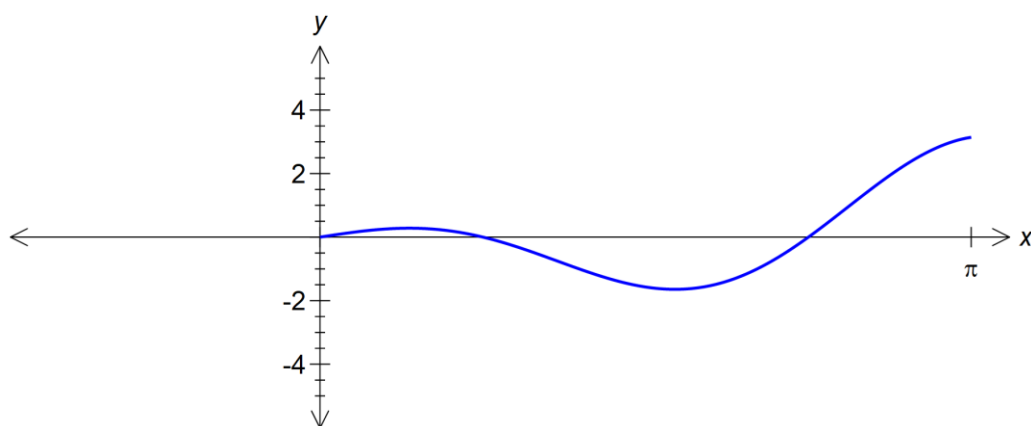
2 marks

- b. Hence, calculate the area.

4 marks

Question 2 (8 marks)

The graph of $y = x \cos(2x)$ over the interval $[0, \pi]$ is shown below.



a. Find the x-intercepts of $y = x \cos(2x)$ over $[0, \pi]$.

2 marks

b. Find the area bounded by the region enclosed by the graph of $y = x \cos(2x)$ and x-axis over $[0, \pi]$.

4 marks

- c. What is the signed area bounded by the region enclosed by the graph of $y = x \cos(2x)$ and x-axis over $[0, \pi]$?

2 marks

Question 3 (6 marks)

- a. Sketch the graph of $y = \arccos(x)$ over the interval $[0, 1]$ identifying all key features.

2 marks

- b. Find the volume, V , of the solid of revolution formed when the graph of $y = \arccos(x)$ is rotated about the y-axis over the interval $[0, 1]$.

4 marks

Question 5 (6 marks)

A curve is defined by the parametric equations

$$x = \frac{4}{3}(t^2 - 1) \text{ and } y = 2t^2$$

a. Find $\frac{dy}{dx}$.

2 marks

b. Find the surface area of revolution formed when the curve defined by the parametric equations, where $0 \leq t \leq 1$, is rotated about the x-axis.

4 marks

END OF KEY TOPIC TEST