Student Name: _____

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Creating VCE Success

SPECIALIST MATHEMATICS 2023

Unit 3

Key Topic Test 16 – Antidifferentiation Techniques
Technology Active

Recommended writing time*: 45 minutes
Total number of marks available: 30 marks

QUESTION BOOK

* The recommended writing time is a guide to the time students should take to complete this test. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this test: pens, pencils, highlighters, erasers, sharpeners and rulers, a CAS and/or scientific calculator
- Students are NOT permitted to bring into the room for this test: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question book of 10 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this test.

SECTION A– Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question

Question 1

$\int_0^1 \frac{\sqrt{1-x^2}}{x-2}\,dx$ correct to two decimal places is

A. -0.46
B. 0.46
C. 0.97
D. -0.81
E. 0.81

Question 2

With a suitable substitution $\int_0^3 x^2\sqrt{x-1}\,dx$ can be expressed as

A. $\int_0^3 \left(u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du$
B. $\int_{-1}^2 \left(u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du$
C. $\int_{-1}^2 \left(u^{\frac{3}{2}} + 2u^{\frac{1}{2}} + 1\right) du$
D. $\int_{-1}^2 u^{\frac{5}{2}}\,du$
E. $\int_{-1}^2 \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du$

Question 3

$\int x\sin(x)\,dx$ is equivalent to

A. $x\cos(x) + \int \cos(x)\ dx$
B. $-x\sin(x) - \int x\ dx$
C. $x\cos(x) - \int \sin(x)\ dx$
D. $-x\cos(x) - \int \cos(x)\ dx$
E. $-x\cos(x) + \int \cos(x)\ dx$

Question 4

Given that $\int_{\frac{\pi}{4}}^{k} \frac{1}{\cos^2(x)\tan(x)}\,dx = \ln\left(\frac{5}{4}\right)$, the value of k can be found by solving

A. $\tan(k) = \ln\left(\frac{5}{4}\right)$

B. $\tan(k) = \frac{4}{5}$

C. $\tan(k) = \frac{5}{4}$

D. $\cos(k) = \frac{4}{5}$

E. $\tan(k) = -\frac{5}{4}$

Question 5

The integral $\int_a^b \cos(2x)\sin(2x)\,dx$ can be calculated using

A. $\frac{1}{2}\int_a^b \sin(4u)\,du$

B. $\int_a^b \sin(4x)\,dx$

C. $2\int_a^b \sin(4x)\,dx$

D. $\frac{1}{2}\int_a^b u\,du$

E. $\frac{1}{2}\int_{2a}^{2b} u\,du$

Question 6

$\int_a^b \frac{2}{(x^2-1)(x+2)}\,dx$, where a and b are real constants, is equivalent to

A. $\int_a^b \left(\frac{2}{3(x+2)} + \frac{1}{x+1} + \frac{1}{3(x-1)}\right)\,dx$

B. $\int_a^b \left(\frac{2}{3(x+2)} - \frac{1}{x+1} + \frac{1}{3(x-1)}\right)\,dx$

C. $\int_{\frac{1}{a}}^{\frac{1}{b}} \left(\frac{2}{3(x+2)} - \frac{1}{x+1} + \frac{1}{3(x-1)}\right)\,dx$

D. $\int_a^b \left(\frac{2}{3(x+2)} - \frac{1}{x+1} + \frac{1}{x-1}\right)\,dx$

E. $\int_a^b \left(\frac{1}{x+1} - \frac{1}{3(x^2-1)}\right)\,dx$

Question 7

An antiderivative of $\int f'(x)\cos\big(f(x)\big)\,dx$ could be

A. $-\sin(f(x))-4$

B. $f(\sin(x))+\frac{1}{2}$

C. $\cos(f(x))$

D. $\sin(x)-2$

E. $\sin(f(x))+2$

Section B Short-answer questions

Instructions for Section B
Answer each question in the space provided. Please provide appropriate workings and use exact answers unless otherwise specified.

Question 1 (8 marks)

a. Use the substitution $x = \tan(\theta)$ to evaluate $\int_0^1 \frac{x^2}{(1+x^2)^{\frac{5}{2}}} dx$.

4 marks

b. Hence, using integration by parts, show that $\int_0^1 \frac{x^2}{(1+x^2)^{\frac{5}{2}}}\, x\, dx = \frac{-5\sqrt{2}}{12} + \frac{2}{3}$

4 marks

Question 2 (6 marks)

Let $f(x) = \frac{x^2-5x+5}{x^2-5x+8}$

a. Show that $f(x)$ can be written in the form $1 + \frac{a}{x^2-5x+8}$, where $a \epsilon R$

2 marks

b. Hence find $\int f(x)\,dx$.

3 marks

c. For what value of k is $\int_{\frac{1}{2}}^{k} f(2x)\,dx = \int_0^1 f(x)\,dx$.
Give your answer correct to two decimal places.

1 mark

Question 3 (9 marks)

a. Let $x^2 = \cos(y)$.
Show that $\frac{dy}{dx} = -\frac{2x}{\sqrt{1-x^4}}$

2 marks

b. Use integration by parts to show that
$\int x\cos^{-1}(x^2)\,dx = \frac{x^2}{2}\cos^{-1}(x^2) - \frac{1}{2}\sqrt{1-x^4} + c \ , c\epsilon R.$

4 marks

c. Hence evaluate $\int_0^1 x \cos^{-1}(x^2)\ dx$.

1 mark

d. Find the value of k for which $\int_0^1 (x \cos^{-1}(x^2) - kx)\ dx = \frac{1}{4}$?

2 marks

END OF KEY TOPIC TEST