

Student Name: \_\_\_\_\_



# **SPECIALIST MATHEMATICS 2023**

## **Unit 3**

### **Key Topic Test 14 – Antidifferentiation Techniques**

#### **Technology Free**

Recommended writing time\*: 45 minutes

Total number of marks available: 30 marks

## **QUESTION BOOK**

\* The recommended writing time is a guide to the time students should take to complete this test. Teachers may wish to alter this time and can do so at their own discretion.

***Conditions and restrictions***

- Students are permitted to bring into the room for this test: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the room for this test: any calculator, blank sheets of paper and/or white out liquid/tape.

***Materials supplied***

- Question and answer book of 7 pages.

***Instructions***

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this test.**

**Question 1 (8 marks)**

Evaluate the following indefinite integrals

a.  $\int \frac{3}{(x-2)(x+1)} dx$

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3 marks

b.  $\int \cos^2(2x) \sin(4x) dx$

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2 marks

c.  $\int x^2 \ln(x) dx$

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3 marks



**Question 3 (5 marks)**

a. Use an appropriate substitution to show that  $\int_{\sqrt{3}}^{\sqrt{5}} x^3 \sqrt{x^4 - 9} dx = \frac{1}{4} \int_0^{16} u du$ .

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3 marks

b. Hence evaluate  $\int_{\sqrt{3}}^{\sqrt{5}} x^3 \sqrt{x^2 - 9} dx$ .

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2 marks

**Question 4 (7 marks)**

The integral  $I_n$  is defined by  $I_n = \int_1^e (\ln x)^n dx$ , for  $n \geq 0$ .

**a.** Evaluate  $I_1$

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3 marks

**b.** Show that  $I_2 = e - 2I_1$ .

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2 marks

**c.** Hence show that  $I_n = e - nI_{n-1}$ .

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2 marks

