Student Name:		



SPECIALIST MATHEMATICS 2023

Unit 3
Key Topic Test 14 – Antidifferentiation Techniques
Technology Free

Recommended writing time*: 45 minutes
Total number of marks available: 30 marks

QUESTION BOOK

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^{*} The recommended writing time is a guide to the time students should take to complete this test. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this test: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the room for this test: any calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

• Question and answer book of 7 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this test.

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Oucsuon 1 (o marks)	Questio	n 1 (8	marks))
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Evaluate the following indefinite integrals

$\mathbf{a.} \int {(x-}$	$\frac{3}{2)(x+1)}dx$	
		3 marks
b. ∫ cos	$s^2(2x)\sin(4x)\ dx$	
c. $\int x^2$	n(x) dx	2 marks

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3 marks

2023 SPECIALIST MATHEMATICS KEY TOPIC TEST

Question 2 (4 marks)
Evaluate $\int_{1}^{2} \frac{2x+1}{x^2-2x+2} dx$, expressing your answer it the form $\ln(a) + b$, where a and b are real numbers.

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Question 3 (5 marks)

a.	Use an appropriate substitution to show that $\int_{\sqrt{3}}^{\sqrt{5}} x^3 \sqrt{x^4 - 9} dx = \frac{1}{4} \int_0^{16} u du$.	
		3 marks
b.	Hence evaluate $\int_{\sqrt{3}}^{\sqrt{5}} x^3 \sqrt{x^2 - 9} dx$.	

2 marks

Question 4 (7	marks)	Ì
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The integral I_n is defined by $I_n =$	$\int_1^e (\ln x)^n dx,$	for $n \ge 0$.
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a.	Evaluate I_1	
		3 marks
b.	Show that $I_2 = e - 2I_1$.	3 marks
		2 marks
c.	Hence show that $I_n = e - nI_{n-1}$.	

2 marks

Question 5 (6 marks) Show that $\frac{1}{\sqrt{3}\sin(x)+3\cos(x)}$ can be written in the form $\frac{\sqrt{3}}{6\sin\left(x+\frac{\pi}{3}\right)}$ Hence, evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{3}\sin(x) + 3\cos(x)} dx.$

END OF KEY TOPIC TEST

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