SPECIALIST MATHEMATICS 2023

Unit 3

Key Topic Test 14 – Differentiation Applications

Technology Active

Recommended writing time*: 45 minutes
Total number of marks available: 30 marks

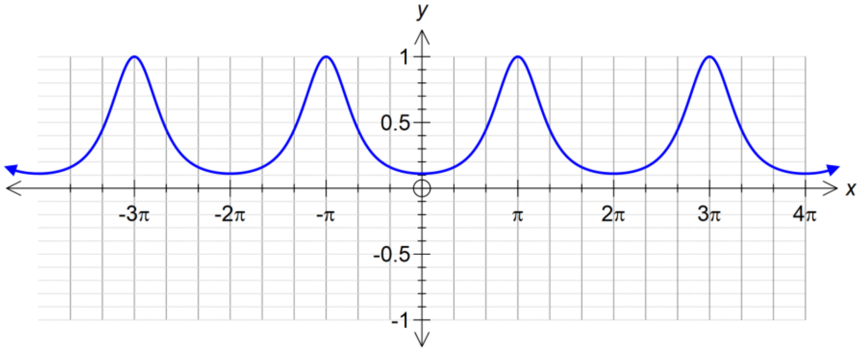
SOLUTIONS

Section A: Multiple-choice questions

Question 1

Answer: **A**

Explanation:
Sketch on CAS

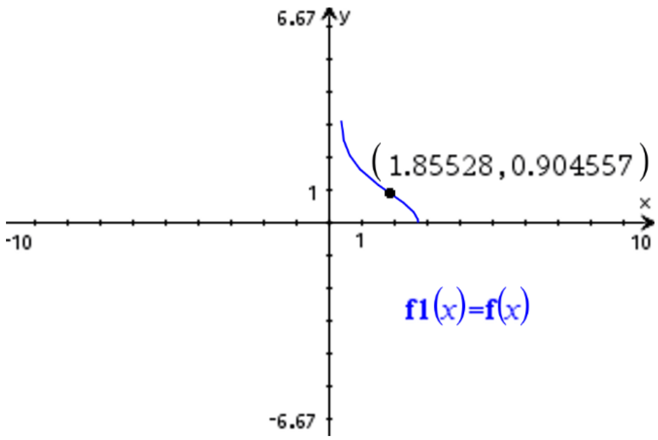


Question 2

Answer: **C**

Explanation:

Sketch on CAS



Question 3

Answer: **B**

Explanation:

$tangentline(f(x), x, 0)$ on CAS

Question 4

Answer: **D**

Explanation:

Sketch all options on CAS for the correct answer.

Question 5

Answer: **B**

Explanation:

Sketch on CAS over the restricted domain or use $fmax(f(x), x)$

Question 6

Answer: **E**

Explanation:

$f'(2) approaches \begin{cases} 1, & x > 2 \\ -1, & x < 2 \end{cases}$

Derivative does not exist at $x = 2$

Question 7

Answer: **D**

Explanation:

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \left(\frac{r}{\sqrt{3}}\right) \quad \left(\tan(60°) = \frac{r}{h}\right)$

$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} = \frac{\sqrt{3}}{\pi r^2} \times 1.8 = \frac{\sqrt{3}}{\pi (0.9)^2} \times 1.8 = 1.23$

Section B: Short-answer questions

Question 1

a.

Define $f(x)=e^{\frac{2}{3}\cdot x^3}$ Done

$\frac{d}{dx}(f(x))$ $2\cdot x^2\cdot e^{\frac{2\cdot x^3}{3}}$

1 mark

b. $f'(x)=0$

$\text{solve}\left(2\cdot x^2\cdot e^{\frac{2\cdot x^3}{3}}=0,x\right)$ $x=0$

Stationary point at $(0,1)$

2 marks

c. For point of inflection $f''(x)=0$

$\frac{d}{dx}\left(2\cdot x^2\cdot e^{\frac{2\cdot x^3}{3}}\right)$ $(4\cdot x^4+4\cdot x)\cdot e^{\frac{2\cdot x^3}{3}}$

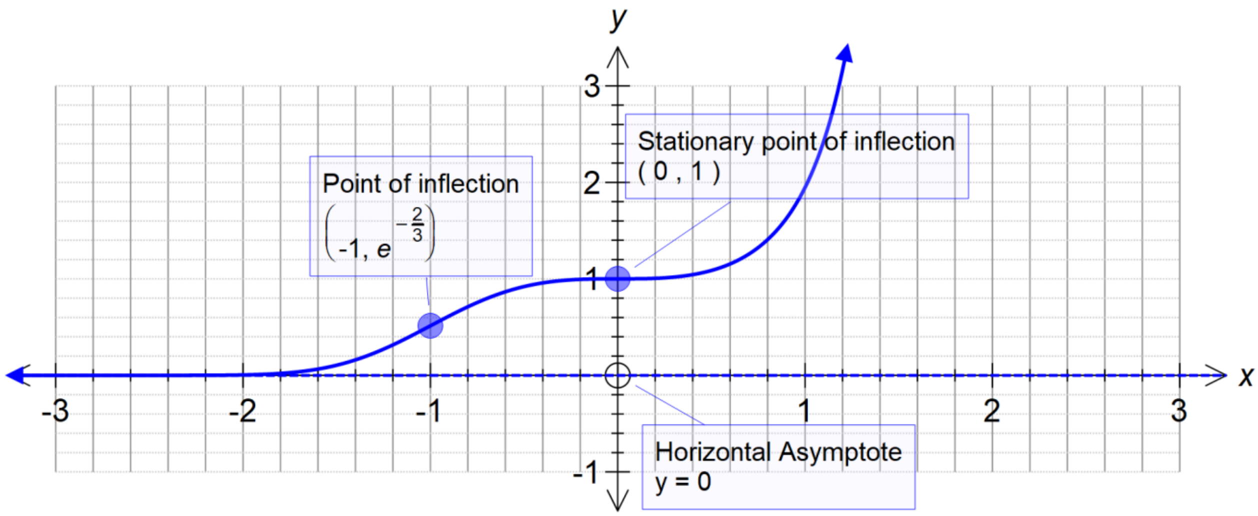
$\text{solve}\left((4\cdot x^4+4\cdot x)\cdot e^{\frac{2\cdot x^3}{3}}=0,x\right)$

$x=-1$ or $x=0$

Inflections points: $\left(-1,e^{-\frac{2}{3}}\right)$ and $(0,1)$

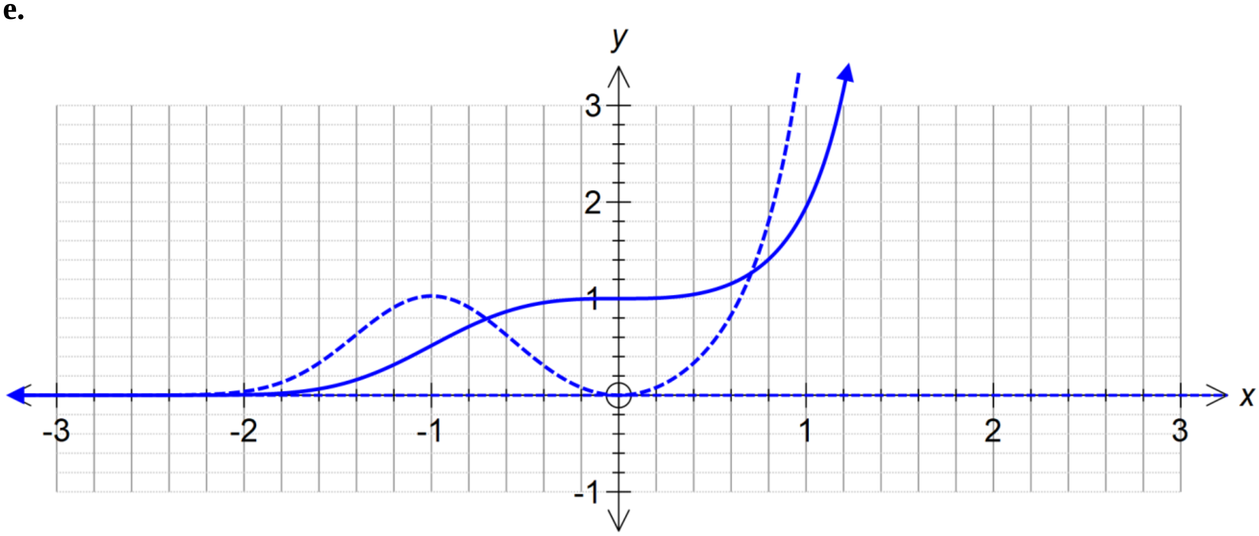
3 marks

d.



3 marks

e.



2 marks

Question 2

a.

Define $f(x)=-(\ln(x))^2-2\cdot\ln(x)+3$ — *Done*

$\frac{d}{dx}(f(x))$ — $\frac{-2\cdot\ln(x)}{x}-\frac{2}{x}$

$\text{solve}\left(\frac{-2\cdot\ln(x)}{x}-\frac{2}{x}=0,x\right)$ — $x=e^{-1}$

$f(x)|x=e^{-1}$ — 4

$(e^{-1},4)$ is a point of maxima.

2 marks

b. $f''(x)=\frac{2\ln(x)}{x^2}$

$f''(x)=0\rightarrow x=1$

Inflection point $(1,3)$

Tangent line:

$y=5-2x$

3 marks

c. $f'(u)=\tan(120°)$

$-\frac{2\ln(u)}{u}-\frac{2}{u}=-\sqrt{3}$

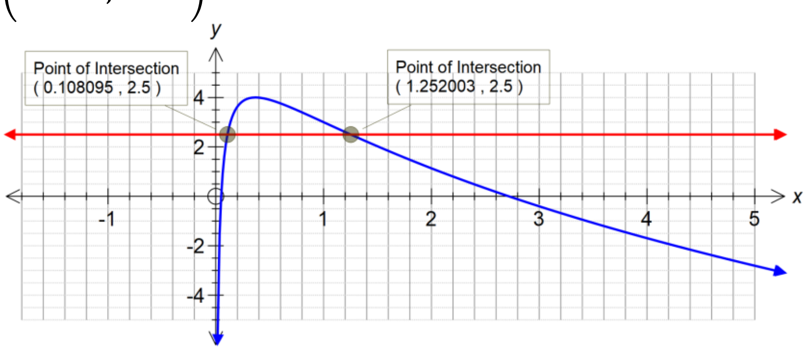
$u=0.6408$ or 1.8894

$v=3.6920$ or 1.3227

3 marks

d. $f(x)=2.5\rightarrow x=e^{-\frac{\sqrt{6}}{2}-1}$ or $e^{\frac{\sqrt{6}}{2}-1}$

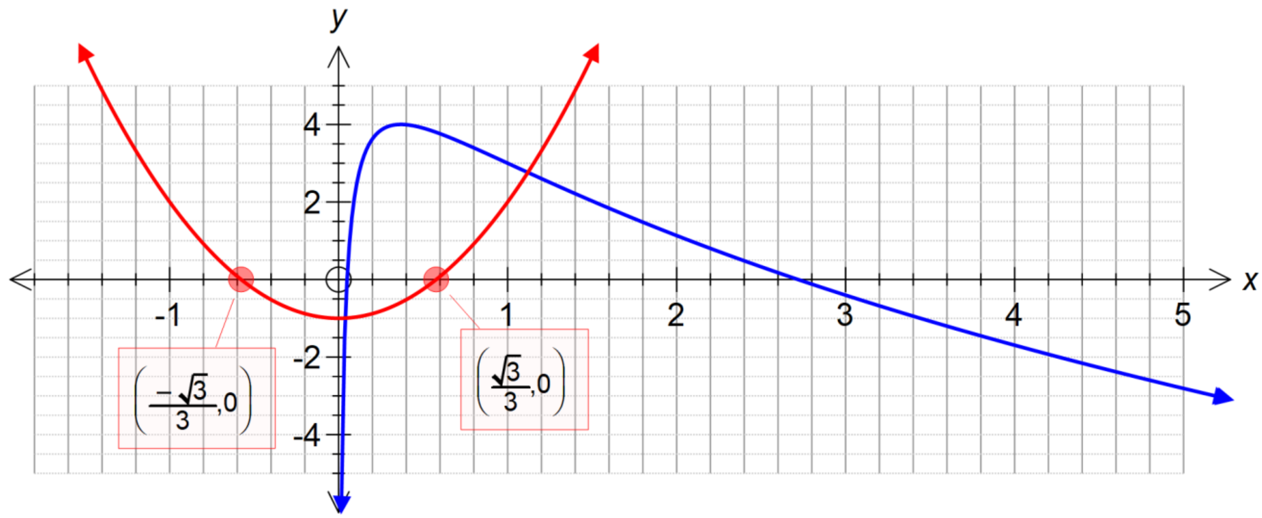
$\left(e^{-\frac{\sqrt{6}}{2}-1},e^{\frac{\sqrt{6}}{2}-1}\right)$



2 marks

e. $Range\ of\ g \subseteq Domain\ of\ f$

$Range\ of\ g \subseteq (0, \infty)$



The parabola must be restricted in a way to give a range of $(0, \infty)$, so

$\left(-\infty, -\frac{\sqrt{3}}{3}\right]$ or $\left[\frac{\sqrt{3}}{3}, \infty\right)$

Since minimum positive value of a is required, $a = \frac{\sqrt{3}}{3}$.

2 marks