Neap

Trial Examination 2023

VCE Specialist Mathematics Units 1&2

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 21 pages

Formula sheet

Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on the answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

Consider the following statement.

$$\text{If } x \neq a \text{ and } x \neq b, \text{ then } x^2 - (a+b)x + ab \neq 0.$$

Which one of the following is the negation of this statement?

A. If $x = a$ and $x = b$, then $x^2 - (a+b)x + ab = 0$.

B. If $x = a$ or $x = b$, then $x^2 - (a+b)x + ab \neq 0$.

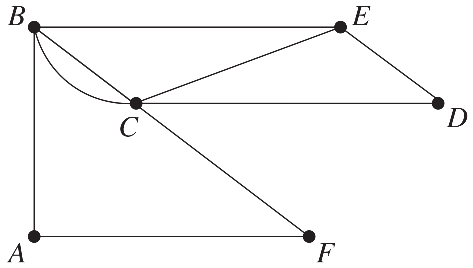
C. If $x = a$ and $x = b$, then $x^2 - (a+b)x + ab \neq 0$.

D. If $x \neq a$ and $x \neq b$, then $x^2 - (a+b)x + ab = 0$.

E. If $x = a$ or $x = b$, then $x^2 - (a+b)x + ab = 0$.

Question 2

Consider the following graph.



Which one of the following is a Hamiltonian path for this graph?

A. *D–E–C–B–A–F*

B. *C–E–D–F–A–B–C*

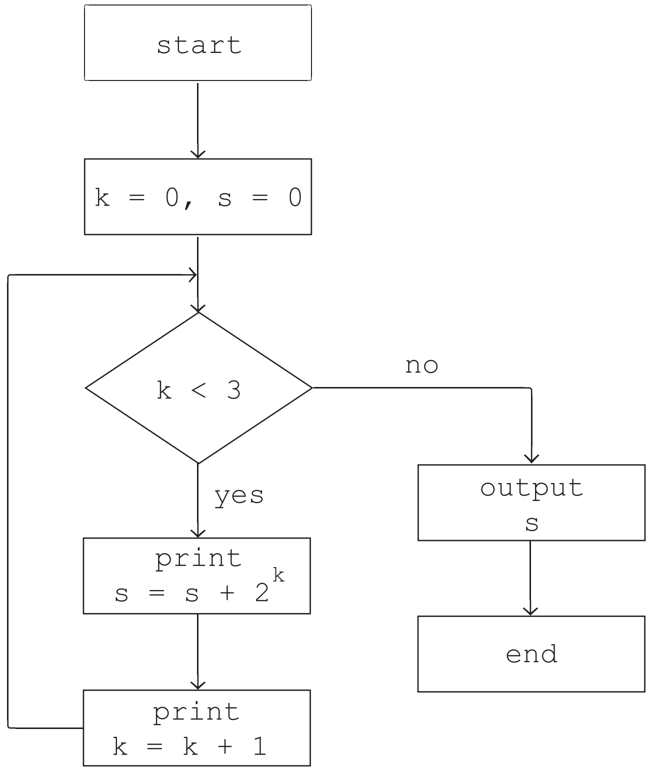
C. *A–B–C–E–D–C–F*

D. *F–C–E–D–C–B–A*

E. *C–B–A–F–C–E–D*

Question 3

Consider the following algorithm flowchart.



What is the output of this algorithm?

A. 1

B. 3

C. 5

D. 7

E. 15

Question 4

To prove the statement 'the function $ax^2 + bx + c = 0$, where $a \neq 0$, has at least two solutions' by contradiction, which one of the following should initially be assumed to be true?

A. The statement has at least two solutions.

B. The statement has at most two solutions.

C. The statement has no solutions.

D. The statement has at most one solution.

E. The statement has two solutions only.

Question 5

a_n is an arithmetic sequence where $a_1 = 1$ and the common difference is 3.

What is the value of n if $a_n = 2005$?

A. 667

B. 668

C. 669

D. 670

E. 671

Question 6

S_n is the sum of the first n terms of an arithmetic sequence, b_n.

If $b_1 = -11$ and $b_4 + b_6 = -6$, what is the value of n when S_n reaches its minimum?

A. 5

B. 6

C. 7

D. 8

E. 9

Question 7

Let W be the statement 'I will perform well in the exam' and S be the statement 'I study eight hours each day'.

Which one of the following is equivalent to the statement 'if I study eight hours each day, I will perform well in the exam'?

A. $W \rightarrow S$

B. $S \rightarrow W$

C. $W \leftrightarrow S$

D. $S \wedge W$

E. $W \vee S$

Question 8

How many ways can the word 'HELLO' be spelled incorrectly?

A. 59

B. 60

C. 96

D. 119

E. 120

Question 9

Four different plant seeds are labelled 1, 2, 3 and 4. Three of the seeds will be selected and planted. Each of the selected seeds will be planted in one of three garden beds. Seed 1 must be planted.

How many ways can the seeds be selected and planted?

A. 6

B. 12

C. 18

D. 24

E. 96

Question 10

Of all the non-repeated six-digit numbers formed by the integers 0, 1, 2, 3, 4 and 5, how many have an integer in the units place that is less than the integer in the tens place?

A. 210

B. 300

C. 464

D. 600

E. 720

Question 11

It is known that the sample of x_1, x_2 and x_3 has a variance of 5.

The variance of the sample of $3x_1$, $3x_2$ and $3x_3$ is

A. 5

B. $5\sqrt{3}$

C. 15

D. 45

E. 135

Question 12

The following table shows the distribution of the random variable X.

X	-1	0	1
$\Pr(X = x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Another random variable is $Y = 2X + 3$.

The expected value of Y is

A. -2

B. -1

C. 1

D. $\frac{7}{3}$

E. 4

Question 13

In triangle ABC, it is known that $a = 4$, $b = 6$ and $\angle C = 120°$.

The length of c is

A. 3

B. 8

C. $2\sqrt{17}$

D. $6\sqrt{2}$

E. $2\sqrt{19}$

Question 14

The graph of the function $y = \sin\left(2x + \frac{\pi}{5}\right)$ is translated $\frac{\pi}{10}$ units to the right.

The function is strictly

A. increasing over the interval $\left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$.

B. decreasing over the interval $\left[\frac{3\pi}{4}, \pi\right]$.

C. increasing over the interval $\left[\frac{5\pi}{4}, \frac{3\pi}{2}\right]$.

D. decreasing over the interval $\left[\frac{3\pi}{2}, 2\pi\right]$.

E. increasing over the interval $\left[\frac{3\pi}{2}, 2\pi\right]$.

Question 15

If $\sin(\alpha)+\cos(\alpha)=-\frac{1}{3}$, the value of $\sin(2\alpha)$ is

A. $-\frac{8}{9}$

B. $-\frac{1}{2}$

C. 0

D. $\frac{1}{2}$

E. $\frac{8}{9}$

Question 16

Which one of the following is **not** a unit vector?

A. $(-1, 0)$

B. $(\cos(x), \sin(x))$

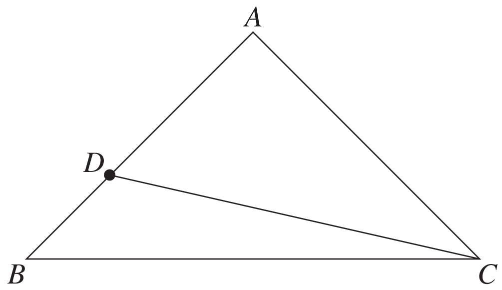
C. $\frac{\underset{\sim}{a}}{|\underset{\sim}{a}|}$, where $|\underset{\sim}{a}| \neq 0$

D. $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

E. $(-1, 1)$

Question 17

Triangle *ABC* is shown in the diagram below. Point *D* lies on *AB*.



If $\overrightarrow{AD} = 2\overrightarrow{DB}$ and $\overrightarrow{CD} = \frac{1}{3}\overrightarrow{CA} + \lambda\overrightarrow{CB}$, the value of λ is

A. $-\frac{1}{3}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

E. $\frac{3}{4}$

Question 18

A hyperbola passes through the point (1, 1) and has the asymptotes $2x + y = 0$ and $2x - y = 0$.

The cartesian form of the hyperbola is

A. $\frac{x^2}{3} - \frac{4y^2}{3} = 1$

B. $\frac{4x^2}{3} - \frac{y^2}{3} = 1$ or $\frac{x^2}{3} - \frac{4y^2}{3} = 1$

C. $\frac{4x^2}{3} - \frac{y^2}{3} = 1$

D. $\frac{4y^2}{3} - \frac{x^2}{3} = 1$

E. $\frac{y^2}{3} - \frac{4x^2}{3} = 1$

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Question 19

The complex number $(m^2 + i)(1 + mi)$ lies on the real axis in an Argand diagram.

The value of m is

A. $-\sqrt{2}$

B. -1

C. 0

D. 1

E. $\sqrt{2}$

Question 20

The value of $i + 2i^2 + 3i^3 + \ldots + 2018i^{2018}$ is

A. $-2018 + 2017i$

B. $1008 - 1008i$

C. $-1010 + 1009i$

D. $1010 - 1009i$

E. $-2018 + 1008i$

END OF SECTION A

SECTION B

Instruction for Section B

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (8 marks)

Prove the following statements using the method indicated in the brackets.

a. If $x^2 + y^2 = 0$, then $x = y = 0$. (contrapositive) 2 marks

b. If $a^3 + b^3 = 2$, then $a + b < 2$. (contradiction) 3 marks

c. When $n \in N^+$, $\frac{1}{1\times 3}+\frac{1}{3\times 5}+...+\frac{1}{(2n-1)\times(2n+1)}=\frac{n}{2n+1}$. (mathematical induction) 3 marks

Question 2 (10 marks)

Let A and B be logic statement variables.

a. Using a truth table, determine if the expression $A \leftrightarrow B = (A \rightarrow B) \wedge (B \rightarrow A)$ is true. 3 marks

b. Using a truth table, determine if the converse of $A \rightarrow B$ is logically equivalent to the contrapositive of $A \rightarrow B$. 3 marks

c. Consider the following function.

$$f(x)=\begin{cases}-x+1, x>0\\ 0, x=0\\ x+3, x<0\end{cases}$$

In the space below, draw an algorithm flowchart to output the value of $f(x)$ with the given input of x. 2 marks

d. Using an algorithm with a while loop, calculate the sum of all the even numbers from 1 to 100 inclusive. 2 marks

Question 3 (11 marks)

a. Consider the vectors $\underset{\sim}{a} = (\sin(\theta), 1)$ and $\underset{\sim}{b} = (1, \cos(\theta))$, $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Find the value of θ when $\underset{\sim}{a} \perp \underset{\sim}{b}$. 2 marks

b. Consider the vectors $\underset{\sim}{c} = (\sin(x), \cos(x))$ and $\underset{\sim}{d} = (\cos(x), \cos(x))$, $x \in R$. Function f is defined as $f(x) = \underset{\sim}{c} \cdot \left(\underset{\sim}{c} + \underset{\sim}{d}\right)$.

i. Express $f(x)$ in terms of $\sin(2x)$. 2 marks

ii. Find the period and maximum value of $f(x)$. 2 marks

c. Consider the vectors $\underset{\sim}{m} = (\cos(\alpha), \sin(\alpha))$ and $\underset{\sim}{n} = (\cos(\beta), \sin(\beta))$. It is known that $|\underset{\sim}{m} - \underset{\sim}{n}| = \dfrac{2\sqrt{5}}{5}$.

i. Find the value of $\cos(\alpha - \beta)$. 2 marks

ii. Prove that $\sin(\alpha) = \sin(\alpha - \beta)\cos(\beta) + \cos(\alpha - \beta)\sin(\beta)$. 1 mark

iii. Hence, find the value of $\sin(\alpha)$ given that $0 < \alpha < \dfrac{\pi}{2}$, $-\dfrac{\pi}{2} < \beta < 0$ and $\sin(\beta) = -\dfrac{5}{13}$. 2 marks

Question 4 (11 marks)

a. Consider the complex numbers $z_1 = 2 + ai$, where $a \in R$, and $z_2 = 3 - i$.

i. Find the value of a if $(z_1 + z_2)^2 \in R$. 2 marks

ii. If $\frac{z_1}{z_2} = b + \frac{1}{4}i$, find the values of a and b. 2 marks

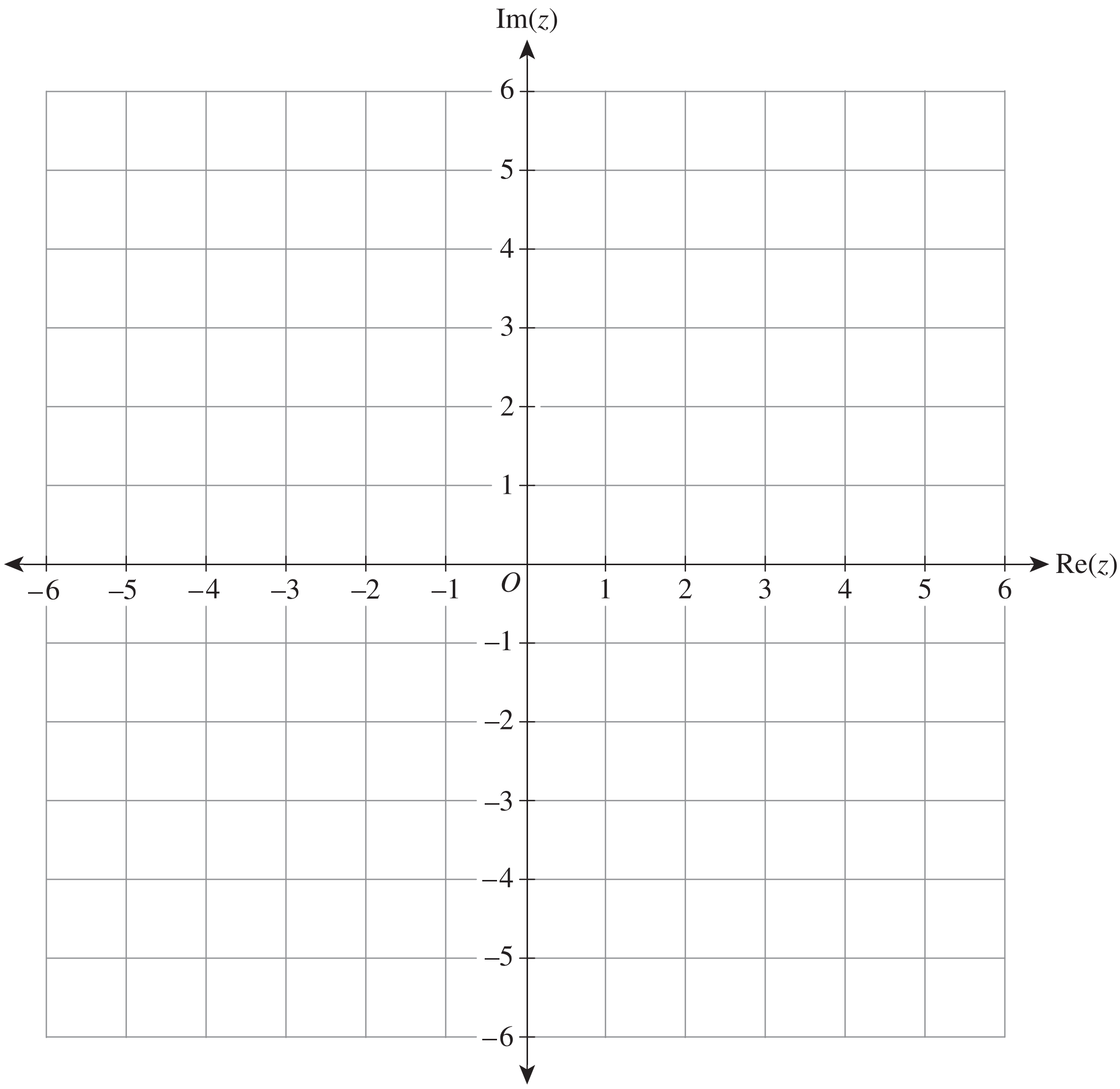
b. The following information is known about the complex number z.

- $|z| = \sqrt{5}$
- It is in the fourth quadrant.
- Its real and imaginary parts are all real integers.

i. Write down all possible values of z. 2 marks

ii. Hence, find the values of m and n if $m-\left(m^2-n\right)i=z^2$. 2 marks

c. On the axes below, sketch the set $A=\left\{\overline{z}\,|z\in c\cap 2\le|z|\le 3\cap -\frac{3\pi}{4}<\text{Arg}(z)\le-\frac{\pi}{4}\right\}$. 3 marks



Question 5 (11 marks)

John has been offered jobs at company A and company B. Company A offers him a monthly salary of \$1500 and an increase of \$230 to his monthly salary at the start of each subsequent year of employment. Company B offers him a monthly salary of \$2000 and an increase of 5% to his monthly salary at the start of each subsequent year of employment.

a. **i.** Write down a simplified expression for John's monthly salary if he works for n years at company A. 1 mark

ii. Write down a simplified expression for John's monthly salary if he works for n years at company B. 1 mark

b. If John intends to work for 10 years, which company should he choose so that he is paid a larger sum over the 10-year period? 3 marks

c. John decides to accept the job at company *A*.

Find the month in which the maximum difference between the salaries of companies *A* and *B* occurs and state this difference, correct to the nearest dollar. 2 marks

After working at company *A* for several years, John is offered a new salary scheme. The new scheme specifies that John will receive the full amount of his salary, a, in the first year of the new scheme. However, from the second year onwards, he will only receive $\frac{2}{3}$ of his salary from the previous year. Additionally, from the second year onwards, John can also receive an amount, b, each year from the company's new investment scheme. Amount b increases by 50% per year.

d. Express John's salary, s_n, after n years of the new salary scheme in terms of a, b and n. 2 marks

e. If $b = \frac{8a}{27}$, during which year of the new scheme will John earn the least amount of money? 2 marks

Question 6 (9 marks)

Vena is a tennis player. The probability of her serving successfully is $\frac{2}{3}$.

a. Find the probability that Vena successfully serves two out of five serves. Give your answer correct to three decimal places. 1 mark

b. If Vena serves five times, find the probability that she serves successfully three times in a row and does not serve successfully the other two times. 2 marks

To improve the efficiency of her serve, Vena undertakes a new training method. Using this method, Vena serves three times. Each successful serve is given a score of 1, and each unsuccessful serve is given a score of 0. Vena is also given an extra score of 1 if two serves in a row are successful. Additionally, she is given an extra score of 3 if all three serves are successful. Let the random variable X be Vena's score after three serves.

c. Find $\Pr(x = 3)$. 2 marks

d. Complete the probability distribution table below for X. 2 marks

X					
$\Pr(X = x)$					

e. Find the expected value of X. 1 mark

f. If Vena repeats this training method 100 times, what is the expected value of her mean score? 1 mark

END OF QUESTION AND ANSWER BOOKLET

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Trial Examination 2023

VCE Specialist Mathematics Units 1&2

Written Examination 2

Multiple-choice Answer Sheet

Student's Name: ______________________________

Teacher's Name: ______________________________

Instructions

Use a **pencil** for **all** entries. If you make a mistake, **erase** the incorrect answer – **do not** cross it out.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than **one** answer is completed for any question.

All answers must be completed like this example: A B C D E

Use pencil only

1	A	B	C	D	E	11	A	B	C	D	E
2	A	B	C	D	E	12	A	B	C	D	E
3	A	B	C	D	E	13	A	B	C	D	E
4	A	B	C	D	E	14	A	B	C	D	E
5	A	B	C	D	E	15	A	B	C	D	E
6	A	B	C	D	E	16	A	B	C	D	E
7	A	B	C	D	E	17	A	B	C	D	E
8	A	B	C	D	E	18	A	B	C	D	E
9	A	B	C	D	E	19	A	B	C	D	E
10	A	B	C	D	E	20	A	B	C	D	E

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Trial Examination 2023

VCE Specialist Mathematics Units 1&2

Written Examinations 1&2

Formula Sheet

Instructions

This formula sheet is provided for your reference. A question and answer booklet is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Mensuration

area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3}Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure

Boolean algebra	$a \wedge b = b \wedge a$ $(a \wedge b) \wedge c = a \wedge (b \wedge c)$ $a \wedge a = a$ $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ $a \wedge 1 = a$	$a \wedge 0 = 0$ $\neg(\neg a) = a$ $\neg(a \wedge b) = \neg a \vee \neg b$ $a \wedge \neg a = 0$ $\neg 0 = 1$
$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\text{cis}(\theta)$		$\lvert z \rvert = \sqrt{x^2 + y^2} = r$
$-\pi < \text{Arg}(z) \le \pi$		$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$	de Moivre's theorem	$z^n = r^n \text{cis}(n\theta)$

Data analysis, probability and statistics

for independent random variables $X_1, X_2 \ldots X_n$	$\text{E}(aX_1 + b) = a\,\text{E}(X_1) + b$ $\text{E}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1\text{E}(X_1) + a_2\,\text{E}(X_2) + \ldots + a_n\text{E}(X_n)$	
	$\text{Var}(aX_1 + b) = a^2\text{Var}(X_1)$ $\text{Var}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1^2\text{Var}(X_1) + a_2^2\text{Var}(X_2) + \ldots + a_n^2\text{Var}(X_n)$	
for independent identically distributed variables $X_1, X_2 \ldots X_n$	$\text{E}(X_1 + X_2 + \ldots + X_n) = n\mu$	
	$\text{Var}(X_1 + X_2 + \ldots + X_n) = n\sigma^2$	
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean	$\text{E}(\bar{X}) = \mu$
	variance	$\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$

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Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e \lvert x \rvert + c$
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a\sec^2(ax)$	$\int \sec^2(ax) dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}(\cot(ax)) = -a\operatorname{cosec}^2(ax)$	$\int \operatorname{cosec}^2(ax) dx = -\frac{1}{a}\cot(ax) + c$
$\frac{d}{dx}(\sec(ax)) = -a\sec(ax)\tan(ax)$	$\int \sec(ax)\tan(ax) dx = \frac{1}{a}\sec(ax) + c$
$\frac{d}{dx}(\operatorname{cosec}(ax)) = -a\operatorname{cosec}(ax)\cot(ax)$	$\int \operatorname{cosec}(ax)\cot(ax) dx = -\frac{1}{a}\operatorname{cosec}(ax) + c$
$\frac{d}{dx}(\sin^{-1}(ax)) = \frac{a}{\sqrt{1-(ax)^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}(\cos^{-1}(ax)) = \frac{-a}{\sqrt{1-(ax)^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}(\tan^{-1}(ax)) = \frac{a}{1+(ax)^2}$	$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
	$\int \frac{1}{ax+b} dx = \frac{1}{a}\log_e \lvert ax+b \rvert + c$

Calculus – continued

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$
Euler's method	If $\frac{dy}{dx} = f(x, y)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h \times f(x_n, y_n)$.
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$
surface area Cartesian about x-axis	$\int_{x_1}^{x_2} 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$
surface area Cartesian about y-axis	$\int_{y_1}^{y_2} 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$
surface area parametric about x-axis	$\int_{t_1}^{t_2} 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$
surface area parametric about y-axis	$\int_{t_1}^{t_2} 2\pi x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Kinematics

acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u + v)t$

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Vectors in two or three dimensions

$\underset{\sim}{r}(t)=x(i)\underset{\sim}{i}+y(t)\underset{\sim}{j}+z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert=\sqrt{x(t)^2+y(t)^2+z(t)^2}$
	$\dot{\underset{\sim}{r}}(t)=\frac{d\underset{\sim}{r}}{dt}=\frac{dx}{dt}\underset{\sim}{i}+\frac{dy}{dt}\underset{\sim}{j}+\frac{dz}{dt}\underset{\sim}{k}$
for $\underset{\sim}{r}_1=x_1\underset{\sim}{i}+y_1\underset{\sim}{j}+z_1\underset{\sim}{k}$ and $\underset{\sim}{r}_2=x_2\underset{\sim}{i}+y_2\underset{\sim}{j}+z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r}_1\cdot\underset{\sim}{r}_2=\lvert\underset{\sim}{r}_1\rvert\lvert\underset{\sim}{r}_2\rvert\cos(\theta)=x_1x_2+y_1y_2+z_1z_2$
	vector cross product $\underset{\sim}{r}_1\times\underset{\sim}{r}_2=\begin{vmatrix}\underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k}\\ x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\end{vmatrix}=(y_1z_2-y_2z_1)\underset{\sim}{i}+(x_2z_1-x_1z_2)\underset{\sim}{j}+(x_1y_2-x_2y_1)\underset{\sim}{k}$
vector equation of a line	$\underset{\sim}{r}(t)=\underset{\sim}{r}_1+t\underset{\sim}{r}_2=(x_1+x_2t)\underset{\sim}{i}+(y_1+y_2t)\underset{\sim}{j}+(z_1+z_2t)\underset{\sim}{k}$
parametric equation of a line	$x(t)=x_1+x_2t \quad y(t)=y_1+y_2t \quad z(t)=z_1+z_2t$
vector equation of a plane	$\underset{\sim}{r}(s,t)=\mathrm{r}_0+s\underset{\sim}{r}_1+t\underset{\sim}{r}_2$ $=(x_0+x_1s+x_2t)\underset{\sim}{i}+(y_0+y_1s+y_2t)\underset{\sim}{j}+(z_0+z_1s+z_2t)\underset{\sim}{k}$
parametric equation of a plane	$x(s,t)=x_0+x_1s+x_2t,\quad y(s,t)=y_0+y_1s+y_2t,\quad z(s,t)=z_0+z_1s+z_2t$
Cartesian equation of a plane	$ax+by+cz=d$

Functions, relations and graphs

The hyperbola with equation $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ has asymptotes given by $y-k=\pm\frac{b}{a}(x-h)$	
$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\mathrm{cosec}^2(x)$
$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$	$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$	$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\sin(2x)=2\sin(x)\cos(x)$	
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$
$\sin^2(ax)=\frac{1}{2}(1-\cos(2ax))$	$\cos^2(ax)=\frac{1}{2}(1+\cos(2ax))$

END OF FORMULA SHEET