2023 VCE Specialist Mathematics Year 12 Trial Examination 2

Kilbaha Education

Quality educational content

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Victorian Certificate of Education
2023

STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS
Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 34 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions and sign your name in the space provided.
- All written responses must be in English.
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

Consider the following statement:
‘If a function is continuous at a point, then it is differentiable at that point.’
Which one of the following is the contrapositive of this statement?

A. If a function is not continuous at a point, then it is not differentiable at that point.

B. If a function is continuous at a point, then it is not differentiable at that point.

C. If a function is not differentiable at a point, then it is not continuous at that point.

D. If a function is not differentiable at a point, then it is continuous at that point.

E. If a function is differentiable at a point, then it is continuous at that point.

Question 2

Several students were investigating the integral $\int x^4 \cos(2x)\,dx$

Alan stated that $\int x^4 \cos(2x)\,dx = \frac{x^4}{2}\sin(2x) - 2\int x^3 \sin(2x)\,dx$

Ben stated that $\int x^4 \cos(2x)\,dx = \frac{x^5}{5}\cos(2x) + \frac{2}{5}\int x^5 \sin(2x)\,dx$

Colin stated that $\int x^4 \cos(2x)\,dx = \left(\frac{x^4}{2} - \frac{3x^2}{2} + \frac{3}{4}\right)\sin(2x) + \left(x^3 - \frac{3x}{2}\right)\cos(2x) + c$

Then

A. Only Alan is correct.

B. Only Ben is correct.

C. Only Colin is correct.

D. Both of Alan and Ben are correct, Colin is incorrect.

E. All of Alan, Ben and Colin are all correct.

Question 3

Several students were investigating the proposition statement $p(n)$: If n is even then n^3+3 is odd.

Alan stated the proposition can be shown to be true using a direct proof.

Ben stated the proposition can be shown to be true using an indirect proof using the contrapositive.

Colin stated the proposition can be shown to be true using a proof by contradiction.

David stated the proposition can be shown to be true using a proof by induction.

Then

A. Only Alan is correct.

B. Only Ben is correct.

C. Only Colin is correct.

D. Only David is correct.

E. All of Alan, Ben and Colin are all correct.

Question 4

Given the vectors $\underset{\sim}{a}=\underset{\sim}{i}+2\underset{\sim}{j}-2\underset{\sim}{k}$, $\underset{\sim}{b}=2\underset{\sim}{i}-\underset{\sim}{j}$ and $\underset{\sim}{c}=-\underset{\sim}{i}+3\underset{\sim}{j}-2\underset{\sim}{k}$, which of the following is **false**?

A. The vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are perpendicular

B. $\underset{\sim}{a}\times\underset{\sim}{c}=\underset{\sim}{b}\times\underset{\sim}{c}$

C. The vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly dependent.

D. $|\underset{\sim}{b}|>|\underset{\sim}{a}|$

E. $|\underset{\sim}{c}|>|\underset{\sim}{b}|$

Question 5

When proving De Moivre's theorem using mathematical induction, in the inductive step it is necessary to assume that

A. $\left(r\,\text{cis}(\theta)\right)^k=r^k\,\text{cis}(k\theta)$ and show that $\left(r\,\text{cis}(\theta)\right)^{k+1}=r^{k+1}\,\text{cis}\left((k+1)\theta\right)$

B. $\left(r\,\text{cis}(\theta)\right)^k=r^k\,\text{cis}(k\theta)$ and show that $\left(r\,\text{cis}(\theta)\right)^{k+1}=r^{k+1}\,\text{cis}\left((k\theta+1)\right)$

C. $\left(r\,\text{cis}(\theta)\right)^k=r^k\,\text{cis}(k\theta)$ and show that $\left(r\,\text{cis}(\theta)\right)^{k+1}=r^{k+1}\,\text{cis}\left((k+\theta)\right)$

D. $\left(r\,\text{cis}(\theta)\right)^{k+1}=r^{k+1}\,\text{cis}\left((k+1)\theta\right)$ and show that $\left(r\,\text{cis}(\theta)\right)^k=r^k\,\text{cis}(k\theta)$

E. $\left(r\,\text{cis}(\theta)\right)^{k+1}=r^{k+1}\,\text{cis}\left((k\theta+1)\right)$ and show that $\left(r\,\text{cis}(\theta)\right)^k=r^k\,\text{cis}(k\theta)$

Question 6

Given the complex number $u = -3 - 3i$, which of the following is **false**?

A. If $n = 4k$ and $k \in Z^+$ then u^n is purely real.

B. If $n = 4k - 2$ and $k \in Z^+$ then u^n is purely imaginary.

C. If $n = 2k + 1$ and $k \in Z^+$ then $\operatorname{Re}\left(u^n\right) = \operatorname{Im}\left(u^n\right)$.

D. $\left|u^n\right| = \sqrt{18^n}$

E. $\arg\left(u^n\right) = -\dfrac{3\pi n}{4}$

Question 7

The area in the Argand plane defined by $\{z : a < z\,\overline{z} \le b, z \in C\} \cap \{z : \dfrac{\pi}{4} \le \operatorname{Arg}(z) \le \dfrac{3\pi}{4}, z \in C\}$

where $b > a > 0$ and $a, b \in R$ is equal to

A. $\dfrac{\pi}{4}\left(b^2 - a^2\right)$

B. $\dfrac{\pi}{4}(b - a)$

C. $\dfrac{\pi}{2}\left(b^2 - a^2\right)$

D. $\dfrac{\pi}{2}(b - a)$

E. $\dfrac{\pi}{2}\left(\sqrt{b} - \sqrt{a}\right)^2$

Question 8

Given $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$ and $\underset{\sim}{n} = 4\underset{\sim}{i} + 2\underset{\sim}{j} - 4\underset{\sim}{k}$, the closest distance between the planes

$\underset{\sim}{r} . \underset{\sim}{n} = 4$ and $-2x - y + 2z = 1$ is

A. $\dfrac{1}{3}$

B. $\dfrac{2}{3}$

C. 1

D. $\dfrac{4}{3}$

E. $\dfrac{5}{3}$

Question 9

The surface area of revolution formed when the curve defined by

$x = t - \sin(t)$ and $y = 1 - \cos(t)$ for $0 \le t \le 2\pi$ is rotated about the x-axis is equal to

A. $16\pi \int_0^{\pi} \sin^3\left(\frac{t}{2}\right) dt$

B. $8\pi \int_0^{\pi} \sin^3\left(\frac{t}{2}\right) dt$

C. $4\pi \int_0^{2\pi} \sin^3\left(\frac{t}{2}\right) dt$

D. $8\pi \int_0^{2\pi} \sin^3\left(\frac{t}{2}\right) dt$

E. $8\pi \int_0^{\frac{\pi}{2}} \left(t - \sin(t)\right) \sin^2\left(\frac{t}{2}\right) dt$

Question 10

The area bounded by the curve $y^2 = 4x$ and the line $x = 4$ is rotated about the y-axis to form a solid of revolution. The volume of this solid is given by

A. $\pi \int_0^{4} \left(4 - \frac{y^2}{4}\right)^2 dy$

B. $\pi \int_0^{4} \left(16 - \frac{y^2}{4}\right)^2 dy$

C. $\pi \int_0^{4} \left(4 - \frac{y^4}{16}\right) dy$

D. $\pi \int_0^{4} \left(16 - \frac{y^4}{16}\right) dy$

E. $2\pi \int_0^{4} \left(16 - \frac{y^4}{16}\right) dy$

For questions 11, 12 and 13, consider the point $(2,1,3)$ the line $\frac{x-3}{2}=\frac{y+2}{2}=1-z$ and the plane $2x-y+2z=6$

Question 11

The point and the line do not intersect, the closest distance of the point to the line is

A. $\frac{2}{3}$

B. 2

C. $\sqrt{14}-2$

D. $\frac{\sqrt{122}}{3}$

E. $3\sqrt{10}$

Question 12

The point and the plane do not intersect, the closest distance of the point to the plane is

A. $\frac{4}{3}$

B. 1

C. $\frac{2}{3}$

D. 2

E. $\sqrt{14}-2$

Question 13

The line and the plane do not intersect, the closest distance of the line to the plane is

A. $\frac{4}{3}$

B. 1

C. $\frac{2}{3}$

D. 2

E. $\sqrt{14}-2$

Question 14

Given that $\frac{dy}{dx} = 2x\tan(y)$, the value of $\frac{d^2y}{dx^2}$ when $x = -1$ and $y = \frac{\pi}{3}$ is equal to

A. $2\sqrt{3} - 8$

B. $-2\sqrt{3}$

C. -8

D. $16\sqrt{3}$

E. $18\sqrt{3}$

Question 15

A parachutist jumps from a plane and falls freely for a short time and then opens the parachute. Let t be the time in seconds after the parachute opens and $x(t)$ be the distance in metres travelled after the parachute opens, and $v(t)$ be the velocity of the parachutist in m/s. Given that the acceleration of the parachutist after the parachute opens is given by $\ddot{x} = g - kv$ where k is a positive constant. If the terminal velocity is v_T and when the parachute opens the speed of descent is $1.5v_T$, then let T be the time until the parachutist has slowed to a speed of $1.1v_T$ and D the distance travelled in this time, then

A. $T = \frac{1}{k}\log_e(5)$ and $D = \frac{g}{5k^2}\left(5\log_e(5) + 2\right)$

B. $T = \frac{1}{k}\log_e(5)$ and $D = \frac{13g}{15k}$

C. $T = \frac{2}{5k}$ and $D = \frac{g}{5k^2}\left(5\log_e(5) + 2\right)$

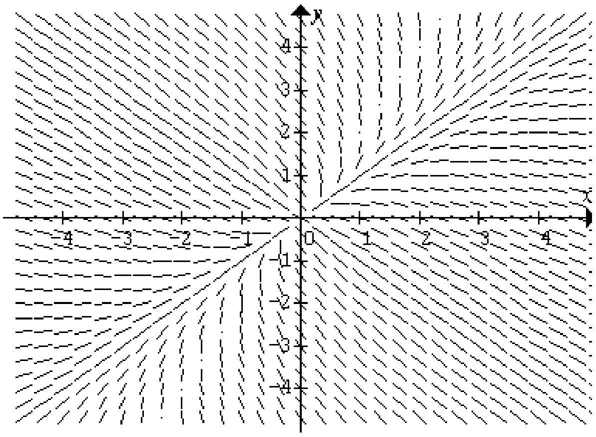
D. $T = \frac{2}{5k}$ and $D = \frac{13g}{15k}$

E. $T = \frac{2}{5k}$ and $D = \frac{26g}{15k}$

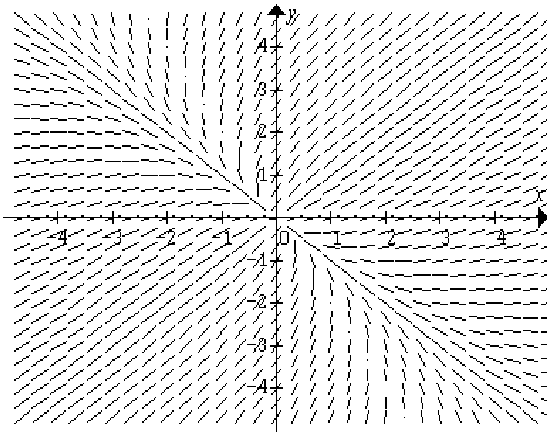
Question 16

The direction field for the differential equation $\dfrac{dy}{dx}=\dfrac{x-2y}{y-2x}$ is best represented by

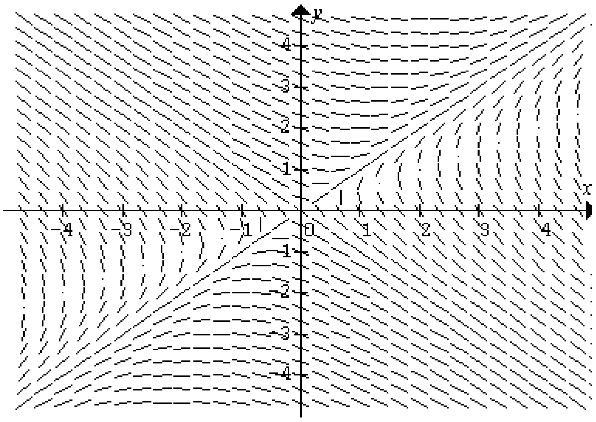
A.



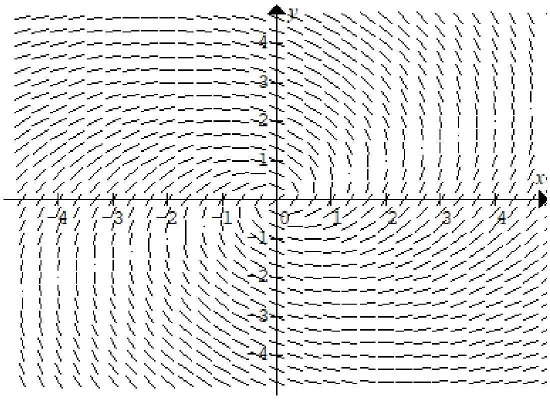
B.



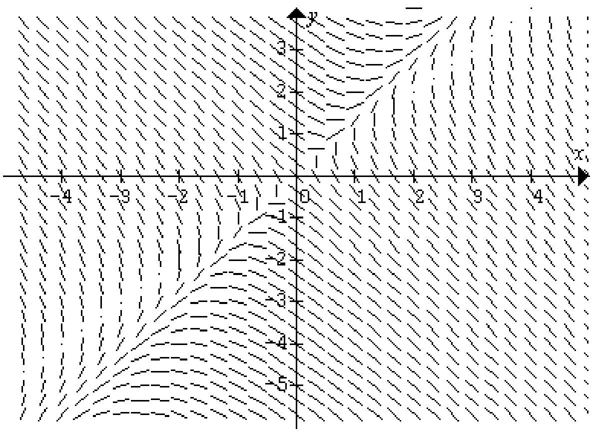
C.



D.



E.



Question 17

The algorithm below, described in pseudocode, finds the y values for differential equation $\frac{dy}{dx} = f(x, y)$, given the initial conditions $y(x0) = y0$ up to xn with a step size of h.

Inputs: $f(x, y)$ the gradient function

$y(x0) = y0$ the initial conditions

xn, the final x value

h, the step size

Define euler$\left(f(x, y), x0, xn, y0, h\right)$

$n \leftarrow \frac{xn - x0}{h}$

$i \leftarrow 0$

$x \leftarrow x0$

$y \leftarrow y0$

While $i < n + 1$ **Do**

$y \leftarrow y + hf(x, y)$

$x \leftarrow x + h$

$i \leftarrow i + 1$

EndWhile

Return y

Consider the algorithm implemented with the following inputs euler$\left(2xy^2, 1, 2, 1, 0.25\right)$

The value of the variable y after **two** iterations of the **While** loop would be closest to

A. -4

B. -0.5

C. 1.5

D. 2.906

E. 83.962

Question 18

The masses of packages of lean minced meat produced by a company are assumed to be normally distributed with a known mean mass of 250 grams and a standard deviation of 12 grams. n packages were chosen randomly and weighed, and a 95% confidence interval for the masses of the packages in grams was found to be $(246.1, 253.9)$. The value of n is closest to

A. 25

B. 26

C. 36

D. 37

E. 38

Question 19

The masses of packages of premium minced meat produced by a company are assumed to be normally distributed with a known mean mass of 500 grams and a standard deviation of 20 grams. A C% confidence interval for the masses of 36 packages of premium minced meat in grams was found to be $(494, 506)$. The value of C is closest to

A. 90

B. 93

C. 95

D. 97

E. 99

Question 20

A farm grows potatoes and carrots. The weights of the potatoes grown on the farm are normally distributed with a mean of 185 grams, with a standard deviation of 15 grams, and the weights of the carrots grown on the farm are normally distributed with a mean of 125 grams, with a standard deviation of 10 grams. Assuming that the weights of carrots and potatoes are independent of each other, then the probability that the difference between the weights of 3 carrots and 2 potatoes differ by less than 2 grams is closest to

A. 0.0288

B. 0.0573

C. 0.4564

D. 0.5436

E. 0.9427

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (12 marks)

Consider the family of functions f with the rule $f(x) = \dfrac{x^2 - c}{x^2 - 4x + c}$ where $c \in R$.

a. Write down the equations of all the straight line asymptotes on the graph of $y = f(x)$ when $c = 3$.

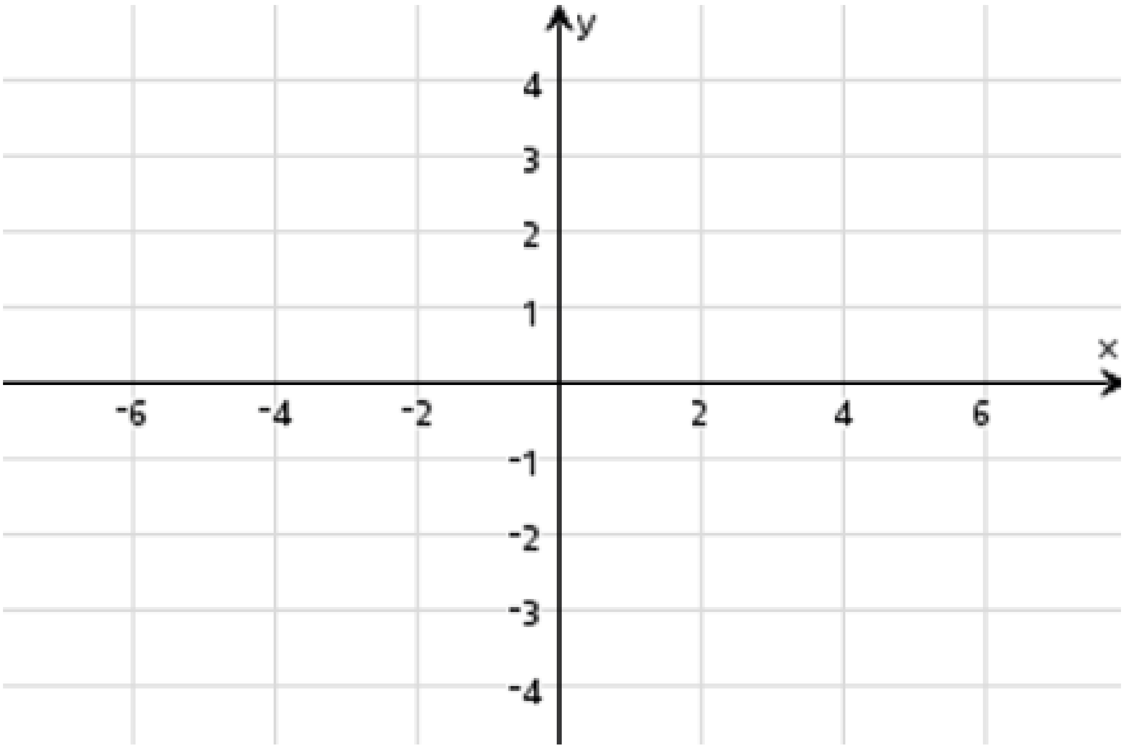
1 mark

b. When $c = 3$ show that the graph of $y = f(x)$ has no turning points.

1 mark

c. Sketch the graph of $y = f(x)$ when $c = 3$ on the set of axes below. Clearly label any asymptotes and the coordinates of all axial intercepts and intercepts with asymptotes if any. Label any inflexion points correct to two decimal places.

2 marks



d.i. Find the value(s) of c for which the graph of $y = f(x)$ has a point of discontinuity.

1 mark

ii. Considering if the graph of $y = f(x)$ crosses the x-axis, complete the table below

1 mark

The graph of $y = f(x)$	values of c
crosses the x-axis twice	
crosses the x-axis once	
does not cross the x-axis	

iii. Considering if the graph of $y = f(x)$, has any vertical asymptotes, complete the table below

1 mark

The graph of $y = f(x)$	values of c
has two vertical asymptotes	
has one vertical asymptote	
has no vertical asymptotes	

iv. Considering if the graph of $y = f(x)$, has any turning points, complete the table below

2 marks

The graph of $y = f(x)$	values of c
has no turning points	
has one maximum and one minimum turning point	

v. Considering if the graph of $y = f(x)$, has any points of inflexion, complete the table below

2 marks

The graph of $y = f(x)$	values of c
has no points of inflexion	
has one point of inflexion	
has two points of inflexion	
has three points of inflexion	

e. When the curve $f(x) = \dfrac{x^2 - c}{x^2 - 4x + c}$ is rotated about the x-axis, between $x = 5$ and $x = 6$ it forms a volume of revolution. The surface area obtained not including the ends, is 11.3836 units^2, determine the value of c correct to one decimal place.

1 mark

___

___

___

___

Question 2 (11 marks)

Let $u = 2 - i$

a. The cubic polynomial $P(z) = z^3 + vz^2 + 5z + w = 0$ where $v, w \in C$ has $P(u^3) = 0$ and the other two roots are purely imaginary. Find the values of v and w.

2 marks

b. The quartic polynomial $Q(z) = z^4 + az^3 + bz^2 + cz + d = 0$ where $a, b, c, d \in R$ has $Q(u) = 0$ and $Q(u^2) = 0$. Find the values of a, b, c and d.

2 marks

c. Let $C=\{z:|z-u|=2\sqrt{2}, z\in C\}$. Find and describe the Cartesian equation of C.

1 mark

d. Let $R=\{z:\text{Arg}(z-u)=\dfrac{3\pi}{4}, z\in C\}$. Express R in the form $y=mx+k$ where $m,k\in Z$, and describe the set R.

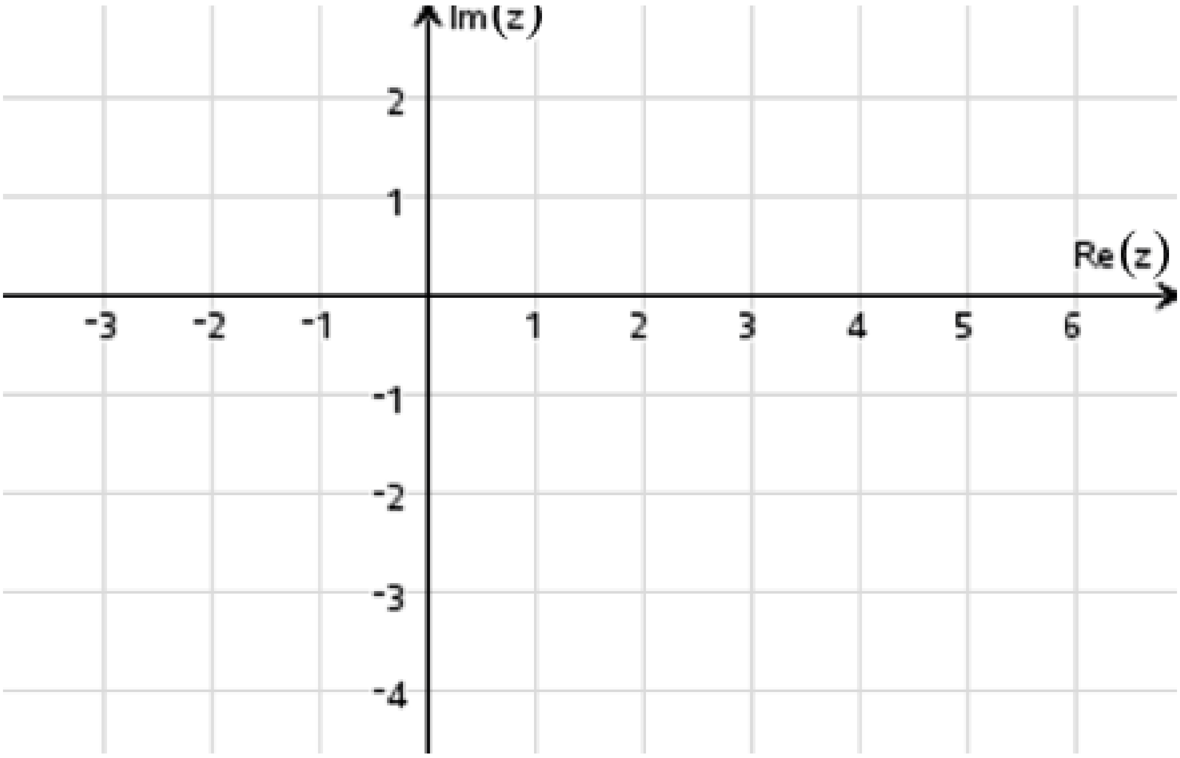
1 mark

e. Find the coordinate(s) of the points of intersection between C and R.

1 mark

f. On the Argand diagram below sketch and label the sets C and R.

2 marks



g. On the Argand diagram in **part f**. clearly shade the region defined by

$\{z : |z-u| \le 2\sqrt{2}, z \in C\} \cap \{z : \frac{\pi}{4} \le \operatorname{Arg}(z-u) \le \frac{3\pi}{4}, z \in C\} \cap \{z : \operatorname{Im}(z) \ge 1, z \in C\}$,

clearly showing all intersection points.

1 mark

h. Find the area of the shaded region in **part g**.

1 mark

Question 3 (11 marks)

Given the three points $A(4,5,-2)$ $B(6,2,-3)$ and $C(a,b,0)$ where $a,b \in Z$.

a. The points *ABC* form an equilateral triangle, write down two simultaneous equations for *a* and *b* and hence verify that $a=7$ and $b=4$.

2 marks

b. Find the equation of the plane containing the points *A*, *B* and *C* and hence determine the area of the triangle *ABC*.

2 marks

c. The points *D*, *E* and *F* are the midpoints of the sides, *AB*, *BC* and *AC* respectively, write down the coordinates of the points *D*, *E* and *F*.

1 mark

d. The point G lies on the plane and is such that $\overrightarrow{AG} = \frac{2}{3}\overrightarrow{AE}$, $\overrightarrow{BG} = \frac{2}{3}\overrightarrow{BF}$ and $\overrightarrow{CG} = \frac{2}{3}\overrightarrow{CD}$, find the coordinates of the point G.

1 mark

e. Determine the equation of the line L in parametric form which passes through the point G and is perpendicular to the plane containing the points A, B and C.

1 mark

f. A point V is such the distances $d\left(\overrightarrow{AV}\right) = d\left(\overrightarrow{BV}\right) = d\left(\overrightarrow{CV}\right) = \sqrt{61}$, determine the possible coordinates of the point V.

2 marks

g. Show that the point V lies on the line L.

1 mark

h. A pyramid has its base as the triangle ABC and its apex the top of the pyramid, at the point V, determine the volume of the pyramid.

1 mark

Question 4 (9 marks)

A motor-bike travels from rest along a long straight line road and accelerates for a time, then brakes and comes to rest. Its velocity v m/s at time t seconds is given by

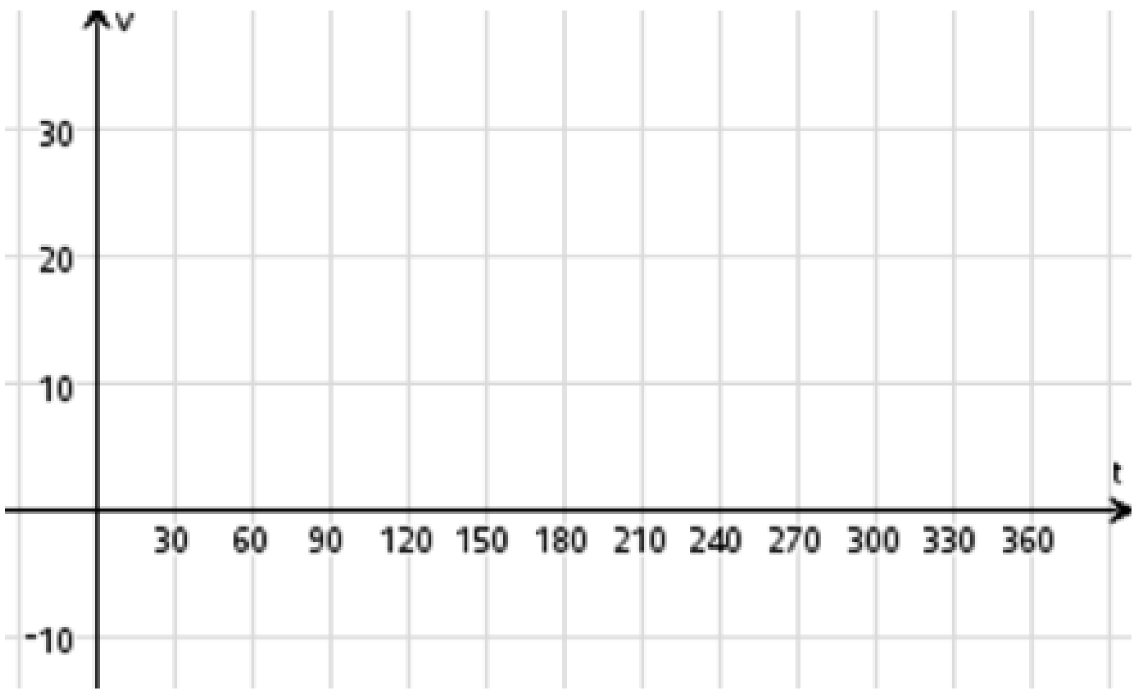
$$v(t)=\begin{cases}\dfrac{75}{\pi}\tan^{-1}\left(\dfrac{t}{120}\right) & 0\le t\le 120\\ a+bt & 120<t\le 240\\ \dfrac{50}{\pi}\cos^{-1}\left(\dfrac{t-240}{120}\right) & 240<t\le 360\end{cases}$$

a. Determine the values of a and b.

2 marks

b. Sketch the velocity-time graph on the axes below.

2 marks



c. Find the instantaneous acceleration after five minutes of motion.
Give your answer in $\text{m}\,\text{s}^{-2}$ correct to 3 decimal places.

1 mark

d. Find the distance travelled over the six minutes of the journey, give your answer correct to the nearest metre.

2 marks

e. Determine the percentage of the time during the journey for which the motor-bike is exceeding the speed limit of 60 km/hr. Given your answer correct to two decimal places.

2 marks

Question 5 (8 marks)

A javelin is thrown by a competitor on level ground. At a time t in seconds measured from the release of the javelin, the position vector $\underset{\sim}{r}(t)$ of the tip of the javelin is given by

$\underset{\sim}{r}(t) = 18t\,\underset{\sim}{i} + 3\sin\left(\frac{\pi t}{5}\right)\underset{\sim}{j} + \left(2 + 11.7t - 5t^2\right)\underset{\sim}{k}$ where $\underset{\sim}{i}$ is a unit vector in the forward direction, $\underset{\sim}{j}$ is a unit vector to the right and $\underset{\sim}{k}$ is a unit vector vertically up. The origin O of the coordinate system is at ground level and all displacements are measured in metres.

a. How long before the javelin strikes the ground?

1 mark

b. Find how far from O does the tip of the javelin hits the ground?
Give your answer in metres correct to one decimal place.

1 mark

c. Find the speed in $\text{m}\,\text{s}^{-1}$ and the acute angle in degrees correct to one decimal places at which the javelin's tip makes when it strikes the ground.

2 marks

At the instant the javelin was thrown a boy throws a ball, its acceleration is given by $\underset{\sim}{a}_B(t) = -9.8\underset{\sim}{k}$ and the initial velocity of the ball is given by $\underset{\sim}{v}_B = \frac{45}{4}\underset{\sim}{i} + \frac{3\sqrt{2}}{4}\underset{\sim}{j}$ and the initial position of the ball is $\underset{\sim}{r}_B = \frac{2273}{80}\underset{\sim}{k}$ where components are measured in metres.

d. Show that that position vector of the ball is given by

$$\underset{\sim}{r}(t) = \frac{45t}{4}\underset{\sim}{i} + \frac{3t\sqrt{2}}{4}\underset{\sim}{j} + \left(\frac{2273}{80} - 4.9t^2\right)\underset{\sim}{k}$$

2 marks

e. Does the ball collide with the javelin? Explain your answer.

2 marks

Question 6 (9 marks)

A company produces party pies and sausage rolls. Over a long time period of time the company has noted that the weight of a party pie is normally distributed with a mean of 47 grams, with a standard deviation of 3 grams and the weight of a sausage roll is normally distributed with a mean of 37.5 grams, with a standard deviation of 2 grams.

a. At a party Jared serves party pies and sausage rolls. The probability that the total weight of p party pies and s sausage rolls exceeding 1578 grams is 0.5. The percentage of the total weight of p party pies and s sausage rolls being less than 1560 grams is 13.397%. Assuming that the weights of party pies and sausage rolls are independent, determine how many party pies and sausage rolls Jared served at the party.

3 marks

Following a change in production of the party pies, the company is not sure if they are putting enough meat in their party pies and are concerned that they could be underweight. To check this they sampled 36 party pies and found that the mean mass of the sample of 46.3 grams, assume that the standard deviation is still 3 grams.

b. Write down suitable hypothesis H_0 and H_1 for this test.

1 mark

c. Determine the p value for the test, correct to four decimal places.

1 mark

d. Does the mean mass of this sample support the alternative hypothesis at the 5% level of significance? Justify your answer.

1 mark

e. What is the maximum value of the mean mass of 36 party pies that will support the alternative hypothesis, at the 5% level of significance.
Give your answer in grams correct to 2 decimal places.

1 mark

f. Following a change in production of the sausage rolls, it is found that the mean weights of the sausage rolls has been reduced to 36.5 grams with the standard deviation remaining the same. A statistical test to check whether there is any evidence of a 1.0 gram reduction in the mean mass of the sausage rolls is undertaken. The test statistic will be the mean weight of 36 sausage rolls. The test is conducted at the 5% level of significance. Determine the value of the Type II error (β) for this test, giving your answer as a percentage correct to one decimal place.

2 marks

END OF SECTION B

EXTRA WORKING SPACE

End of question and answer book for the
2023 Kilbaha VCE Specialist Mathematics Trial Examination 2

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SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics formulas

Mensuration

area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3}Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)

$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r\,\text{cis}(\theta)$	$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	
$-\pi < \text{Arg}(z) \le \pi$	$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$	
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$	de Moivre's theorem	$z^n = r^n \text{cis}(n\theta)$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \text{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x)$ $= 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$
$\sin^2(ax) = \frac{1}{2}(1 - \cos(2ax))$	$\cos^2(ax) = \frac{1}{2}(1 + \cos(2ax))$

Data analysis, probability and statistics

<table>
<tr><td rowspan="2">for independent random variables $X_1, X_2 \ldots X_n$</td><td colspan="2">$E(aX_1+b)=aE(X_1)+b$
$E(a_1X_1+a_2X_2+\ldots a_nX_n)$
$=a_1E(X_1)+a_2E(X_2)+\ldots+a_nE(X_n)$</td></tr>
<tr><td colspan="2">$\text{Var}(aX_1+b)=a^2\text{Var}(X_1)$
$\text{Var}(a_1X_1+a_2X_2+\ldots a_nX_n)$
$=a_1^2\text{Var}(X_1)+a_2^2\text{Var}(X_2)+\ldots+a_n^2\text{Var}(X_n)$</td></tr>
<tr><td rowspan="2">for independent identically distributed variables $X_1, X_2 \ldots X_n$</td><td colspan="2">$E(X_1+X_2+\ldots+X_n)=n\mu$</td></tr>
<tr><td colspan="2">$\text{Var}(X_1+X_2+\ldots+X_n)=n\sigma^2$</td></tr>
<tr><td>approximate confidence interval for μ</td><td colspan="2">$\left(\bar{x}-z\dfrac{s}{\sqrt{n}}\,,\,\bar{x}+z\dfrac{s}{\sqrt{n}}\right)$</td></tr>
<tr><td rowspan="2">distribution of sample mean $\bar{X}$</td><td>mean</td><td>$E(\bar{X})=\mu$</td></tr>
<tr><td>variance</td><td>$\text{Var}(\bar{X})=\dfrac{\sigma^2}{n}$</td></tr>
</table>

Vectors in two and three dimensions

<table>
<tr><td>$\underset{\sim}{r}(t)=x(t)\underset{\sim}{i}+y(t)\underset{\sim}{j}+z(t)\underset{\sim}{k}$</td><td>$\left|\underset{\sim}{r}(t)\right|=\sqrt{x(t)^2+y(t)^2+z(t)^2}$</td></tr>
<tr><td></td><td>$\dot{\underset{\sim}{r}}(t)=\dfrac{d\underset{\sim}{r}}{dt}=\dfrac{dx}{dt}\underset{\sim}{i}+\dfrac{dy}{dt}\underset{\sim}{j}+\dfrac{dz}{dt}\underset{\sim}{k}$</td></tr>
<tr><td rowspan="2">for $\underset{\sim}{r_1}=x_1\underset{\sim}{i}+y_1\underset{\sim}{j}+z_1\underset{\sim}{k}$
and $\underset{\sim}{r_2}=x_2\underset{\sim}{i}+y_2\underset{\sim}{j}+z_2\underset{\sim}{k}$</td><td>vector scalar product
$\underset{\sim}{r_1}\cdot\underset{\sim}{r_2}=\left|r_1\right|\left|r_2\right|\cos(\theta)=x_1x_2+y_1y_2+z_1z_2$</td></tr>
<tr><td>vector cross product
$\underset{\sim}{r_1}\times\underset{\sim}{r_2}=\begin{vmatrix}\underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k}\\ x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\end{vmatrix}=(y_1z_2-y_2z_1)\underset{\sim}{i}+(x_2z_1-x_1z_2)\underset{\sim}{j}+(x_1y_2-x_2y_1)\underset{\sim}{k}$</td></tr>
<tr><td>vector equation of a line</td><td>$\underset{\sim}{r}(t)=\underset{\sim}{r_1}+t\underset{\sim}{r_2}=(x_1+x_2t)\underset{\sim}{i}+(y_1+y_2t)\underset{\sim}{j}+(z_1+z_2t)\underset{\sim}{k}$</td></tr>
<tr><td>parametric equation of line</td><td>$x(t)=x_1+x_2t \quad y(t)=y_1+y_2t \quad z(t)=z_1+z_2t$</td></tr>
<tr><td>vector equation of a plane</td><td>$\underset{\sim}{r}(s,t)=\underset{\sim}{r_0}+s\underset{\sim}{r_1}+t\underset{\sim}{r_2}$
$=(x_0+x_1s+x_2t)\underset{\sim}{i}+(y_0+y_1s+y_2t)\underset{\sim}{j}+(z_0+z_1s+z_2t)\underset{\sim}{k}$</td></tr>
<tr><td>parametric equation of a plane</td><td>$x(s,t)=x_0+x_1s+x_2t \quad y(s,t)=y_0+y_1s+y_2t \quad z(s,t)=z_0+z_1s+z_2t$</td></tr>
<tr><td>Cartesian equation of a plane</td><td>$ax+by+cz=d$</td></tr>
</table>

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq-1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int\frac{1}{x}dx=\log_e\left(\lvert x\rvert\right)+c$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int\sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int\cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$	$\int\sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$
$\frac{d}{dx}\left(\cot(ax)\right)=-a\operatorname{cosec}^2(ax)$	$\int\operatorname{cosec}^2(ax)\,dx=-\frac{1}{a}\cot(ax)+c$
$\frac{d}{dx}\left(\sec(ax)\right)=a\sec(ax)\tan(ax)$	$\int\sec(ax)\tan(ax)\,dx=\frac{1}{a}\sec(ax)+c$
$\frac{d}{dx}\left(\operatorname{cosec}(ax)\right)=-a\operatorname{cosec}(ax)\cot(ax)$	$\int\operatorname{cosec}(ax)\cot(ax)\,dx=-\frac{1}{a}\operatorname{cosec}(ax)+c$
$\frac{d}{dx}\left(\sin^{-1}(ax)\right)=\frac{1}{\sqrt{1-(ax)^2}}$	$\int\frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}(ax)\right)=\frac{-1}{\sqrt{1-(ax)^2}}$	$\int\frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}(ax)\right)=\frac{a}{1+(ax)^2}$	$\int\frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int(ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq-1$
	$\int(ax+b)^{-1}\,dx=\frac{1}{a}\log_e\left(\lvert ax+b\rvert\right)+c$

Calculus- continued

product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}dx=uv-\int v\frac{du}{dx}dx$
Euler's method	If $\frac{dy}{dx}=f(x)$, $x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf\left(x_n,y_n\right)$
arc length parametric	$\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about the x-axis	$\int_{x_1}^{x_2}2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about the y-axis	$\int_{y_1}^{y_2}2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$
surface area parametric about the x-axis	$\int_{t_1}^{t_2}2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$
surface area parametric about the y-axis	$\int_{t_1}^{t_2}2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

Kinematics

acceleration	$a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v=u+at$	$s=ut+\frac{1}{2}t^2$
	$v^2=u^2+2as$	$s=\frac{1}{2}(u+v)t$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

									Letter
Figures									
Words									

SIGNATURE ______________________________

SECTION A

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
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