# VCE Specialist Mathematics Year 12 Trial Examination 1



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# Victorian Certificate of Education 2023

## STUDENT NUMBER

					L	etter
Figures						
Words						

# **SPECIALIST MATHEMATICS**

# **Trial Written Examination 1**

Reading time: 15 minutes Total writing time: 1 hour

# **QUESTION AND ANSWER BOOK**

## Structure of book

Number of questions	Number of questions to be answered	Number of marks
10	10 be answered	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or white out liquid/tape.

## Materials supplied

• Question and answer book of 22 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.

## **Instructions**

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

## **Instructions**

Answer all questions in the spaces provided.

Unless otherwise specified an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s<sup>2</sup>, where g = 9.8.

<b>Question 1</b> (4 marks)
Solve the equation $z^4 + 16 = 0$ , $z \in C$ , giving your answers in rectangular $a + bi$ form.

**Question 2** (3 marks)

Solve the differential equation	$\frac{dy}{dx} =$	$\frac{x\left(9+4y^2\right)}{6}$	given	$y(0) = \frac{3}{2}.$
Express your answer in the form	m y =	= f(x).		

**Question 3** (3 marks)

Let  $f: D \to R$ ,  $f(x) = \frac{x}{x^2 - 4}$ , where *D* is the maximal domain of *f*.

i. Find f'(x) and hence show that the graph of f has no turning points.

1 mark

ii. Given that  $f''(x) = \frac{2x(x^2 + 12)}{(x^2 - 4)^3}$  find the coordinates of any points of inflexion and

sketch the graph of  $f(x) = \frac{x}{x^2 - 4}$  on the axes below, labelling the equations of all asymptotes.

2 marks

-4 -3 -2 -1 1 2 3 4

## **Question 4** (3 marks)

A particle moves in a straight line. At a time t seconds, the particle has a displacement of x metres and a velocity of v ms<sup>-1</sup> and an acceleration a ms<sup>-2</sup>. Initially the particle is at the origin and has a velocity of 2 ms<sup>-1</sup>. If the acceleration  $a = -\frac{4}{3}e^{-\frac{2x}{3}}$  and the velocity of the particle is always positive.

positive.							
a.	Show that $v = 2e^{-\frac{x}{3}}$						
		1 mark					

b.	After three seconds, the particle has a displacement of $\log_e(s)$ , find the value of s.	
		2 marks


## **Question 5** (3 marks)

The weights of a piece of corn are normally distributed with a mean of 100 gm with a variance of 12 gm<sup>2</sup>. Z has the standard normal distribution and given that Pr(Z < 1.5) = 0.933.

a.	Determine the probability that the total weight of three independent pieces of corn v between 291 and 300 gms. Give your answer correct to three decimal places.				
	between 271 and 300 gms. Give your answer correct to time decimal places.	2 marks			
b.	An <b>approximate</b> 95% confidence interval for $n$ pieces of corn was found to be 99 t gm. Given that $Pr(-2 < Z < 2) = 0.95$ , where Z is a standard normal random varial Determine the value of $n$ .				
	Determine the value of m	1 mark			

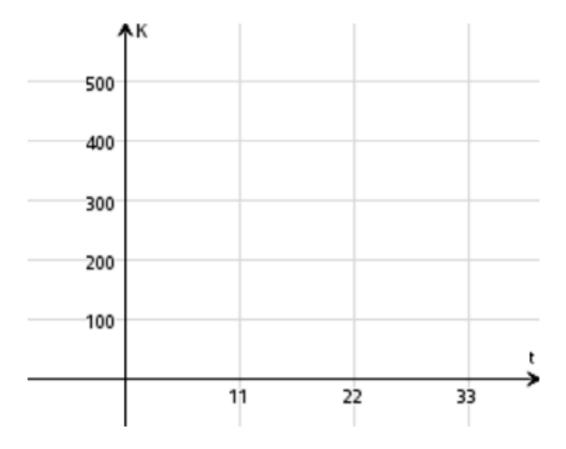
## **Question 6** (5 marks)

On a plantation in the year 2000 there were initially 50 kangaroos. The population growth of the number of kangaroos K on the plantation is given by  $\frac{dK}{dt} = \frac{K}{5} \left( 1 - \frac{K}{500} \right)$ , where t is the time in years after 2000.

a.	Solve the differential equation to find <i>K</i> in terms of <i>t</i> .		
	1	4 mark	

**b.** It is known that the number of kangaroos on the plantation was increasing most rapidly around the year 2011. Sketch the graph of *K* versus *t* on the axes below, clearly labelling the point of inflexion, the equations of any asymptotes and coordinates of any axial intercepts.

1 mark



$\Omega_{11}$	estion	7	(1	marks	a )
Qu	testion	. /	(4	marks	5)

When the area bounded by the curve the curve $y = \sqrt{3}x + 4$ the x-axis $x = 0$ and $x = 2$ is rotated				
about the <i>x</i> -axis it forms a volume of revolution, find the surface area of this volume of revolution.				

023 Kilbaha VCE Specialist Mathematics Irial Examination 1	Page 12
Question 8 (4 marks)	
rove by induction that $5^{2n} + 3n - 1$ is divisible by 9 for $n \in \mathbb{N}$ .	
Tove by induction that 3 1 3n 1 is divisible by 5 for n C11.	

**Question 9** (6 marks)

A plane has the vector equation  $\underline{r}(s,t) = (3+4t+3s)\underline{i} + (-2+t+2s)\underline{j} + (1+2t-s)\underline{k}$ 

a. Show that the equation of the plane can be written in the form x-2y-z=6.

2 marks

b.	One line has the vector equation $\underline{r}(t) = 2\underline{i} - \underline{j} + z_0\underline{k} + t(\underline{i} - 2\underline{j} + c\underline{k})$ , find the values of $z_0$
	and c for which the line and the plane do not intersect.

2 marks

c.	Another line has the equation $\frac{x-3}{2} = \frac{y+1}{b} = z-2$ , this line intersects the plane making an			
	angle of $30^{\circ}$ with the plane. Determine the value(s) of b.			
	6 · · · · · · · · · · · · · · · · · · ·	2 marks		

**Question 10** (5 marks)

Let  $I_n = \int_0^1 x^{2n+1} e^{x^2} dx$  for integers  $n \ge 0$ .

**a.** Show that  $I_0 = \frac{1}{2}(e-1)$ 

1 mark

**b.** Using integration by parts on  $I_n$ , show that  $I_{n+1} = \frac{e}{2} - (n+1)I_n$ 

3 marks

2023 Kilbaha VCE Specialist Mathematics Trial Examination 1	Page 16
c. Hence evaluate $I_3$ .	
	1 mark

EXTRA WORKING SPACE		

# End of question and answer book for the 2023 Kilbaha VCE Specialist Mathematics Trial Examination 1

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# **SPECIALIST MATHEMATICS**

# Written examination 1

# **FORMULA SHEET**

## **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

# **Specialist Mathematics formulas**

## Mensuration

area of a	$r^2$ (0. sin (0))	volume of	$\frac{4}{3}\pi r^3$
circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	a sphere	3"
volume of	$\pi r^2 h$	area of	$\frac{1}{2}bc\sin(A)$
a cylinder	$\pi r n$	a triangle	$2^{2^{\operatorname{csin}(T)}}$
volume of	$1_{\pi r^2 h}$	sine rule	a = b = c
a cone	$\int \frac{1}{3}\pi r^2 h$		$\sin(A)^{-}\sin(B)^{-}\sin(C)$
volume of	1	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$
a pyramid	$\frac{1}{3}Ah$		

# Algebra, number and structure ( complex numbers )

$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r\cos(\theta)$	$ z  = \sqrt{x^2 + y^2} = r$
$-\pi < \operatorname{Arg}(z) \le \pi$	$z_1 z_2 = r_1 r_2 \operatorname{cis} (\theta_1 + \theta_2)$
$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$	de Moivre's $z^n = r^n \operatorname{cis}(n\theta)$ theorem

# Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \csc^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x)$ = $2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$
$\sin^2(ax) = \frac{1}{2}(1-\cos(2ax))$	$\cos^2(ax) = \frac{1}{2}(1 + \cos(2ax))$

# Data analysis, probability and statistics

for independent random variables $X_1, X_2,, X_n$	$E(aX_1 + b) = aE(X_1)$ $E(a_1X_1 + a_2X_2 +a_n)$ $= a_1E(X_1) + a_2E(X_2)$ $Var(aX_1 + b) = a^2Va$ $Var(a_1X_1 + a_2X_2 +$ $= a_1^2Var(X_1) + a_2^2Va$	$(X_n)$ $(X_n) + + a_n E(X_n)$ $(X_1)$
for independent identically distributed variables	$E(X_1 + X_2 + \dots + X_n)$	,
$X_1, X_2 X_n$ approximate confidence interval for $\mu$	$Var(X_1 + X_2 + + X_n)$ $(\overline{x} - z \frac{s}{\sqrt{n}}, \overline{x} + z \frac{s}{\sqrt{r}})$	```
distribution of sample mean $\overline{X}$	mean	$E(\bar{X}) = \mu$
	variance	$\operatorname{Var}\left(\bar{X}\right) = \frac{\sigma^2}{n}$

# **Vectors in two and three dimensions**

$\underline{r}(t) = x(t)\underline{i} + y(t)\underline{j} + z(t)\underline{k}$	$\left \underline{r}(t)\right  = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\underline{r}}(t) = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\dot{\underline{i}} + \frac{dy}{dt}\dot{\underline{j}} + \frac{dz}{dt}\dot{\underline{k}}$
for $r_1 = x_1 \underline{i} + y_1 \underline{j} + z_1 \underline{k}$	vector scalar product
and $r_2 = x_2 i + y_2 j + z_2 k$	$r_1 \cdot r_2 =  r_1  r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$
22 22 22	vector cross product
	$ \vec{r}_{1} \times \vec{r}_{2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_{1} & y_{1} & z_{1} \end{vmatrix} = (y_{1}z_{2} - y_{2}z_{1})\underline{i} + (x_{2}z_{1} - x_{1}z_{2})\underline{j} + (x_{1}y_{2} - x_{2}y_{1})\underline{k} $
	$\begin{vmatrix} x_2 & y_2 & z_2 \end{vmatrix}$
vector equation of a line	$\underline{r}(t) = \underline{r}_1 + t\underline{r}_2 = (x_1 + x_2 t)\underline{i} + (y_1 + y_2 t)\underline{j} + (z_1 + z_2 t)\underline{k}$
parametric equation of line	$x(t) = x_1 + x_2t$ $y(t) = y_1 + y_2t$ $z(t) = z_1 + z_2t$
vector equation of a plane	$\underline{r}(s,t) = \underline{r}_0 + s\underline{r}_1 + t\underline{r}_2$
	$= (x_0 + x_1 s + x_2 t) \underline{i} + (y_0 + y_1 s + y_2 t) \underline{j} + (z_0 + z_1 s + z_2 t) \underline{k}$
parametric equation of a plane	$x(s,t) = x_0 + x_1 s + x_2 t$ $y(s,t) = y_0 + y_1 s + y_2 t$ $z(s,t) = z_0 + z_1 s + z_2 t$
Cartesian equation of a plane	ax + by + cz = d

# **Calculus**

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c \ , \ n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_{e}(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e( x ) + c$
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a\sec^2(ax)$	$\int \sec^2(ax)dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}(\cot(ax)) = -a\csc^2(ax)$	$\int \csc^2(ax) dx = -\frac{1}{a}\cot(ax) + c$
$\frac{d}{dx}(\sec(ax)) = a\sec(ax)\tan(ax)$	$\int \sec(ax)\tan(ax)dx = \frac{1}{a}\sec(ax) + c$
$\frac{d}{dx}(\csc(ax)) = -a\csc(ax)\cot(ax)$	$\int \csc(ax)\cot(ax)dx = -\frac{1}{a}\csc(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(ax)\right) = \frac{1}{\sqrt{1-(ax)^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a}\right) + c, \ a > 0$
$\frac{d}{dx}\left(\cos^{-1}\left(ax\right)\right) = \frac{-1}{\sqrt{1-\left(ax\right)^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1} \left(\frac{x}{a}\right) + c, \ a > 0$
$\frac{d}{dx}\left(\tan^{-1}\left(ax\right)\right) = \frac{a}{1+\left(ax\right)^{2}}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1} \left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c, \ n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a} \log_e ( ax+b ) + c$

# **Calculus- continued**

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
integration by parts	$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$
Euler's method	If $\frac{dy}{dx} = f(x)$ , $x_0 = a$ and $y_0 = b$ ,
arc length parametric	then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n, y_n)$ $\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
surface area Cartesian about the <i>x</i> -axis	$\int_{x_1}^{x_2} 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}  dx$
surface area Cartesian about the <i>y</i> -axis	$\int_{y_1}^{y_2} 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}  dy$
surface area parametric about the <i>x</i> -axis	$\int_{t_1}^{t_2} 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
surface area parametric about the <i>y</i> -axis	$\int_{t_1}^{t_2} 2\pi x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$

## **Kinematics**

acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$	$= v \frac{dv}{dx} = \frac{d}{dx} \left( \frac{1}{2} v^2 \right)$	
constant acceleration formulas	v = u + at	$s = ut + \frac{1}{2}t^2$	
Tommanas	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u+v)t$	

## **END OF FORMULA SHEET**