VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2022

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | Letter | |

SPECIALIST MATHEMATICS

Written examination 1

Tuesday 31 May 2022

Reading time: 2.00 pm to 2.15 pm (15 minutes)
Writing time: 2.15 pm to 3.15 pm (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (3 marks)

Consider the relation $x^2e^{y-1} + 4ye^x = 9e$.

Find $\frac{dy}{dx}$ at the point (1, 2).

Question 2 (3 marks)

Given the complex numbers $z = 1 - \sqrt{3}i$ and $w = -2 - 2i$, find $\frac{z^3}{w^2}$ in cartesian form.

DO NOT WRITE IN THIS AREA

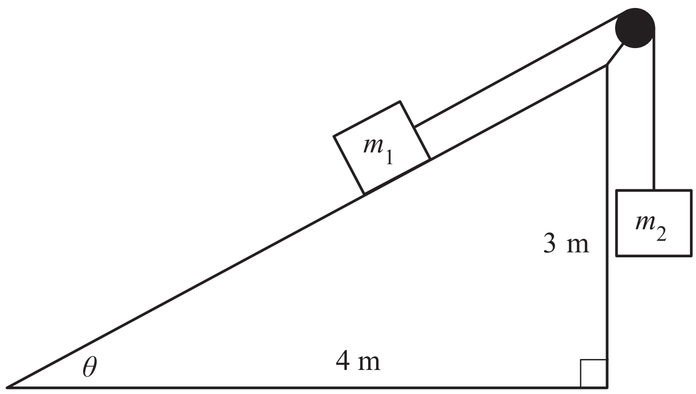
Question 3 (3 marks)

The region enclosed by the graph of $y = x^2 - 2x + 1$ and the straight line that passes through the x and y intercepts of this parabola is rotated about the x-axis to form a solid of revolution.

Find the volume of this solid of revolution.

Question 4 (5 marks)

A smooth plane is inclined at an angle θ to the horizontal. Two particles of mass m_1 kilograms and m_2 kilograms are connected by a light inextensible string that passes over a smooth pulley, as shown in the diagram below. The particle of mass m_1 kilograms lies on the plane.



a. Given that the system is in equilibrium, express m_2 in terms of m_1. 2 marks

b. The string is cut and the particle of mass m_1 kilograms starts to slide down the plane.

Find how far, in metres, the particle of mass m_1 kilograms has slid down the plane from its initial position when the particle has a velocity of 6 ms^{-1}. 3 marks

Question 5 (3 marks)

Find the centre and the radius of the circle defined by $3|z+i|=|z|$, where $z \in C$.

Question 6 (3 marks)

Relative to a fixed origin, the position of a 2 kg mass after t seconds is given by $\underset{\sim}{r}(t) = 32\sqrt{t}\,\underset{\sim}{i} + 6t^2\,\underset{\sim}{j} - 3e^{2t-8}\,\underset{\sim}{k}$, $t \geq 0$, where components are measured in metres.

Find the magnitude of the resultant force, in newtons, after it has acted on the mass for four seconds, giving your answer as an integer.

Question 7 (4 marks)

The path of a moving particle after t seconds is given by $\underset{\sim}{r}(t) = 4\sec(t)\underset{\sim}{i} + 2\tan(t)\underset{\sim}{j}$ for $t \in \left[0, \frac{\pi}{2}\right)$, where components are measured in metres.

a. Find the cartesian equation of the path. 2 marks

b. Find the speed of the particle when $t = \frac{\pi}{6}$. 2 marks

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Question 8 (6 marks)

A printer can use three types of ink cartridges: Standard, Deluxe and Elite. The Standard cartridge has a mean print run (the average number of pages that can be printed before all of the ink has been used up) of 2000 pages with a standard deviation of 40 pages. The mean print run of the Deluxe cartridge is 2500 pages with a standard deviation of 30 pages. The mean print run of the Elite cartridge is μ_E pages with a standard deviation of σ_E pages. The print runs of each type of cartridge are normally distributed and independent.

a. Find the expected print run and the standard deviation when one Standard cartridge and one Deluxe cartridge are used in succession (that is, one is used after the other is finished). 2 marks

b. One of each of the three types of cartridges is used in succession.

For what values of σ_E is the standard deviation of the print run for this situation less than or equal to 60? 2 marks

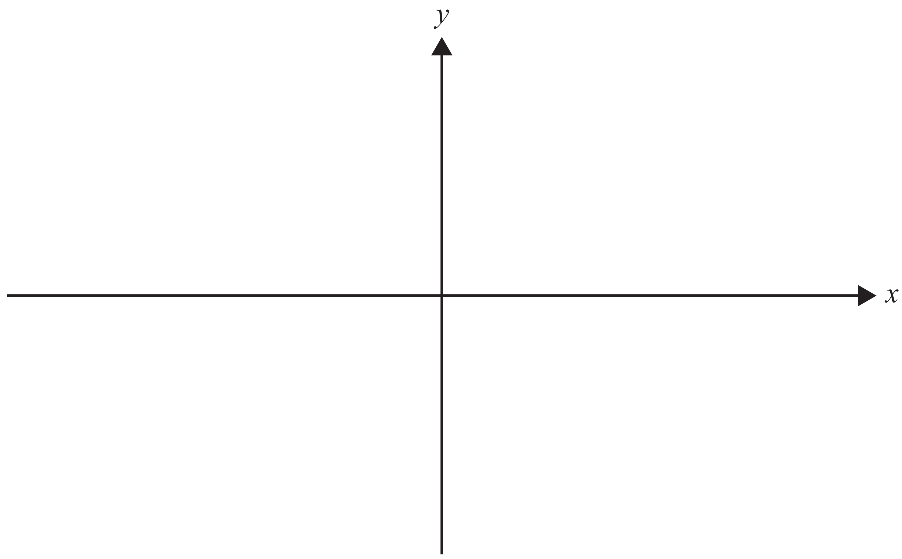
Question 8 – continued

c. A person knows that the standard deviation of the print run of the Standard cartridge is 40 but this person does not know the mean. A random sample of n Standard cartridges is selected and these cartridges are used in the printer in succession. The 95% confidence interval, using z correct to two decimal places, for the mean print run based on this sample is (1960.4, 1999.6).

Find the value of n. 2 marks

Question 9 (5 marks)

a. Given that there is a local minimum turning point at $x = 0$, sketch the graph of $y = \dfrac{9}{(x+1)(x-2)^2}$ on the axes provided below. Label any asymptotes with their equations and axes intercepts with their coordinates. 2 marks



b. Find the area bounded by the graph of $y = \dfrac{9}{(x+1)(x-2)^2}$, the coordinate axes and the line $x = 1$. 3 marks

DO NOT WRITE IN THIS AREA

Question 10 (5 marks)

a. Consider $f:\left[0, \dfrac{3}{2}\right] \to R,\ f(x) = \dfrac{\sqrt{3}}{2}\left(3\arcsin\left(\dfrac{\sqrt{3}x}{3}\right) + x\sqrt{3-x^2}\right)$.

Show that $f'(x) = \sqrt{9-3x^2}$. 2 marks

b. A particle moves along a curve such that its position vector at time t is

$$\underset{\sim}{r}(t) = \frac{t^3}{6}\underset{\sim}{i} + \frac{\sqrt{3}}{2}\left(3\arcsin\left(\frac{\sqrt{3}t}{3}\right) + t\sqrt{3-t^2}\right)\underset{\sim}{j}.$$

Find the distance that the particle travels along the curve from $t = 0$ to $t = \dfrac{3}{2}$. 3 marks

END OF QUESTION AND ANSWER BOOK

VICTORIA
State
Government

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2022

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Circular functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\mathrm{cosec}^2(x)$
$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$	$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$	$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\operatorname{cis}(\theta)$	
$\lvert z \rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \operatorname{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}\left(\theta_1 - \theta_2\right)$
$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$ $\mathrm{E}(aX + bY) = a\mathrm{E}(X) + b\mathrm{E}(Y)$ $\operatorname{var}(aX + b) = a^2\operatorname{var}(X)$
for independent random variables X and Y	$\operatorname{var}(aX + bY) = a^2\operatorname{var}(X) + b^2\operatorname{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\overline{X}$	mean $\mathrm{E}\left(\overline{X}\right) = \mu$ variance $\operatorname{var}\left(\overline{X}\right) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\lvert x\rvert+c$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$	$\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right)=\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right)=\frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right)=\frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$
	$\int (ax+b)^{-1} dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx}=f(x)$, $x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf(x_n)$
acceleration	$a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2}\sqrt{1+\left(f'(x)\right)^2}\,dx$ or $\int_{t_1}^{t_2}\sqrt{\left(x'(t)\right)^2+\left(y'(t)\right)^2}\,dt$

Vectors in two and three dimensions

$\underset{\sim}{r}=x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert=\sqrt{x^2+y^2+z^2}=r$
$\dot{\underset{\sim}{r}}=\frac{d\underset{\sim}{r}}{dt}=\frac{dx}{dt}\underset{\sim}{i}+\frac{dy}{dt}\underset{\sim}{j}+\frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2=r_1r_2\cos(\theta)=x_1x_2+y_1y_2+z_1z_2$

Mechanics

momentum	$\underset{\sim}{p}=m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R}=m\underset{\sim}{a}$

END OF FORMULA SHEET