Neap

Trial Examination 2022

VCE Specialist Mathematics Units 3&4

Written Examination 2

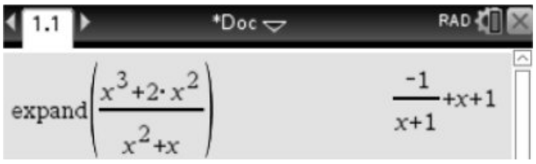
Suggested Solutions

SECTION A – MULTIPLE-CHOICE QUESTIONS

1	A	B	C	**D**	E
2	A	B	C	D	**E**
3	A	B	**C**	D	E
4	**A**	B	C	D	E
5	A	B	**C**	D	E
6	A	**B**	C	D	E
7	A	**B**	C	D	E
8	A	B	**C**	D	E
9	A	B	**C**	D	E
10	A	B	**C**	D	E

11	A	B	C	**D**	E
12	A	**B**	C	D	E
13	**A**	B	C	D	E
14	A	B	C	D	**E**
15	**A**	B	C	D	E
16	A	**B**	C	D	E
17	A	B	**C**	D	E
18	**A**	B	C	D	E
19	A	B	C	**D**	E
20	**A**	B	C	D	E

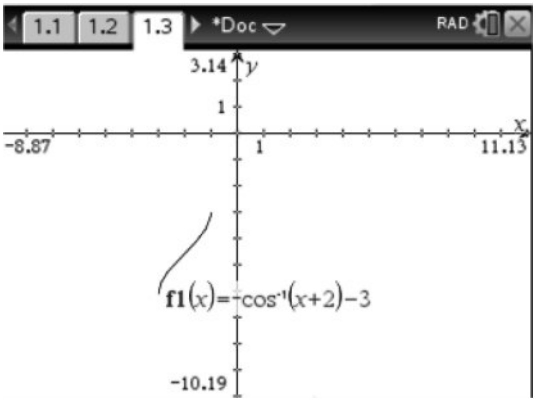
Question 1 D



The graph has one non-vertical asymptote at $y = x + 1$ and one vertical asymptote at $x = -1$.

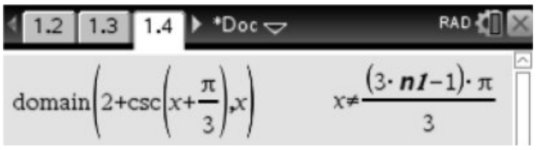
Note: The graph has a discontinuity, but not an asymptote, at $x = 0$.

Question 2 E



$$-1 \le x - 2a \le 1$$
$$2a - 1 \le x \le 2a + 1$$
$$-3 \le x \le -1 \Rightarrow a = -1$$
$$\therefore f(x) = -\cos^{-1}(x+2) - 3$$
$$R_f = [-\pi - 3, -0 - 3] = [-\pi - 3, -3]$$

Question 3 C



$R \setminus \left\{\frac{(3n-1)\pi}{3}\right\} = R \setminus \left\{n\pi - \frac{\pi}{3}\right\}$ is the same as $R \setminus \left\{n\pi - \frac{\pi}{3} + \pi\right\}$.

Since each number is half a revolution away from the next number:

$$\therefore R \setminus \left\{n\pi + \frac{2\pi}{3}\right\} = R \setminus \left\{\frac{(3n+2)\pi}{3}\right\}, n \in Z$$

Note: None of the options show the answer in the form produced by CAS, and **A** *is incorrect since the domain is the complementary set.*

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Question 4 A

A is correct.

$z = a + bi$

$\text{Re}(z+1) = a+1$

$\text{Im}(\bar{z}) = -b$

$\therefore a+1 = -b$

Since $a+1=3$ and $-b=3$:

$\therefore z = 2-3i$

B is incorrect. This option represents $a+1=2$ and $-b=-1$.

C is incorrect. This option represents $a+1=4$ and $-b=-4$.

D is incorrect. This option represents $a+1=4$ and $-b=2$.

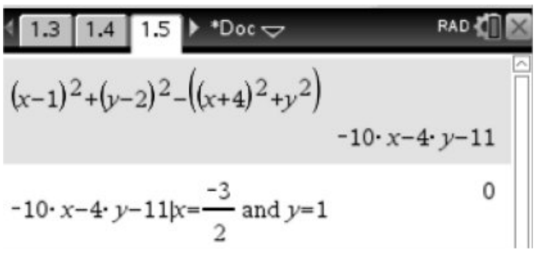
E is incorrect. This option represents $a+1=2$ and $-b=-2$.

Question 5 C

C is correct.

$z = x + yi$

$\sqrt{(x-1)^2+(y-2)^2} = \sqrt{(x+4)^2+y^2}$



The point $\left(\frac{-3}{2}, 1\right)$ satisfies $-10x-4y-11=0$.

A, **B**, **D** and **E** are incorrect. These options do not satisfy $-10x-4y-11=0$.

Question 6 B

$$\frac{z^{2k+1}}{w^k} = \frac{\left((-1+i)^2\right)^k(-1+i)}{i^k}$$

$$= \frac{(-2i)^k(-1+i)}{i^k}$$

$$= (-2)^k(-1+i)$$

$$= -\left(2^k\right)(-1+i) \quad \text{(since } k \text{ is odd)}$$

$$= \left(2^k\right)(1-i)$$

Note: The valid answer should satisfy $-\pi < \text{Arg} \leq \pi$.

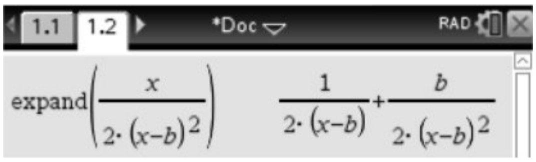
Question 7 B

B is correct. The height between point A and side BC is $\underset{\sim}{c}+\frac{1}{2}\underset{\sim}{a}$. Height is perpendicular to the side; hence, the scalar product is 0.

A, **C** and **D** are incorrect as $\underset{\sim}{a}-\underset{\sim}{b}+\underset{\sim}{c}=\underset{\sim}{0}$.

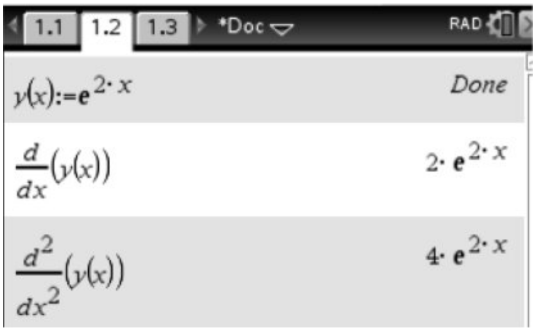
E is incorrect as $|2\underset{\sim}{b}|=|\underset{\sim}{b}|+|\underset{\sim}{c}|\neq|\underset{\sim}{b}+\underset{\sim}{c}|$.

Question 8 C



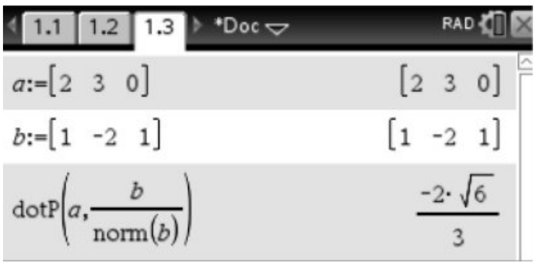
Note that $A=\frac{1}{2}$ and $B=\frac{b}{2}$.

Question 9 C



It can be verified that $2\frac{d^2y}{dx^2}+3\frac{dy}{dx}-14y=8e^{2x}+6e^{2x}-14e^{2x}=0$.

Question 10 C



Note: The correct answer is the value of $\underset{\sim}{a}\cdot\hat{\underset{\sim}{b}}$.

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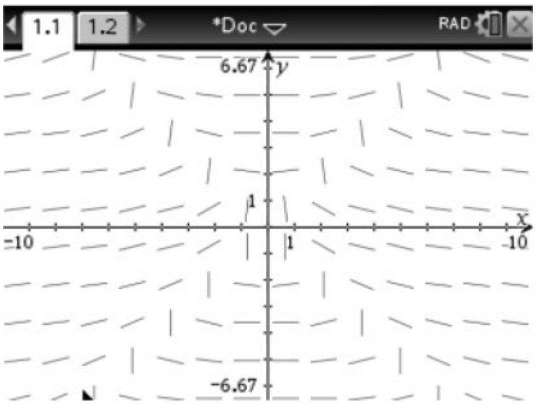
Question 11 D

D is correct, and **B** and **C** are incorrect. When $y > x$ in the first quadrant, the slopes are always positive. Among options **B**, **C** and **D**, option **C** does not satisfy this requirement. When $y > x$ in the second quadrant, the slopes are always negative. Option **D** satisfies this requirement. This can be verified using CAS.



A is incorrect. For $y = x$ or $y = -x$, $\frac{dy}{dx}$ is undefined.

E is incorrect. For $x = 0$, $\frac{dy}{dx} = 0$.

Question 12 B

Let $u = 2x - 1$.

$2x = u + 1 \Rightarrow 4x + 1 = 2(u+1) + 1 = 2u + 3$

$du = 2dx \Rightarrow dx = \frac{1}{2}du$

$$\int \frac{4x+1}{\sqrt{2x-1}}dx = \frac{1}{2}\int \frac{2u+3}{\sqrt{u}}du$$

Question 13 A

$f(x) = \cos\left(e^x\right)$

$h = 0.1$

$x_{n+1} = x_n + h,\ y_{n+1} = y_n + hf(x_n)$

$x_0 = 0,\ y_0 = 1 \Rightarrow y_1 = y_0 + 0.1\left(\cos\left(e^0\right)\right) = 1 + 0.1(\cos(1))$

$x_1 = 0.1 \Rightarrow y_2 = 1 + 0.1\cos(1) + 0.1\cos\left(e^{0.1}\right)$

$$\begin{aligned} x_2 = 0.2 \Rightarrow y_3 &= 1 + 0.1\cos(1) + 0.1\cos\left(e^{0.1}\right) + 0.1\cos\left(e^{0.2}\right) \\ &= 1 + 0.1\left(\cos(1) + \cos\left(e^{0.1}\right) + \cos\left(e^{0.2}\right)\right) \end{aligned}$$

Question 14 E

$$110 \times \cos(60°) - 10 = 3a$$

$$a = 15 \text{ ms}^{-2}$$

$$\begin{aligned} v &= u + at \\ &= 0 + 15 \times 6 \\ &= 90 \text{ ms}^{-1} \end{aligned}$$

$$\begin{aligned} \underset{\sim}{p} &= m\underset{\sim}{v} \\ &= 3 \times 90 \\ &= 270 \text{ kg ms}^{-1} \end{aligned}$$

Question 15 A

The system will move in the direction of the heavier mass.

Let the tension be T. The equations of motion for each mass will be:

$$20g - T = 20a$$

$$T - 12g = 12a$$

Adding the equations side by side gives:

$$8g = 32a$$

$$\begin{aligned} a &= \frac{g}{4} \\ &= \frac{9.8}{4} \\ &= 2.45 \text{ ms}^{-2} \end{aligned}$$

Question 16 B

The magnitudes of three of the forces are known. Their vector sum is:

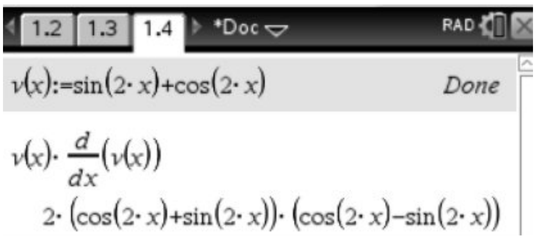
$$4\sqrt{2}\cos(45°)\underset{\sim}{i} + 4\sqrt{2}\sin(45°)\underset{\sim}{j} + 5\underset{\sim}{i} - 3\underset{\sim}{j} = 9\underset{\sim}{i} + \underset{\sim}{j}$$

Since the system is in equilibrium:

$$\underset{\sim}{F} = -\left(9\underset{\sim}{i} + \underset{\sim}{j}\right)$$

$$\begin{aligned} F &= \left|\underset{\sim}{F}\right| \\ &= \sqrt{9^2 + 1^2} \\ &= \sqrt{82} \end{aligned}$$

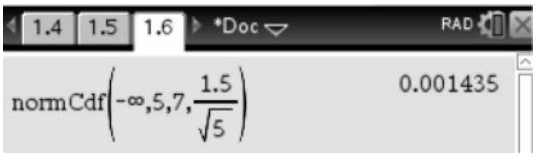
Question 17 C



$$
\begin{aligned}
F &= ma \\
&= m \times v\frac{dv}{dx} \\
&= 2 \times 2\left(\cos^2(2x) - \sin^2(2x)\right) \\
&= 4\cos(4x)
\end{aligned}
$$

$F_{\max} = 4$ N $\left(\text{since } \cos(4x) \le 1\right)$

Question 18 A

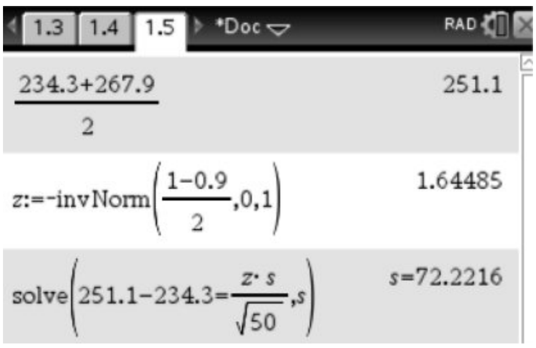


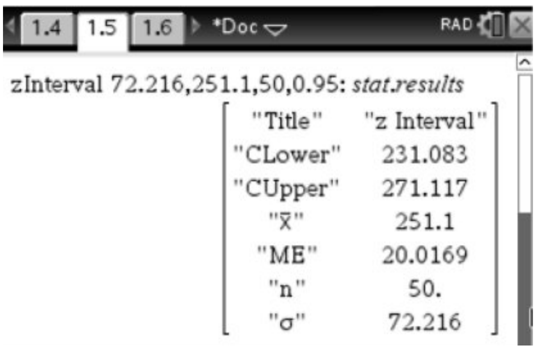
Question 19 D

A type II error is accepting the null hypothesis when it is false. In this context, a type II error would be stating that the kettle boils water in 90 seconds when it actually does not boil water in 90 seconds.

Question 20 A

$$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right) = (234.3,\ 267.9)$$





SECTION B

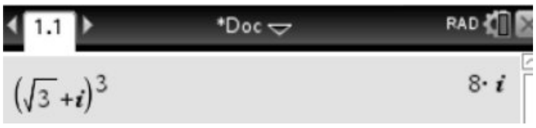
Question 1 (14 marks)

a. $u = 2\,\text{cis}\left(\dfrac{\pi}{6}\right)$ A1

$v = 2\,\text{cis}\left(\dfrac{5\pi}{6}\right)$ A1

$w = 2\,\text{cis}\left(-\dfrac{\pi}{2}\right)$ A1

b.

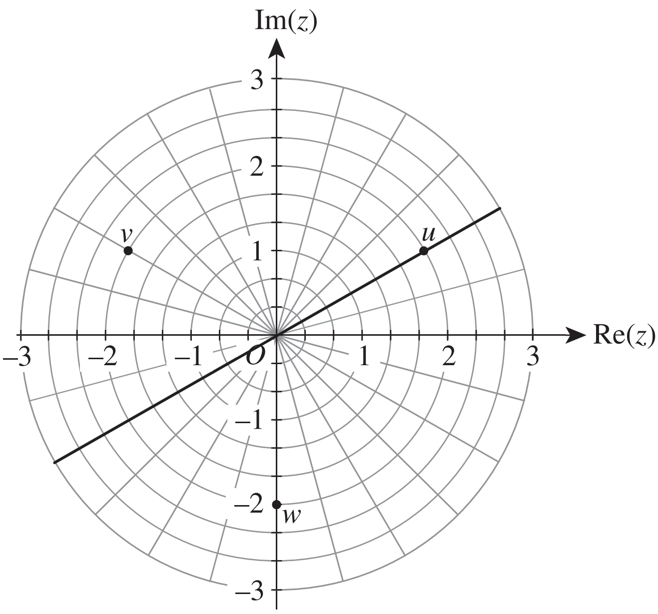


$k = 8i$ A1

c. **i.** Let $z = x + yi$.

$\sqrt{3}y - x = 0$ A1

$y = \dfrac{1}{\sqrt{3}}x$ is a line that makes an angle of $\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right) = 30°$ with the positive x-axis.



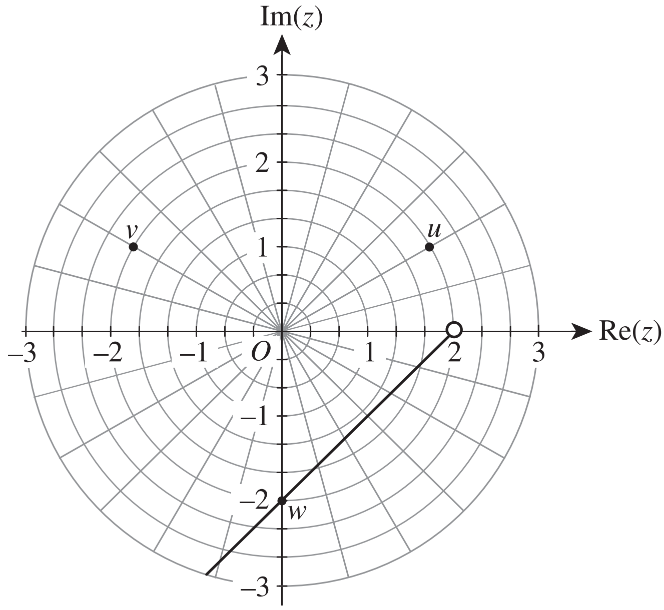
correct line A1

ii. The line is the perpendicular bisector of v and w.

$\therefore z_1 = 0 - 2i$ or $z_1 = -2i$ A1

$z_2 = -\sqrt{3} + i$ A1

iii.



correct ray A1

passing through w and excluding (2, 0) A1

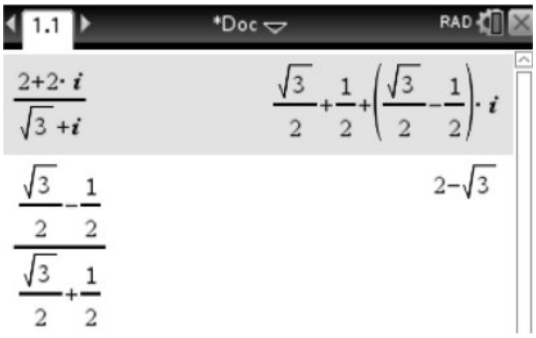
d. **i.** $2 + 2i = 2\operatorname{cis}\left(\frac{\pi}{4}\right)$

$u = 2\operatorname{cis}\left(\frac{\pi}{6}\right)$

$\frac{2+2i}{u} = \frac{2}{2}\operatorname{cis}\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$ M1

$= \operatorname{cis}\left(\frac{\pi}{12}\right)$ A1

ii.



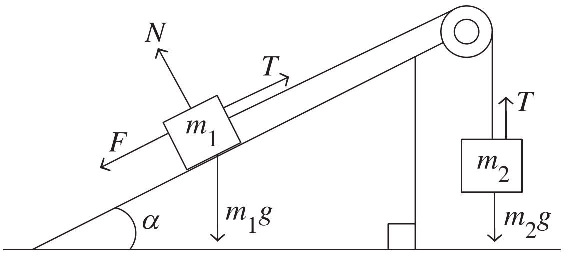
$$\frac{2+2i}{u}=\frac{\sqrt{3}+1}{2}+\left(\frac{\sqrt{3}-1}{2}\right)i$$ M1

$$\text{Arg}\left(\frac{2+2i}{u}\right)=\frac{\pi}{12}$$

$$\tan\left(\frac{\pi}{12}\right)=\frac{\frac{\sqrt{3}-1}{2}}{\frac{\sqrt{3}+1}{2}}=2-\sqrt{3}$$ A1

Question 2 (15 marks)

a. **i.**



first pair of correct forces A1
second pair of correct forces A1
third pair of correct forces A1

ii. mass m_1: $T-F-m_1g\sin\alpha=m_1a$ A1

mass m_2: $m_2g-T=m_2a$ A1

Adding the equations side by side gives:

$$-F-m_1g\sin\alpha+m_2g=m_1a+m_2a$$

$$-F+g\left(m_2-m_1\sin\alpha\right)=a\left(m_1+m_2\right)$$ M1

$$\therefore F=g\left(m_2-m_1\sin\alpha\right)-\left(m_1+m_2\right)a$$

b. If the system is in equilibrium, $F=0$ and $a=0$. M1

$$g\left(m_2-m_1\sin\alpha\right)=0$$

$$\sin\alpha=\frac{m_2}{m_1}$$ A1

c. From **part a.ii.**:

$F = g(m_2 - m_1 \sin\alpha) - (m_1 + m_2)a$

Substituting $F = \frac{m_1}{5}$ and $m_2 = \frac{m_1}{2}$ gives:

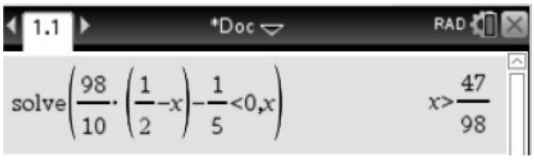
$\frac{m_1}{5} = g\left(\frac{m_1}{2} - m_1 \sin\alpha\right) - \left(m_1 + \frac{m_1}{2}\right)a$ M1

$\frac{1}{5} = g\left(\frac{1}{2} - \sin\alpha\right) - \frac{3}{2}a$

$\frac{3}{2}a = g\left(\frac{1}{2} - \sin\alpha\right) - \frac{1}{5}$

$g\left(\frac{1}{2} - \sin\alpha\right) - \frac{1}{5} < 0 \quad (\text{since } a < 0 \ (\text{object with mass } m_1 \text{ falls}))$ M1

$\sin\alpha > \frac{47}{98}$ A1

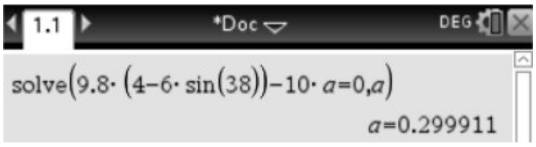


d. **i.** $0 = g(m_2 - m_1 \sin\alpha) - (m_1 + m_2)a$

$0 = 9.8(4 - 6\sin(38°)) - 10a$ M1

$a = 0.2999\ldots > 0$

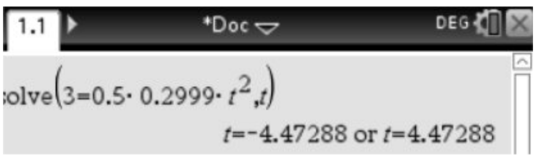
Therefore, the object with mass m_2 will fall to the floor. A1



ii. $s = ut + \frac{1}{2}at^2$

$3 = 0 + \frac{1}{2}(0.2999\ldots)t^2$ M1

$t = 4.47$ seconds A1



Note: Consequential on answer to ***Question 2d.i.***

Question 3 (9 marks)

a. $\frac{dx}{dt} = \text{inflow} - \text{outflow}$

$= 10 \times 2 - \frac{4x}{30 + (2-4)t}$ M1

$= \frac{2x}{t-15} + 20$

b. **i.** $x = \frac{4}{9}(t-60)(t-15)$

$= \frac{4}{9}\left(t^2 - 75t + 900\right)$

$\frac{dx}{dt} = \frac{4}{9}\left(2t - 75\right)$

$= \frac{8}{9}t - \frac{100}{3}$ M1

$\frac{2x}{t-15} + 20 = \frac{\frac{8}{9}(t-60)(t-15)}{(t-15)} + 20$

$= \frac{8}{9}(t-60) + 20$

$= \frac{8}{9}t - \frac{100}{3}$ M1

$\therefore \frac{dx}{dt} = \frac{2x}{t-15} + 20$

$t = 0 \Rightarrow x = \frac{4}{9} \times 60 \times 15 = 400$ M1

Therefore, $x = \frac{4}{9}(t-60)(t-15)$ satisfies the differential equation.

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ii. $x \geq 0$

$0 \leq t < 15$ A1

Note: t = 15 does not satisfy $\frac{dx}{dt} = \frac{2x}{t-15} + 20.$

c. Differentiating both sides of the equation $\frac{dx}{dt} = \frac{2x}{t-15} + 20$ with respect to t gives:

$$\frac{d^2x}{dt^2} = \frac{2\frac{dx}{dt}(t-15) - 2x \times 1}{(t-15)^2}$$ M1

$$= \frac{2\left(\frac{2x}{t-15} + 20\right)(t-15) - 2x}{(t-15)^2}$$ M1

$$= \frac{2\big(2x + 20(t-15)\big) - 2x}{(t-15)^2}$$

$$= \frac{4x + 40(t-15) - 2x}{(t-15)^2}$$

$$= \frac{2x + 40(t-15)}{(t-15)^2}$$

$$= \frac{\frac{8}{9}(t-60)(t-15) + 40(t-15)}{(t-15)^2}$$ M1

$$= \frac{8(t-60) + 360}{9(t-15)}$$ M1

$$= \frac{8t-120}{9t-135}$$

Question 4 (13 marks)

a. $3x^2 + 3y^2\dfrac{dy}{dx} = 4y + 4x\dfrac{dy}{dx}$ M1

$$\frac{dy}{dx} = \frac{4y - 3x^2}{3y^2 - 4x}$$ A1

b. $\left.\dfrac{4y - 3x^2}{3y^2 - 4x}\right|_{(2,2)} = -1$ A1

$$y - 2 = -1(x - 2)$$

$$y = -x + 4$$ A1

Note: Consequential on answer to ***Question 4a.***

c. $x = \dfrac{4t}{1+t^3}$ and $y = \dfrac{4t^2}{1+t^3}$

$$x^3 = \frac{64t^3}{(1+t^3)^3}$$

$$y^3 = \frac{64t^6}{(1+t^3)^3}$$

$$x^3 + y^3 = \frac{64t^3(1+t^3)}{(1+t^3)^3} = \frac{64t^3}{(1+t^3)^2}$$ M1

$$4xy = 4 \times \frac{4t}{1+t^3} \times \frac{4t^2}{1+t^3} = \frac{64t^3}{(1+t^3)^2}$$ M1

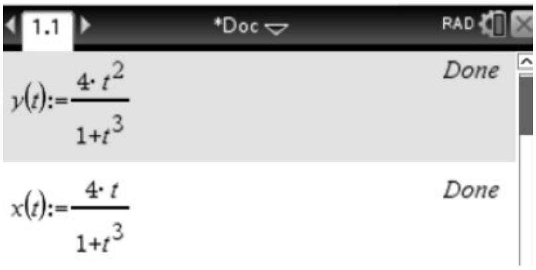
$$\therefore x^3 + y^3 = 4xy$$

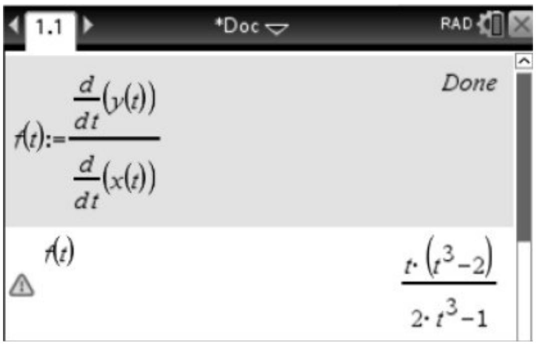
d. $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

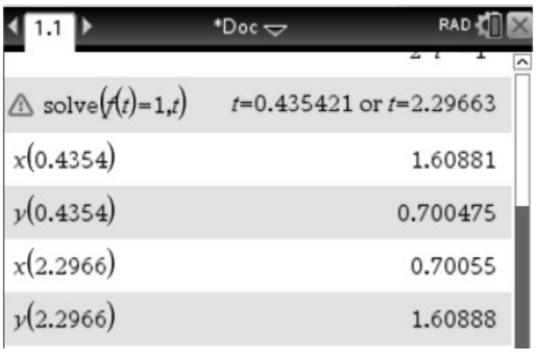
$= \dfrac{t\left(t^3 - 2\right)}{2t^3 - 1}$ M1

$\dfrac{dy}{dx} = 1 \Rightarrow t = 0.435\ldots$ or $t = 2.296\ldots$ A1

(1.61, 0.70) and (0.70, 1.61) A1







e. **i.** The total area can be found by subtracting the area between $t = 0$ and $t = a$ from the area between $t = a$ and $t = \infty$.

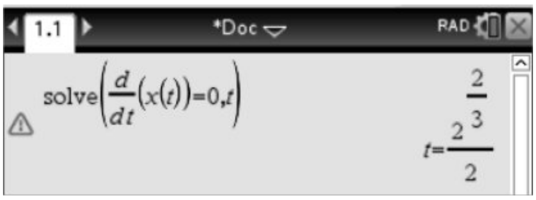
However, the curve does not always trace in the positive direction between $t = 0$ and $t = \infty$.

Therefore, the value of a single definite integral between $t = a$ and $t = \infty$ will be negative.

In order to find the t value when the curve traces in the negative direction, the value when the derivative is undefined must be found.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \Rightarrow \frac{dx}{dt} = 0$$

$a = 2^{-\frac{1}{3}}$ A1



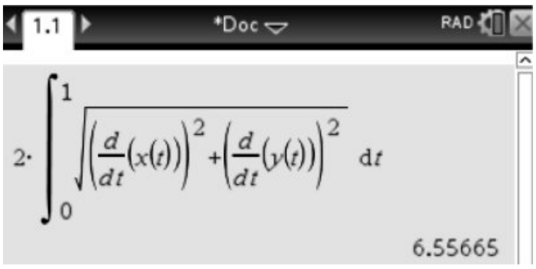
ii. Substituting $a = 2^{-\frac{1}{3}}$ into the given expression, the area will be:

$\frac{8}{3}$ A1

*Note: Consequential on answer to **Question 4e.i.***

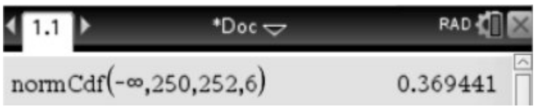
f. $2\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ M1

$= 6.56$ A1

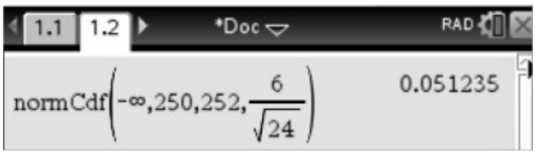


Question 5 (9 marks)

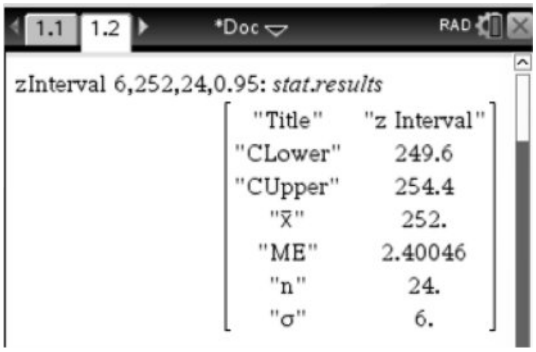
a. 0.369 A1



b. 0.051 A1



c. (249.6, 254.4) A1



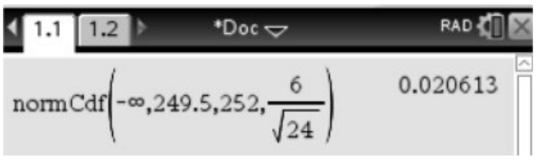
d. **i.** $H_0 : \mu = 250$

$H_1 : \mu < 250$

both H_0 *and* H_1 A1

ii. p-value $= \Pr(\bar{X} < 247) = 0.0206$ A1

p-value $< 0.05 \Rightarrow$ machine is faulty A1



e. $\Pr(\bar{X} > 250) = 0.99$

$\Pr(Z > a) = 0.99 \Rightarrow a = -2.3263\ldots$ A1

$$\frac{250 - 252}{\frac{s}{\sqrt{24}}} = a$$ M1

$s = 4.2$ A1