VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2021

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER Letter

SPECIALIST MATHEMATICS

Written examination 2

Wednesday 2 June 2021

Reading time: 10.00 am to 10.15 am (15 minutes)
Writing time: 10.15 am to 12.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 23 pages
- Formula sheet
- Answer sheet for multiple-choice questions

Instructions
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

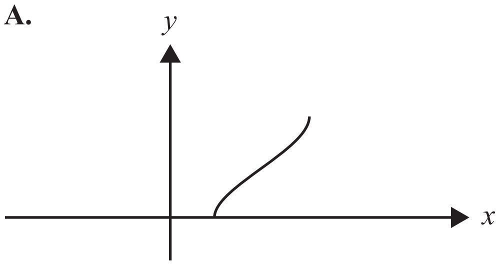
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

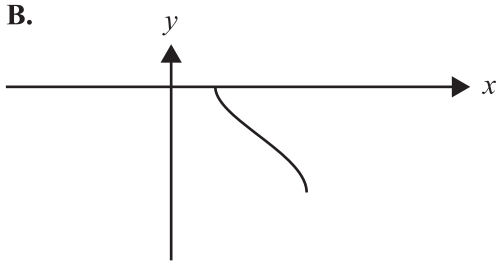
Question 1

The graph of $y = \cos^{-1}(2 - bx)$, where b is a positive real constant, could be

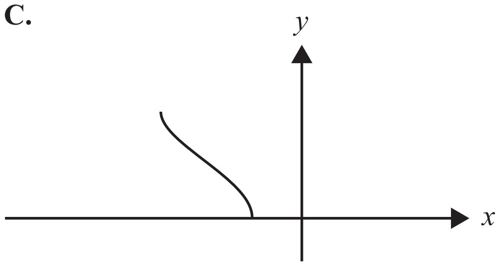
A.



B.



C.



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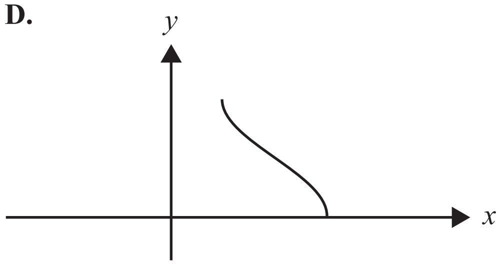
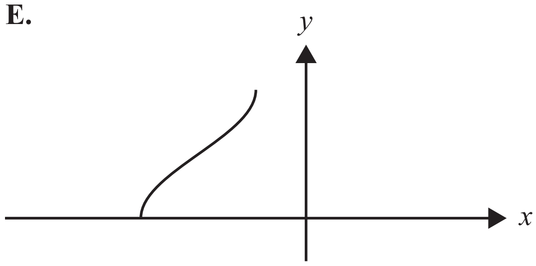


E.



Question 2

Consider the function with rule $f(x) = |x-3| + |x+3| - a$, where a is a real constant.

The graph of $\dfrac{1}{f(x)}$ will have three asymptotes if the set of values of a is

A. $\{-3, 3\}$

B. $\{\ \}$

C. $[6, \infty)$

D. $(-\infty, 6)$

E. $[-3, 3]$

Question 3

The graph of $f(x) = \dfrac{1}{x^2 + sx + t}$, where s and t are real constants, will have no vertical asymptotes when

A. $t = \dfrac{s^2}{4}$

B. $s > \dfrac{t^2}{4}$

C. $t > \dfrac{s^2}{4}$

D. $s < \dfrac{t^2}{4}$

E. $t < \dfrac{s^2}{4}$

Question 4

The expression $\dfrac{ax+b}{(2x-1)^2(x+1)}$ has partial fraction form $\dfrac{1}{(x+1)} - \dfrac{2}{(2x-1)} - \dfrac{1}{(2x-1)^2}$.

The values of a and b, where a and b are non-zero real constants, are respectively

A. 12 and 21

B. 7 and 16

C. −5 and 4

D. −7 and 2

E. 3 and 6

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5

The complex number $z = b + i$, where $b \in R$, has the property $\text{Arg}(z^7) = \text{Arg}(z)$.

A possible value of b is

A. $-\sqrt{3}$

B. 0

C. $\frac{1}{\sqrt{3}}$

D. 1

E. $\sqrt{3}$

Question 6

If z is a non-zero complex number, then $\cos(\arg(z))$ is equal to

A. $\left|\frac{\text{Re}(z)}{z}\right|$

B. $\frac{\text{Re}(z)}{\sqrt{z\bar{z}}}$

C. $\frac{\text{Im}(z)}{|z|}$

D. $\sin\left(\frac{\text{Im}(z)}{z}\right)$

E. $\frac{\text{Re}(z)}{|z^2|}$

Question 7

With a suitable substitution, $\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \frac{\cos(2x)}{\sin^2(2x)}\,dx$ can be expressed as

A. $\frac{1}{2}\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{u^2}\,du$

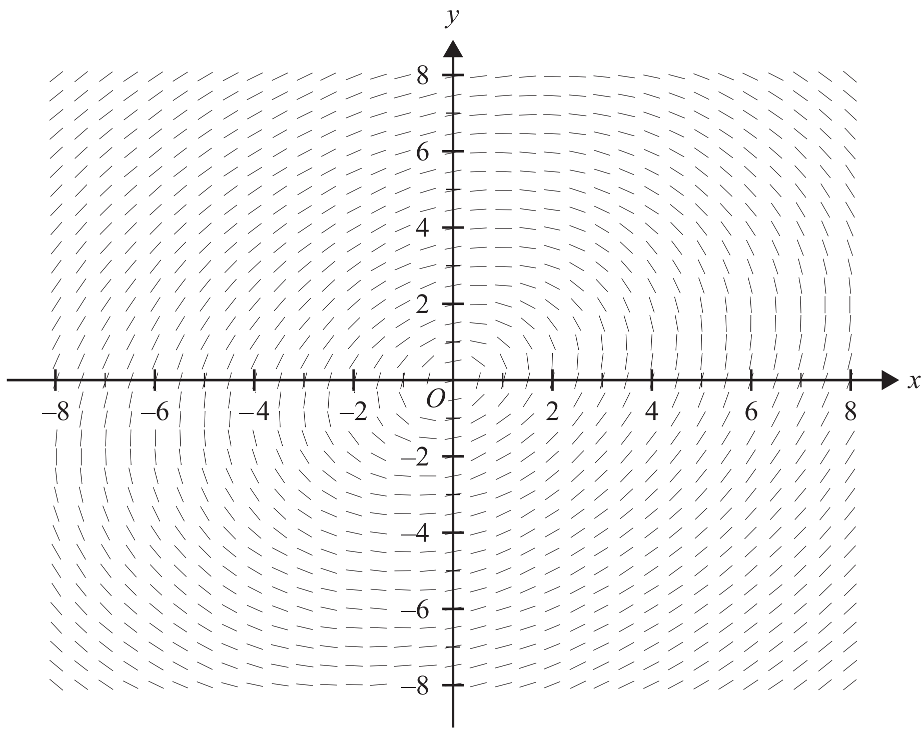
B. $2\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{u^2}\,du$

C. $2\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} u^2\,du$

D. $2\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \frac{1}{u^2}\,du$

E. $\frac{1}{2}\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} u\,du$

Question 8



The differential equation that has the diagram above as its direction field is

A. $\dfrac{dy}{dx} = \dfrac{4x+y}{x-4y}$

B. $\dfrac{dy}{dx} = \dfrac{x-4y}{4x+y}$

C. $\dfrac{dy}{dx} = \dfrac{4x-y}{x+4y}$

D. $\dfrac{dy}{dx} = \dfrac{x-4y}{4x-y}$

E. $\dfrac{dy}{dx} = \dfrac{4x-y}{x-4y}$

Question 9

Consider the differential equation $\frac{dy}{dx} = 2x + 1$, where $y(x_0) = y_0$.

Using Euler's method with a step size of h, which one of the following is equal to y_2?

A. $y_0 + 4x_0h + 2h^2 + 2h$

B. $y_0 + 4x_0h + 2h$

C. $y_0 + 4x_0h + h^2 + 3h$

D. $y_0 + 2x_0h + h$

E. $y_0 + 4x_0h + 3h^2 + h$

Question 10

A tank initially contains 20 litres of water in which 0.25 kg of salt is dissolved. Fresh water flows in at a rate of 6 litres per minute and the solution, kept uniform by stirring, flows out at a rate of 8 litres per minute.

The mass of salt, x kilograms, in solution in the tank after t minutes is related to t by

A. $\int \frac{dx}{5x} = \int \frac{8}{t-10}\,dt$

B. $\int \frac{dx}{x} = \int \frac{4}{10-t}\,dt$

C. $\int \frac{dx}{x} = \int \frac{8}{10-t}\,dt$

D. $\int \frac{dx}{x} = \int \frac{4}{t-10}\,dt$

E. $\int x\,dx = \int \frac{4}{t-10}\,dt$

Question 11

Let $\underset{\sim}{a}$ and $\underset{\sim}{b}$ be arbitrary non-zero vectors.

Which one of the following statements is always true?

A. $|\underset{\sim}{a} + \underset{\sim}{b}| \geq |\underset{\sim}{a} - \underset{\sim}{b}|$

B. $|\underset{\sim}{a} - \underset{\sim}{b}| \leq |\underset{\sim}{a}| + |\underset{\sim}{b}|$

C. $|\underset{\sim}{a} - \underset{\sim}{b}| < |\underset{\sim}{a}| - |\underset{\sim}{b}|$

D. $|\underset{\sim}{a} - \underset{\sim}{b}| < |\underset{\sim}{a}| + |\underset{\sim}{b}|$

E. $|\underset{\sim}{a} - \underset{\sim}{b}| > |\underset{\sim}{a}| - |\underset{\sim}{b}|$

Question 12

A particle has position vector $\underset{\sim}{r}(t) = t^3\underset{\sim}{i} + 3t^2\underset{\sim}{j}$.

The total distance travelled by the particle from $t = 1$ to $t = 4$, correct to two decimal places, is

A. 17.37

B. 36.05

C. 65.89

D. 77.42

E. 78.26

Question 13

The scalar resolute of $\underset{\sim}{a} = \underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ in the direction of $\underset{\sim}{b} = 4\underset{\sim}{i} - 4\underset{\sim}{j} + 7\underset{\sim}{k}$ is

A. $-\frac{3}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{19}{9}$

E. $\frac{19}{6}$

Question 14

A jet of mass 80 000 kg accelerates from rest in a straight line along a horizontal runway to a take-off speed of 75 ms^{-1}. The thrust force of the engines is 200 000 N and the resistance forces acting on the jet total $100v$ newtons, where v is the speed of the jet in metres per second.

The distance, in metres, needed for the jet to take off is closest to

A. 1125

B. 1154

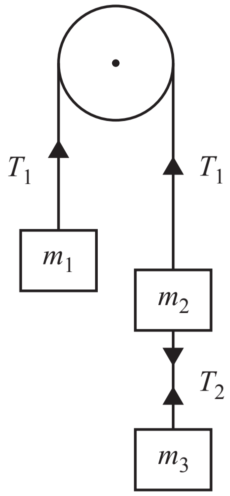
C. 1520

D. 1853

E. 1976

Question 15

Three masses of m_1, m_2 and m_3 kilograms, attached by light inextensible strings, hang in equilibrium over a smooth pulley. Tensions of magnitude T_1 and T_2 newtons are present in the strings, as shown in the diagram below.



The value of $\frac{T_2}{T_1}$ is given by

A. $\frac{m_1 - m_2}{m_1 + m_2}$

B. $\frac{m_1 - m_2}{m_1 - m_3}$

C. $\frac{m_1 - m_2}{m_3}$

D. $\frac{m_1 - m_3}{m_2}$

E. $\frac{m_1 - m_2}{m_1}$

Question 16

A particle of mass 2 kg has a horizontal velocity component of magnitude 12 ms^{-1} and a vertical velocity component of magnitude 5 ms^{-1}.

The magnitude, in kg ms^{-1}, of the momentum of the particle is

A. 13
B. 17
C. 24
D. 26
E. 34

Question 17

A particle moves in a straight line. At time t seconds, where $t \geq 0$, its displacement x metres from the origin and its velocity v metres per second are such that $v = 9 + x^2$.

If initially $x = 3$ m, then t is equal to

A. $9x + \frac{1}{3}x^3$

B. $9x + \frac{1}{3}x^3 - 36$

C. $\frac{1}{3}\arctan\left(\frac{x}{3}\right) - \frac{\pi}{12}$

D. $\arctan\left(\frac{x}{3}\right) - \frac{\pi}{4}$

E. $\arctan\left(\frac{x}{3}\right) + 3$

Question 18

It is believed that the average life span of people in Okinawa is 80 years. A random sample of the life spans of 200 of these people had an average of 84 years. A one-tailed statistical test, using a 5% level of significance, is to be carried out to investigate if this result provides evidence that the average life span of people in Okinawa is greater than 80 years.

Which one of the following statements would form suitable null and alternative hypotheses for this test?

A. $H_0 : \mu > 80 \qquad H_1 : \mu > 84$
B. $H_0 : \mu > 84 \qquad H_1 : \mu > 80$
C. $H_0 : \mu = 80 \qquad H_1 : \mu \neq 80$
D. $H_0 : \mu = 84 \qquad H_1 : \mu > 80$
E. $H_0 : \mu = 80 \qquad H_1 : \mu > 80$

Question 19

X and Y are independent random variables with variances of 3 and 7 respectively.

Given the random variable $Z = 2X - 3Y + 5$, the standard deviation of Z is closest to

A. 5.20
B. 5.66
C. 8.66
D. 8.94
E. 11.40

Question 20

In order to gain an accurate assessment of a patient's blood pressure, a doctor asks the patient to take 16 blood pressure readings at random times. The readings are measured in millimetres of mercury.

The mean and the standard deviation of these readings are $\bar{x} = 125$ and $s = 10$ respectively.

Assuming the patient's blood pressure readings are normally distributed with mean μ and with $\sigma = 10$, a 99% confidence interval for μ is closest to

A. (99.2, 150.7)
B. (118.6, 131.4)
C. (112.1, 137.8)
D. (95.0, 135.0)
E. (120.1, 129.8)

END OF SECTION A
TURN OVER

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (10 marks)

A curve is defined parametrically by

$$x = \sec(t), \qquad y = \operatorname{cosec}(t), \qquad \text{where } t \in \left(0, \frac{\pi}{2}\right)$$

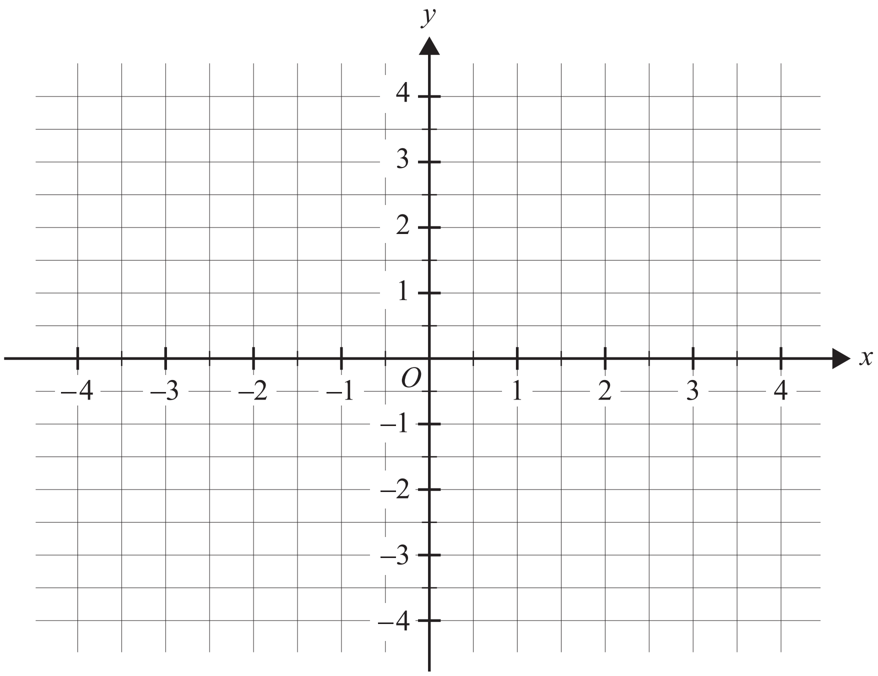
a. Show that the curve can be represented in cartesian form by the relation $y = \dfrac{x}{\sqrt{x^2 - 1}}$. 2 marks

b. State the domain and range of the relation given by $y = \dfrac{x}{\sqrt{x^2 - 1}}$ for this curve. 2 marks

c. Use the derivative of the relation to show that the gradient of the curve is negative at all points on the curve. 2 marks

SECTION B – Question 1 – continued

d. Sketch the graph of the relation on the axes below, labelling any asymptotes with their equations. 2 marks



e. **i.** Write down a definite integral in terms of x, which gives the arc length of the curve from $x = 1.5$ to $x = 4$. 1 mark

ii. Find this arc length correct to two decimal places. 1 mark

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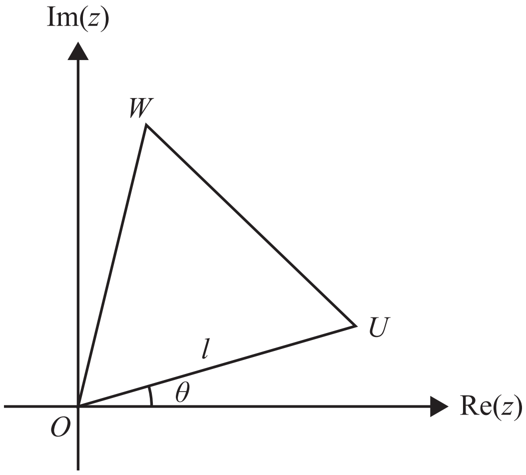
Question 2 (9 marks)

Points O, U and W in the complex plane represent complex numbers 0, u and w respectively, where $\text{Arg}(u) = \theta$. When plotted on an Argand diagram, the points are the vertices of an equilateral triangle of side length l, as shown below.



a. Express w in polar form in terms of l and θ. 1 mark

b. Point P represents the complex number p and is the midpoint of U and W.

i. Express p in polar form in terms of l and θ. 2 marks

ii. Express p^{12} in the form $\frac{a}{b}u^n$, where a, b and n are integers. 3 marks

c. Show that $\frac{w}{u} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$. 1 mark

d. Point U has coordinates $(a, 17)$ and point W has coordinates $(b, 41)$.

Find the values of a and b. 2 marks

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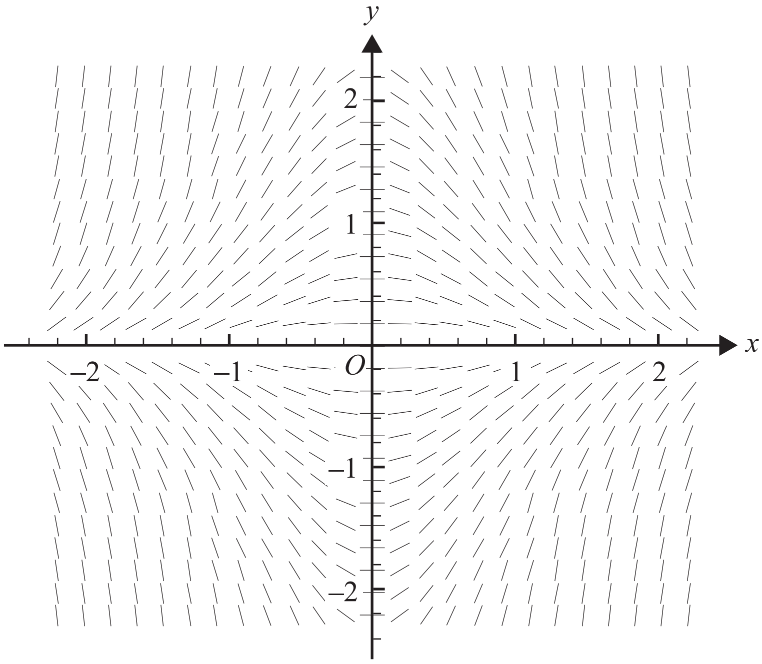
Question 3 (10 marks)

a. A family of curves is defined by the differential equation

$$\frac{dy}{dx} = -2xy$$

i. Plot the solution curve of this differential equation on the slope field below for the initial condition $y(0) = 1.5$ 1 mark



ii. Solve the differential equation above to show that $y = e^{-x^2}$ for the initial condition $y(0) = 1$. 2 marks

iii. Find the coordinates of any points of inflection of the curve given by $y = e^{-x^2}$. 2 marks

b. Consider the family of curves defined by the differential equation

$$\frac{dy}{dx} = -kxy, \qquad \text{where } k > 0 \text{ and } y(0) = 1$$

i. Use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y. 2 marks

ii. Hence, show that the y-coordinate of any points of inflection of curves in the family does not depend on k. 3 marks

DO NOT WRITE IN THIS AREA

Question 4 (12 marks)

The position vector of a golf ball t seconds after it is hit is modelled by $\underset{\sim}{r}(t) = at\,\underset{\sim}{i} + (bt - 5t^2)\,\underset{\sim}{j}$, where $a, b \in Z$, $t \geq 0$ and where $\underset{\sim}{i}$ is the unit vector in the positive horizontal direction and $\underset{\sim}{j}$ is the unit vector in the positive vertical direction.

Displacement components are measured in metres.

The golf ball is hit from ground level and, after 4 s, it is 4 m above ground level and descending as it passes over the top of a tree. The tree is at a horizontal distance of 160 m from where the ball was hit.

a. Show that $a = 40$ and $b = 21$. 2 marks

b. What acute angle to the horizontal direction does the tangent to the path of the golf ball make as the golf ball passes over the top of the tree? Give your answer in degrees, correct to one decimal place. 2 marks

c. What is the maximum height, in metres, reached by the golf ball? 2 marks

d. Find the cartesian equation of the path of the golf ball, giving your answer in the form $cy = dx - x^2$, where c and d are integers. 2 marks

A person is standing at a horizontal distance of 163 m from where the golf ball was hit. The golf ball passes directly over the head of this person.

e. If the person is 1.8 m tall, how far directly above the person's head does the ball pass? Give your answer in metres, correct to two decimal places. 2 marks

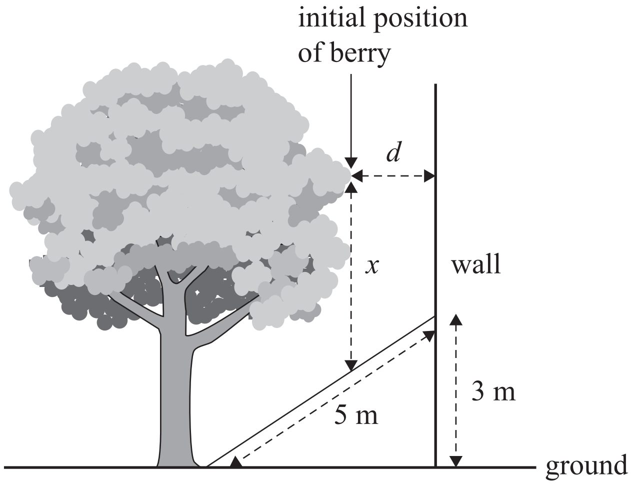
f. How far, in metres, is the golf ball from this person's feet when it hits the ground? 2 marks

DO NOT WRITE IN THIS AREA

Question 5 (9 marks)

A berry, a small round fruit, of mass 0.025 kg, initially at rest, falls from a tree from a position that is d metres from a wall. Assume that the berry falls vertically and the only force acting on it is gravity.

After 0.5 s, the berry hits a plank leaning against the wall, as shown below. The length of the plank is 5 m and it leans against the wall with its upper end at a height 3 m above the ground, which is horizontal. The plank and the path of the berry lie in a vertical plane.



a. Find the distance, x metres, that the berry has fallen when it hits the plank. 2 marks

b. What is the speed, in metres per second, of the berry when it hits the plank? 1 mark

c. Show that the magnitude of the component of the velocity of the berry in the direction along the plank at the instant that it hits the plank is $2.94\ \text{ms}^{-1}$. 1 mark

The berry does not bounce or deform, but immediately begins to slide down the plank with an initial speed of 2.94 ms^{-1}, subject to gravity and a constant resistance of magnitude R newtons that opposes the direction of motion. The berry reaches the end of the plank at a speed of 1.06 ms^{-1} after 2 s.

d. Find R. 3 marks

e. Find the horizontal distance, d metres, from the wall to the initial position from which the berry began to fall. 2 marks

Question 6 (10 marks)

A real estate agent's income is comprised of a fixed monthly payment of \$5000 plus a commission of \$4000 for each house sold. The agent may sell up to three houses in a month with the following probability distribution, where H is the number of houses sold.

h	0	1	2	3
$\Pr(H = h)$	0.2	0.6	0.19	0.01

a. Find the agent's expected total income each month. Give your answer in dollars, correct to the nearest dollar. 2 marks

b. What extra amount of commission should the agent request for each house sold so that their expected monthly income increases by \$2000? Give your answer in dollars and cents, correct to the nearest cent. 2 marks

To prepare a certain house for sale, the agent recommends that the garden be landscaped and replanted. The time taken for a contractor to landscape the garden is normally distributed with a mean of 18 hours and a standard deviation of four hours. The time taken for replanting is independent of the time taken for landscaping and is normally distributed with a mean of 24 hours and a standard deviation of six hours.

c. What is the probability, correct to four decimal places, that the garden can be landscaped and replanted within a week? Assume that the contractor will work eight hours per day for five days of the week. 2 marks

d. What period of time should the agent allow the contractor to complete the tasks of landscaping and replanting, in order to be 90% confident that these tasks will be completed within this period of time? Give your answer in hours, correct to two decimal places. 1 mark

e. Before the annual income review, the agent believes that the company's remuneration level is below the industry average of \$115 000 per annum, with an industry standard deviation of \$12 000. To test this hypothesis, the agent takes a random sample of 36 of the company's employees and finds their mean income to be \$110 800 per annum.

Assuming an industry standard deviation of \$12 000, carry out a one-tailed statistical test at the 5% level of significance to ascertain whether the sample mean of \$110 800 per annum supports the agent's belief. 3 marks

END OF QUESTION AND ANSWER BOOK

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2021

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \text{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}\left(\theta_1 - \theta_2\right)$
$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\text{E}(aX + b) = a\text{E}(X) + b$ $\text{E}(aX + bY) = a\text{E}(X) + b\text{E}(Y)$ $\text{var}(aX + b) = a^2\text{var}(X)$
for independent random variables X and Y	$\text{var}(aX + bY) = a^2\text{var}(X) + b^2\text{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\overline{X}$	mean $\text{E}\left(\overline{X}\right) = \mu$ variance $\text{var}\left(\overline{X}\right) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c, n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\lvert x\rvert+c$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$	$\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right)=\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c, a>0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right)=\frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c, a>0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right)=\frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$
	$\int (ax+b)^{-1} dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx}=f(x)$, $x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf(x_n)$
acceleration	$a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2}\sqrt{1+\left(f'(x)\right)^2}\,dx$ or $\int_{t_1}^{t_2}\sqrt{\left(x'(t)\right)^2+\left(y'(t)\right)^2}\,dt$

Vectors in two and three dimensions

$\underset{\sim}{r}=x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$
$\lvert \underset{\sim}{r}\rvert=\sqrt{x^2+y^2+z^2}=r$
$\dot{\underset{\sim}{r}}=\frac{d\underset{\sim}{r}}{dt}=\frac{dx}{dt}\underset{\sim}{i}+\frac{dy}{dt}\underset{\sim}{j}+\frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1\cdot\underset{\sim}{r}_2=r_1r_2\cos(\theta)=x_1x_2+y_1y_2+z_1z_2$

Mechanics

momentum	$\underset{\sim}{p}=m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R}=m\underset{\sim}{a}$

END OF FORMULA SHEET