Neap

Trial Examination 2021

VCE Specialist Mathematics Units 1&2

Written Examination 1

Suggested Solutions

Question 1 (5 marks)

a. $2|x-5|=10$

$|x-5|=5$

$x-5=-5$ or $x-5=+5$

$x=0$ or $x=10$ A1

b. $3x-|x|=-4$

$3x-x=-4$ if $x\geq 0$

$2x=-4$

$x=-2$

As $x\geq 0$, $x=-2$ is not valid. M1

$3x-(-x)=-4$ if $x<0$

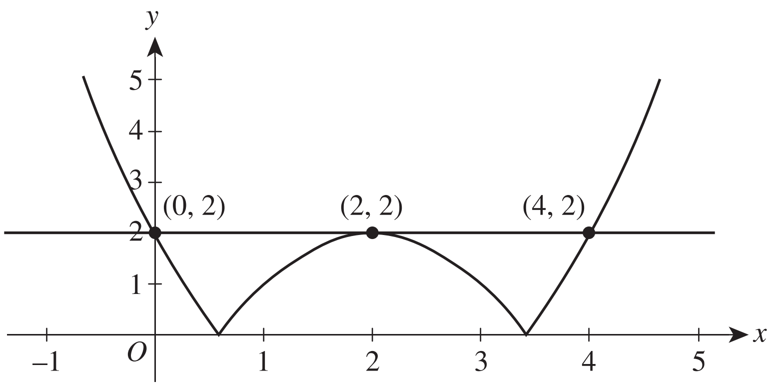
$4x=-4$

$x=-1$ A1

c. $|x^2-4x+2|\geq 2$

$|(x-2)^2-2|\geq 2$

Sketch the graphs of $y=|(x-2)^2-2|$ and $y=2$.



correct sketch of graphs M1

$x\in(-\infty,0]\cup\{2\}\cup[4,\infty)$ A1

Question 2 (8 marks)

a. i. $t_2=4, t_n=t_{n-1}+5$

$t_2=4, t_3=t_2+5, t_3=9$

$t_3=9, t_4=t_3+5, t_4=14$

$t_2=4, t_1=t_2-5, t_1=-1$

The first four terms of the sequence are −1, 4, 9 and 14. A1

ii. $S_n=\frac{n}{2}\left(2a+(n-1)d\right)$

$S_{10}=\frac{10}{2}(2\times -1+(10-1)\times 5)$

$=215$ A1

b. $t_3 = 6,\ t_6 = 15$

$d = \dfrac{15-6}{6-3}$ M1

$= 3$

$t_{13} = t_3 + 10d$

$= 6 + 10 \times 3$

$= 36$ A1

OR

Using simultaneous equations:

$t_3 = 6 = a + 2d$

$\Rightarrow 6 = a + 2g$ (1)

$t_6 = 15 = a + 5d$

$\Rightarrow 15 = a + 5d$ (2)

$(2) - (1)$

$\Rightarrow 3d = 9$

$d = 3$

Substitute $d = 3$ into (1).

$6 = a + 2 \times 3$

$a = 0$ M1

$t_n = 0 + (n-1) \times 3$

$t_{13} = 12 \times 3$

$= 36$ A1

c. i. $r = \dfrac{x-6}{x}$

$= \dfrac{2x-7}{x-6}$

$(x-6)^2 = x(2x-7)$

$x^2 + 5x - 36 = 0$ M1

$(x-4)(x+9) = 0$

As $x > 0$:

$x = 4$ A1

ii. From **part c.i.**, $x = 4$.

$\Rightarrow t_1 = 4,\ t_2 = -2,\ t_3 = 1, \ldots$

$\therefore r = -\dfrac{1}{2}$ and $a = 4$ M1

$S_\infty = \dfrac{a}{1-r}$

$= \dfrac{4}{1-\left(-\dfrac{1}{2}\right)} = \dfrac{4}{\dfrac{3}{2}}$

$= \dfrac{8}{3}$ A1

Note: Consequential on answer to ***Question 2c.i.***

Question 3 (6 marks)

a. **i.** Locus is a circle with centre (–1, 5) and radius of 4.

$(x+1)^2+(y-5)^2=16$ A1

ii. $x+1=4\cos(\theta)$ and $y-5=4\sin(\theta)$

$x=4\cos(\theta)-1$ and $y=4\sin(\theta)+5$

$P\big(4\cos(\theta)-1,\ 4\sin(\theta)+5\big)$ A1

b.

$$AP=4-BP$$

$$\sqrt{(x-1)^2+y^2}=4-\sqrt{(x+1)^2+y^2}$$

$$(x-1)^2+y^2=16-8\sqrt{(x+1)^2+y^2}+(x+1)^2+y^2$$

$$(x-1)^2-(x+1)^2=16-8\sqrt{(x+1)^2+y^2}$$ M1

$$-4x=16-8\sqrt{(x+1)^2+y^2}$$

$$x=-4+2\sqrt{(x+1)^2+y^2}$$ M1

$$x+4=2\sqrt{(x+1)^2+y^2}$$

$$(x+4)^2=4\big((x+1)^2+y^2\big)$$ M1

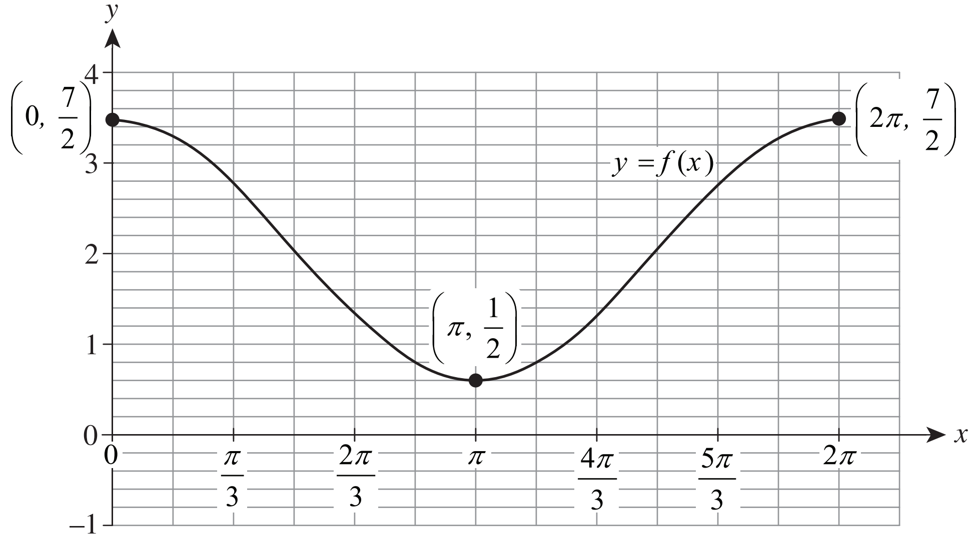
$$x^2+8x+16=4y^2+4x^2+8x+4$$

$$3x^2+4y^2=12$$

$$\frac{x^2}{4}+\frac{y^2}{3}=1$$ A1

Question 4 (4 marks)

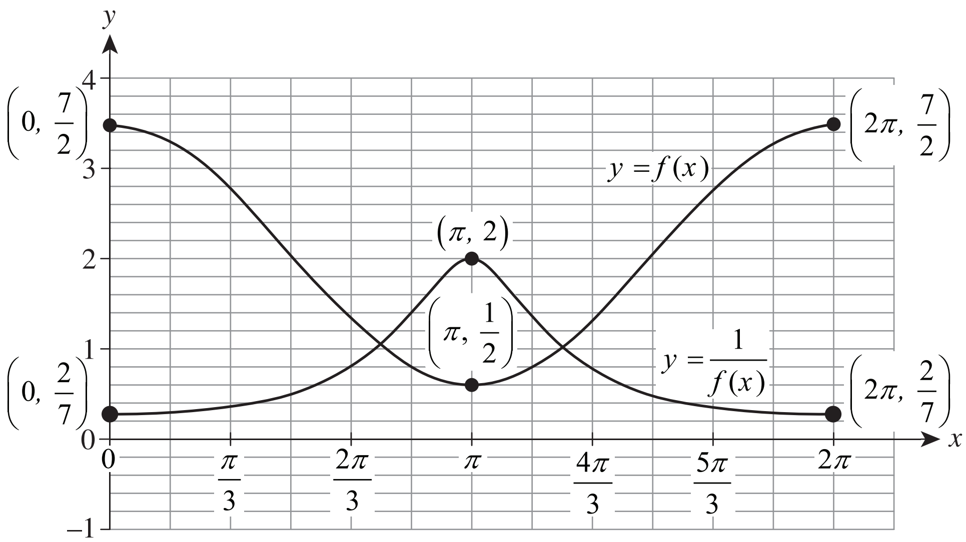
a.



correct shape, including gradient of zero at end points A1

correct coordinates of turning point and end points A1

b.



correct shape, including gradient of zero at end points A1

correct location and coordinates of end points A1

Question 5 (6 marks)

a.

$$2^{x+1} = 20$$

$$2 \times 2^x = 20$$

$$2^x = 10$$

Now suppose that x is rational. Since $x > 1$:

$x = \frac{m}{n}$, where $m, n \in N$. M1

$$2^{\frac{m}{n}} = 10$$

$$2^m = 10^n$$

$2^m = 5^n \times 2^n$ M1

So, 2^m and 2^n are both even numbers, while 5^n must be odd.

The LHS is even and the RHS is odd, giving a contradiction. Hence, x is not rational, and must be irrational. A1

b. Let $P(n)$ be the proposition $11^n - 5^n$ is divisible by 6.

$P(1)$ is the proposition $11^1 - 5^1 = 6$ and is therefore divisible by 6.

Let k be any natural number, and assume that $P(k)$ is true.

$\to 11^k - 5^k = 6m$ for some $m \in Z$. M1

$P(k+1) = 11^{k+1} - 5^{k+1}$ M1

$= 11 \times 11^k - 5 \times 5^k$

$= (6+5) \times 11^k - 5 \times 5^k$

$= 6 \times 11^k + 5 \times 11^k - 5 \times 5^k$

$= 6 \times 11^k + 5(11^k - 5^k)$

$= 6 \times 11^k + 5 \times 6m$

$= 6(11^k + 5m)$ A1

As $P(k+1)$ is divisible by 6, $11^n - 5^n$ is divisible by 6 for $n \in N$.

Question 6 (6 marks)

a. $\overrightarrow{OA} = \sqrt{3}\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\overrightarrow{OB} = m\underset{\sim}{i} + n\underset{\sim}{j}$.

$\overrightarrow{OA}.\overrightarrow{OB} = \sqrt{3}m + 3n$ M1

If $\angle AOB = 90°$, then $\overrightarrow{OA}.\overrightarrow{OB} = 0$.

$\sqrt{3}m + 3n = 0$

$\sqrt{3}m = -3n$

$\frac{m}{n} = -\frac{3}{\sqrt{3}}$

$= -\sqrt{3}$ A1

b. **i.** $\overrightarrow{OA} = \sqrt{3}\underset{\sim}{i} + 3\underset{\sim}{j}$

$\left|\overrightarrow{OA}\right| = \sqrt{\left(\sqrt{3}\right)^2 + (3)^2}$

$= \sqrt{12}$

$m = 0 \to \overrightarrow{OB} = n\underset{\sim}{j}$

If $\left|\overrightarrow{OA}\right| = \left|\overrightarrow{OB}\right|$, then $\overrightarrow{OB} = \sqrt{12}$. M1

$n = \pm\sqrt{12}$

$= \pm 2\sqrt{3}$ A1

ii. Assuming $n = 2\sqrt{3}$, $\overrightarrow{OB} = 2\sqrt{3}\underset{\sim}{j}$.

$\angle AOB = 90°$

$A = \frac{1}{2}bh$

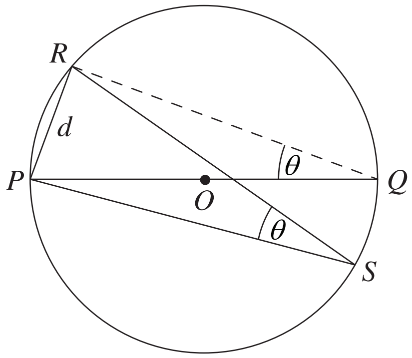
$= \frac{1}{2}\left|\overrightarrow{OA}\right| \cdot \left|\overrightarrow{OB}\right|$ M1

$= \frac{1}{2} \times 2\sqrt{3} \times 2\sqrt{3}$

$= 6 \text{ units}^2$ A1

Question 7 (5 marks)

a.



$\angle PQR = \angle PSR = \theta$ (angles on the same arc are equal) M1

$\angle QRP = 90°$ (angle in a semi-circle is a right angle) M1

$\sin(\theta) = \frac{d}{2r}$ M1

$d = 2r\sin(\angle RSP)$

b. $\overrightarrow{PS} = \overrightarrow{PO} + \overrightarrow{OS}$

$\overrightarrow{RQ} = \overrightarrow{RS} + \overrightarrow{SQ}$

$\overrightarrow{PO} = \overrightarrow{OQ}$ and $\overrightarrow{RO} = \overrightarrow{OS}$ (radii on same diameters) M1

$\Rightarrow \overrightarrow{PS} = \overrightarrow{OQ} + \overrightarrow{RO}$

$\therefore \overrightarrow{PS} = \overrightarrow{RQ}$ M1