VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2020

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | | Letter | |

SPECIALIST MATHEMATICS
Written examination 1

Thursday 19 November 2020

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 10 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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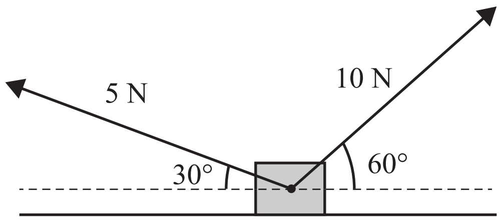
Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (5 marks)

A 2 kg mass is initially at rest on a smooth horizontal surface. The mass is then acted on by two constant forces that cause the mass to move horizontally. One force has magnitude 10 N and acts in a direction 60° upwards from the horizontal, and the other force has magnitude 5 N and acts in a direction 30° upwards from the horizontal, as shown in the diagram below.



a. Find the normal reaction force, in newtons, that the surface exerts on the mass. 2 marks

b. Find the acceleration of the mass, in ms^{-2}, after it begins to move. 2 marks

c. Find how far the mass travels, in metres, during the first four seconds of motion. 1 mark

Question 2 (4 marks)

Evaluate $\int_{-1}^{0} \frac{1+x}{\sqrt{1-x}}\,dx$. Give your answer in the form $a\sqrt{b}+c$, where a, b, $c \in R$.

Question 3 (3 marks)

Find the cube roots of $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$. Express your answers in polar form using principal values of the argument.

Question 4 (4 marks)

Solve the inequality $3 - x > \frac{1}{|x-4|}$ for x, expressing your answer in interval notation.

Question 5 (4 marks)

Let $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + m\underset{\sim}{j} - \underset{\sim}{k}$, where m is an integer.

The vector resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$ is $-\dfrac{11}{18}\left(\underset{\sim}{i} + m\underset{\sim}{j} - \underset{\sim}{k}\right)$.

a. Find the value of m. 3 marks

b. Find the component of $\underset{\sim}{a}$ that is perpendicular to $\underset{\sim}{b}$. 1 mark

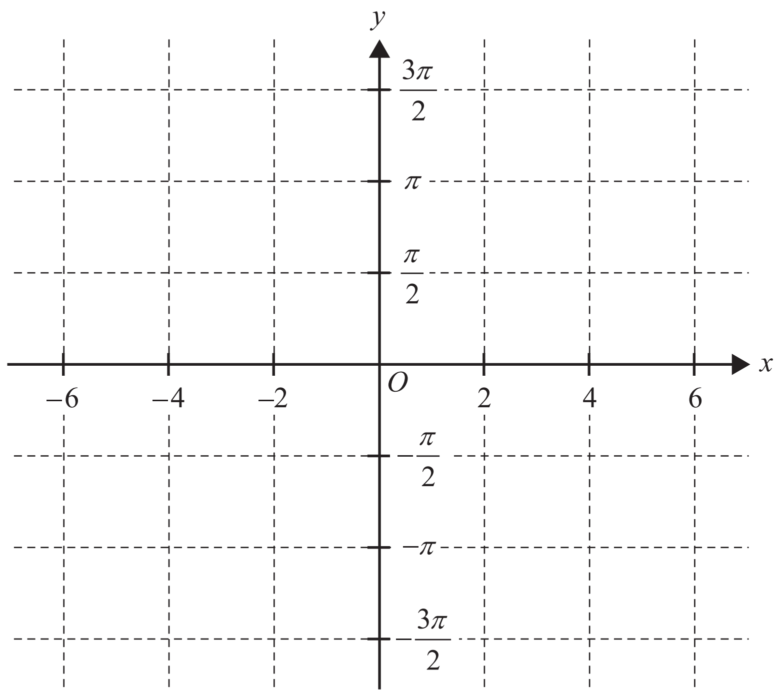
Question 6 (5 marks)

Let $f(x) = \arctan(3x - 6) + \pi$.

a. Show that $f'(x) = \dfrac{3}{9x^2 - 36x + 37}$. 1 mark

b. Hence, show that the graph of f has a point of inflection at $x = 2$. 2 marks

c. Sketch the graph of $y = f(x)$ on the axes provided below. Label any asymptotes with their equations and the point of inflection with its coordinates. 2 marks



Question 7 (5 marks)

Consider the function defined by

$$f(x) = \begin{cases} mx + n, & x < 1 \\ \dfrac{4}{1 + x^2}, & x \geq 1 \end{cases}$$

where m and n are real numbers.

a. Given that $f(x)$ and $f'(x)$ are continuous over R, show that $m = -2$ and $n = 4$. 2 marks

b. Find the area enclosed by the graph of the function, the x-axis and the lines $x = 0$ and $x = \sqrt{3}$. 3 marks

Question 8 (5 marks)

Find the volume, V, of the solid of revolution formed when the graph of $y = 2\sqrt{\frac{x^2 + x + 1}{(x+1)(x^2+1)}}$ is rotated about the x-axis over the interval $\left[0, \sqrt{3}\right]$. Give your answer in the form $V = 2\pi\left(\log_e(a) + b\right)$, where $a, b \in R$.

Question 9 (5 marks)

Consider the curve defined parametrically by

$$x = \arcsin(t)$$

$$y = \log_e(1+t) + \frac{1}{4}\log_e(1-t)$$

where $t \in [0, 1)$.

a. $\left(\frac{dy}{dt}\right)^2$ can be written in the form $\frac{1}{a(1+t)^2} + \frac{1}{b(1-t^2)} + \frac{1}{c(1-t)^2}$, where a, b and c are real numbers.

Show that $a = 1$, $b = -2$ and $c = 16$. 2 marks

b. Find the arc length, s, of the curve from $t = 0$ to $t = \frac{1}{2}$. Give your answer in the form $s = \log_e(m) + n\log_e(p)$, where $m, n, p \in Q$. 3 marks

END OF QUESTION AND ANSWER BOOK

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SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	[–1, 1]	[–1, 1]	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$	
$\lvert z \rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}\left(\theta_1 - \theta_2\right)$
$z^n = r^n \,\text{cis}(n\theta)$ (de Moivre’s theorem)	

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e \|x\| + c$
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a\sec^2(ax)$	$\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a}\log_e \|ax+b\| + c$
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1+\left(f'(x)\right)^2}\, dx$ or $\int_{t_1}^{t_2} \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\, dt$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\|\underset{\sim}{r}\| = \sqrt{x^2+y^2+z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

END OF FORMULA SHEET