Neap

Trial Examination 2020

VCE Specialist Mathematics Units 1&2

Written Examination 1

Suggested Solutions

Question 1 (7 marks)

a. $t_2 = 4 \times 3 - 2$ M1

$= 10$

$t_3 = 4 \times 10 - 2$

$= 38$ A1

b. **i.** $\dfrac{x}{x-2} = \dfrac{x+3}{x}$ M1

$x^2 = (x-2)(x+3)$

$x^2 = x^2 + x - 6$

$0 = x - 6$

$x = 6$ A1

ii. sequence:

$x - 2,\ x,\ x + 3$

$x = 6 \to 4, 6, 9$

$r = \dfrac{6}{4} = \dfrac{9}{6}$

$= \dfrac{3}{2}$ A1

iii. $S_n = \dfrac{a(r^n - 1)}{r - 1}$

$= \dfrac{4\left(\left(\frac{3}{2}\right)^n - 1\right)}{\frac{3}{2} - 1}$ M1

$= \dfrac{4\left(\frac{3}{2}\right)^n - 4}{\frac{1}{2}}$

$= 8\left(\dfrac{3}{2}\right)^n - 8$

$= 2^3 \cdot \dfrac{3^n}{2^n} - 8$ A1

$= \dfrac{3^n}{2^{n-3}} - 8$

Question 2 (7 marks)

a. $f(0) = |0-4| + |0-6| - 4$

$= 6$ A1

b. $|x-4| + |x-6| - 4 = 0$

Case 1: $x \geq 6$

$(x-4) + (x-6) - 4 = 0$

$2x = 14$

$x = 7$

Case 2: $4 < x < 6$

$(x-4) + -(x-6) - 4 = 0$

$-4 + 6 - 4 = 0$

$-2 = 0$

$\therefore$ no solution

Case 3: $x \leq 4$ M1

$-(x-4) + -(x-6) - 4 = 0$

$-2x + 6 = 0$

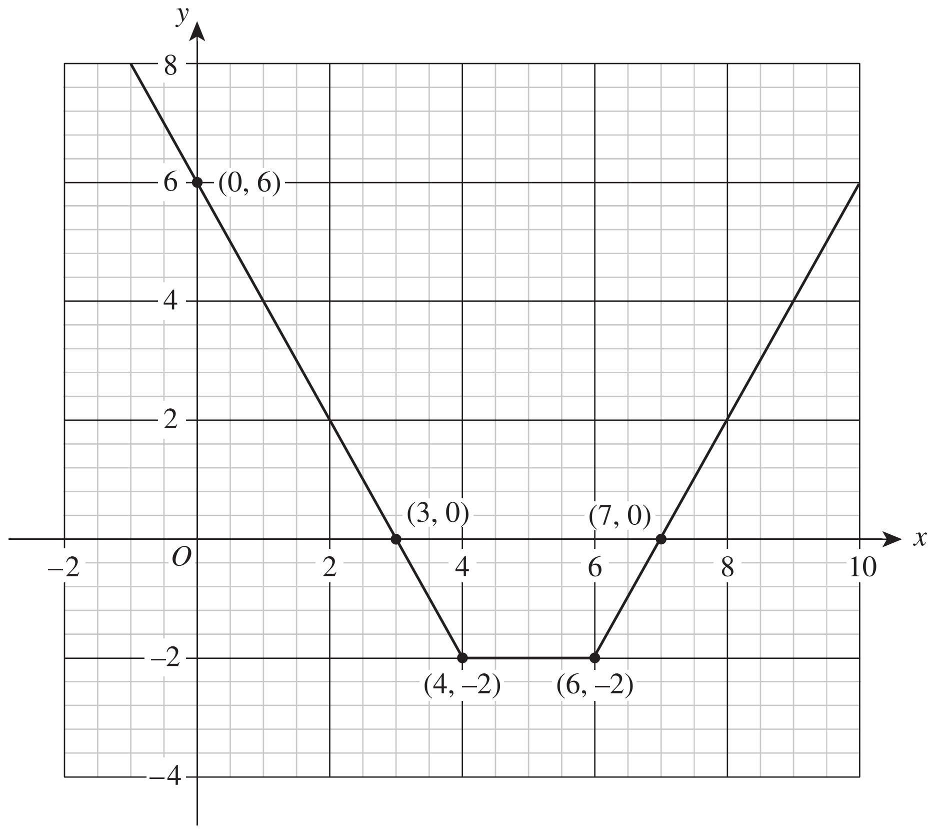
$x = 3$

Solutions:

$x = 3$ A1

$x = 7$ A1

c.



3 marks

intercept points, correct position and coordinate A1

cusp points, correct position and coordinate A1

correct shape A1

Question 3 (6 marks)

a. $t = 0 \rightarrow x = 1,\ y = -1$

$t = 3 \rightarrow x = 8,\ y = 28$

domain:

$x \in [1, 8)$ A1

range:

$y \in [-1, 27)$ A1

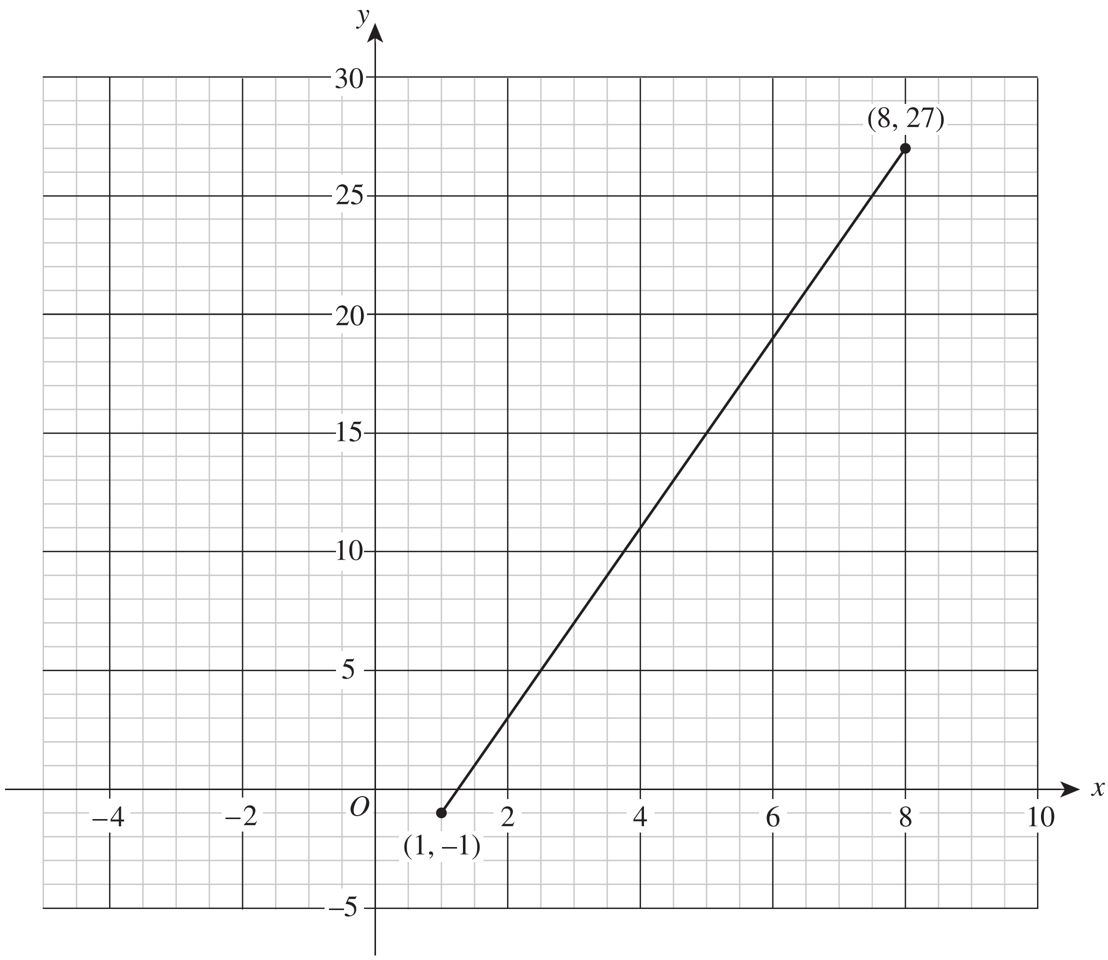
b. $x = 2^t$ and $y = 2^{t+2} - 5$

$y = 2^2 \times 2^t - 5$

$= 4 \times 2^t - 5$ M1

$= 4x - 5$ A1

c.



2 marks

correct endpoints A1

correct shape A1

Question 4 (5 marks)

a. Assume the contradictory position that $\sqrt{x} \le 2x$, for $x > 0$.

$x \le 4x^2$

$1 \le 4x$ M1

$x \ge \frac{1}{4}$

The original assumption is incorrect and therefore $0 < x < \frac{1}{4}$. A1

b. Assume $2^{3n} - 1 = 7k$ where $n, k \in Z^+$. M1

For $n = 1 \rightarrow \text{LHS} = 2^{3 \times 1} - 1 = 7 \rightarrow k = 1$

$\therefore$ true for $n = 1$

For $n + 1 \rightarrow \text{LHS} = 2^{3(n+1)} - 1$ M1

$$\begin{aligned}\text{LHS} &= 2^{3n+3} - 1\\ &= 8 \times 2^{3n} - 1\\ &= 8(2^{3n} - 1) + 7\\ &= 8(7k) + 7\\ &= 7(8k + 1)\end{aligned}$$

As $8k + 1$ is an integer, $7(8k + 1)$ is divisible by 7. A1

As the rule is true for $n = 1$ and consecutive cases, the rule is true for all n for $n \in Z^+$.

Question 5 (3 marks)

$$\begin{aligned}m_{AB} &= \frac{6-2}{2-1}\\ &= 4\end{aligned}$$

$$\begin{aligned}\rightarrow m_{AP} &= 0.5 \times 4\\ &= 2\end{aligned}$$ A1

Let the point $P(x, y)$.

$$\rightarrow m_{AP} = \frac{y-2}{x-1}$$ M1

$$\begin{aligned}\frac{y-2}{x-1} &= 2\\ y - 2 &= 2(x-1)\\ y &= 2x\end{aligned}$$ A1

Question 6 (7 marks)

a. **i.** $\overrightarrow{OC} = \underset{\sim}{i} + 2\underset{\sim}{j}$ A1

$\overrightarrow{OD} = 5\underset{\sim}{i} + \underset{\sim}{j}$ A1

ii. $$\begin{aligned}\overrightarrow{CD} &= \overrightarrow{OD} - \overrightarrow{OC}\\ &= 5\underset{\sim}{i} + \underset{\sim}{j} - (\underset{\sim}{i} + 2\underset{\sim}{j})\end{aligned}$$ M1

$$\begin{aligned}\left|\overrightarrow{CD}\right| &= \sqrt{4^2 + (-1)^2}\\ &= \sqrt{17}\end{aligned}$$ A1

b. $\overrightarrow{EF} = \overrightarrow{OF} - \overrightarrow{OE}$

$= p\overrightarrow{OB} - p\overrightarrow{OA}$

$= p(25\underset{\sim}{i} + 5\underset{\sim}{j} - 10\underset{\sim}{i} - 20\underset{\sim}{j})$

$= p(15\underset{\sim}{i} - 15\underset{\sim}{j})$ M1

$\left|\overrightarrow{EF}\right| = \sqrt{p^2(15)^2 + (-15)^2}$ M1

$= p\sqrt{450}$

$= p\sqrt{9 \times 5 \times 5 \times 2}$

$= 15p\sqrt{2}$

unit vector $\left|\overrightarrow{EF}\right| = 1$

$15p\sqrt{2} = 1$

$p = \dfrac{\sqrt{2}}{30}$ A1

Question 7 (5 marks)

a. Using the alternate segment theorem:

$\angle BCX = 50°$ A1

b. Using alternate angles on parallel lines:

$\angle CBD = 50°$ A1

c. As $\angle ABC$ is twice the bisected angle,

$\angle ABC = 2 \times 50° = 100°$. A1

d. Let $ED = x \rightarrow DB = 2x$ M1

$ED \times DB = AD \times DC$

$x \times 2x = 6 \times 4$

$2x^2 = 24$

$x = \sqrt{12}$ A1

$= 2\sqrt{3}$ cm