Victorian Certificate of Education – Free Trial Examinations

STUDENT NUMBER									Letter

SPECIALIST MATHEMATICS

Free Trial Written Examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 9 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

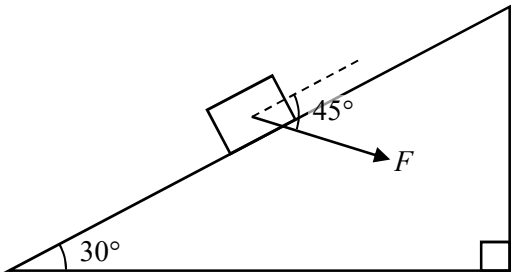
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (3 marks)

A 12 kg mass on a smooth plane inclined at 30° is held in equilibrium by a force of F newtons, acting 45° to the inclined plane, as shown below.



a. On the diagram above, show all other forces acting on the mass and label them. 1 mark

b. Find F, in newtons, expressing your answer in the form $a\sqrt{b}\,g$, where $a, b \in \mathbb{N}$. 2 marks

Question 2 (3 marks)

Given that $\cos(2\theta) = \frac{3}{8}$, where $\theta \in \left(\frac{3\pi}{4}, \pi\right)$, find $\text{cis}(\theta)$ in cartesian form.

Question 3 (4 marks)

Find the equation of the line perpendicular to the curve given by $e^{xy} = y^2 - 3$ at the point $(0, 2)$.

Question 4 (3 marks)

The masses of pears grown on a particular farm vary normally with a mean of 200 grams and a standard deviation of 15 grams. The pears are sold to supermarkets in boxes of 25 pears.

Given that $\Pr(-2 < Z < 2) = 0.955$, correct to three decimal places, where $Z \sim \mathrm{N}(0,1)$, calculate the probability that the mean mass per pear in a randomly selected 25-pear box exceeds 206 grams. Give your answer correct to **three** decimal places.

Question 5 (4 marks)

Let $g : \mathbb{C} \to \mathbb{C},\ g(z) = z^4 - 2z^3 + 8z - 16$.

Find all solutions of $g(z) = 0$ for z in cartesian form given that $z = 2\,\mathrm{cis}\left(\frac{\pi}{3}\right)$ is a solution.

Question 6 (4 marks)

Find $\int_1^2 \frac{4}{x^2(x+2)}dx$, expressing your answer in the form $a + \log_e\left(\frac{b}{c}\right)$, where $a, b, c \in \mathbb{N}$.

Question 7 (4 marks)

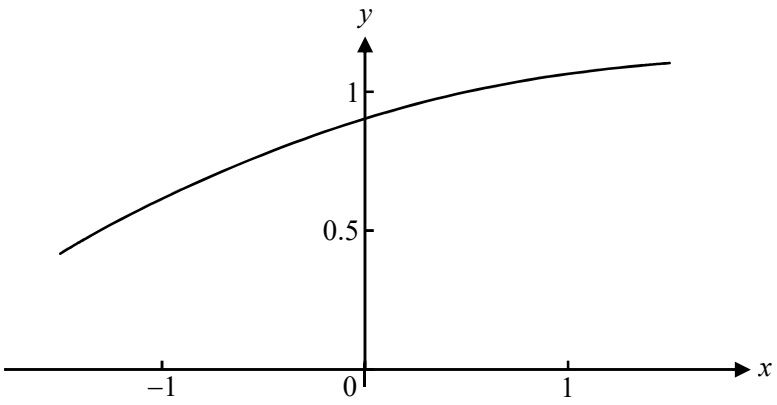
Relative to a fixed origin O, the points A and B are defined by the position vectors $\underset{\sim}{a} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} - \underset{\sim}{j} + m\underset{\sim}{k}$ respectively, where $m \in \mathbb{R}$.

Given that the cosine of the angle AOB is $\frac{-1}{\sqrt{6}}$, find the area of the triangle AOB.

Question 8 (5 marks)

a. Show that $\frac{d}{dx}\left[\log_e\left(x+\sqrt{x^2+k}\right)\right]=\frac{1}{\sqrt{x^2+k}}$. 1 mark

Let $h(x)=\frac{\sqrt{x+2}}{\sqrt[4]{x^2+6}}$. Part of the graph of h is shown below.



b. Find the volume generated when the region bounded by the graph of h, the x-axis, and the lines $x=-\sqrt{2}$ and $x=\sqrt{2}$, is rotated about the x-axis. Express your answer in the form $\pi\log_e(b)$, where $b>1$. 4 marks

Question 9 (6 marks)

The velocity vector, in ms^{-1}, of a 5 kg body moving relative to an origin O at time t seconds is given by $\dot{\underset{\sim}{r}}(t) = 2\sin^2(t)\,\underset{\sim}{i} + 4\cos^2(t)\,\underset{\sim}{j}$, where $t \in [0, \pi]$.

a. **i.** Express $\dot{\underset{\sim}{r}}(t)$ in terms of $\cos(2t)$. 1 mark

ii. Hence, find $\underset{\sim}{r}(t)$ given that $\underset{\sim}{r}(0) = \underset{\sim}{i} + 2\underset{\sim}{j}$. 2 marks

b. Find the value(s) of t for which the magnitude of the net force acting on the body is $5\sqrt{5}$ N. 3 marks

Question 10 (4 marks)

Find the arc length of the curve given by $y = \dfrac{e^x + e^{-x}}{2}$ from $x = -1$ to $x = 1$.

END OF QUESTION AND ANSWER BOOK