
Section A – Multiple-choice answers

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|----|---|-----|---|-----|---|-----|---|
| 1. | C | 6. | E | 11. | D | 16. | E |
| 2. | E | 7. | B | 12. | A | 17. | C |
| 3. | B | 8. | A | 13. | A | 18. | C |
| 4. | D | 9. | A | 14. | B | 19. | D |
| 5. | B | 10. | C | 15. | A | 20. | A |

Section A - Multiple-choice solutions

Question 1

For $y = \tan^{-1}\left(\frac{x}{a}\right)$, $\text{range} = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

The graph of $y = \tan^{-1}(x)$ has been dilated by a factor of 'a' from the y-axis. This does not affect the range.

The answer is C.

Question 2

$$\frac{x^2 - 2}{(x-1)^2(x^2+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+2}$$

The answer is E.

Question 3

Since $z = 3\text{cis}\left(\frac{\pi}{5}\right)$,

then $z^6 = 3^6\text{cis}\left(\frac{6\pi}{5}\right)$.

So $\arg(z^6) = \frac{6\pi}{5}$.

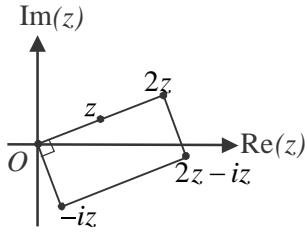
Since $-\pi < \text{Arg}(z^6) \leq \pi$ (formula sheet)

then $\text{Arg}(z^6) = -\frac{4\pi}{5}$

The answer is B.

Question 4

Do a quick sketch.



The complex number z has been rotated 90° in a clockwise direction to become the complex number $-iz$.

Hence the line segment joining O and $2z$ lies at an angle of 90° to the line segment joining O and $-iz$.

The quadrilateral is therefore a rectangle.

$$\begin{aligned} \text{So area} &= |2z||-iz| \\ &= 2|z||z| \text{ since } |-iz| = |z| \\ &= 2|z|^2 \end{aligned}$$

The answer is D.

Question 5

$$\begin{aligned} 1 + \sqrt{3}i &= 2\text{cis}\left(\frac{\pi}{3} + 2k\pi\right) \\ &= 2\text{cis}\left(\frac{\pi + 6k\pi}{3}\right) \\ &= 2\text{cis}\left(\frac{\pi(1 + 6k)}{3}\right) \end{aligned}$$

$$\begin{aligned} r &= \sqrt{1^2 + (\sqrt{3})^2} \\ &= 2 \\ \theta &= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) \\ &= \frac{\pi}{3} + 2k\pi, \quad k \in \mathbb{Z} \end{aligned}$$

$$\begin{aligned} \text{So } z^n &= 1 + \sqrt{3}i \\ &= 2\text{cis}\left(\frac{\pi(1 + 6k)}{3}\right) \\ z &= 2^{\frac{1}{n}}\text{cis}\left(\frac{\pi(1 + 6k)}{3n}\right) \end{aligned}$$

Since z_1 is a solution to the equation $z^n = 1 + \sqrt{3}i$,

$$\text{then } z_1 = 2^{\frac{1}{n}}\text{cis}\left(\frac{\pi(1 + 6k)}{3n}\right)$$

For option A, if $n = 2$, then $|z_1| = 2^{\frac{1}{2}} = \sqrt{2}$ so option A can be true.

For option B, we have $\text{Arg}(z_1) = 0$.

We require $\pi(1 + 6k) = 0$

$$\pi \neq 0, \quad 1 + 6k = 0$$

$$k = -\frac{1}{6}$$

but $k \in \mathbb{Z}$ so option B cannot be true.

The answer is B.

Question 6

$$x = \sqrt{t}, \quad y = e^{2t}, \quad 0 \leq t \leq 3.$$

$$\begin{aligned} \text{length of curve} &= \int_0^3 \sqrt{(x'(t))^2 + (y'(t))^2} dt \\ &= \int_0^3 \sqrt{\left(\frac{1}{2}t^{-\frac{1}{2}}\right)^2 + (2e^{2t})^2} dt \\ &= 402.6839\dots \end{aligned}$$

The closest answer is 402.7.

The answer is E.

Question 7

$$\begin{aligned} &\int_{-2}^0 (x+2)\sqrt{1-x} dx \\ &= \int_3^1 (3-u)\sqrt{u} \times -1 \frac{du}{dx} dx \\ &= \int_1^3 (3u^{\frac{1}{2}} - u^{\frac{3}{2}}) du \end{aligned}$$

The answer is B.

$$\text{Let } u = 1 - x$$

$$\frac{du}{dx} = -1$$

$$\text{Also } x = 1 - u$$

$$x + 2 = 3 - u$$

$$\text{When } x = 0, u = 1$$

$$\text{When } x = -2, u = 3$$

Question 8

$$\frac{dy}{dx} = 15\sqrt{x}(x+2), \quad y(1) = y_0 = 26$$

Using the formula on the formula sheet, and noting that $h = -0.2$,

$$x_0 = 1, \quad y_0 = 26$$

$$\begin{aligned} x_1 = 0.8, \quad y_1 &= 26 - 0.2(15\sqrt{1}(1+2)) \\ &= 17 \end{aligned}$$

$$\begin{aligned} x_2 = 0.6, \quad y_2 &= 17 - 0.2(15\sqrt{0.8}(0.8+2)) \\ &= 9.4868\dots \end{aligned}$$

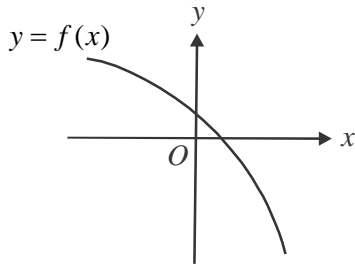
The closest answer is 9.5.

The answer is A.

Question 9

Given that $f'(x) < 0$ and $f''(x) < 0$ then the graph of f has a negative gradient and is concave down over its entire domain.

An example of such a graph is shown.



Both the function and the gradient of f will be strictly decreasing. There can be no non-stationary points of inflection because the graph does not change its concavity i.e. $f''(x)$ is negative over the entire domain.

The answer is A.

Question 10

Along the x -axis, the gradient is zero.

For option A, $\frac{dy}{dx} \neq x^2 + y$ except at the origin so reject option A.

For option B, the same applies so reject option B.

For option C, $\frac{dy}{dx} = y^2$, this satisfies the x -axis markings where $\frac{dy}{dx} = 0$ for all values of x .

When $y = 1$, the gradient is approximately 1 as is the case when $y = -1$.

For other values of y the differential equation also holds.

The answer is C.

Question 11

For option A, a rhombus has all four sides of equal length so $|\underline{a}| = |\underline{b}|$.

For option B, $\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}|\cos(\theta)$.

Since θ , which is the angle between \underline{a} and \underline{b} , is greater than 90° ,

then $\cos(\theta) < 0$

So $\underline{a} \cdot \underline{b} < 0$

For option C, $|\underline{a}| = |\underline{b}|$ so $|\underline{a}| = |-\underline{b}|$.

For option E, the scalar product of the diagonal vectors $\underline{a} - \underline{b}$ and $\underline{a} + \underline{b}$ is zero because these two vectors are perpendicular in a rhombus.

For option D, $|\underline{a}| + |\underline{b}| \neq 0$.

The answer is D.

Question 12

vector resolute of \underline{a} perpendicular to \underline{b}

$$\begin{aligned} &= \underline{a} - (\underline{a} \cdot \hat{\underline{b}}) \hat{\underline{b}} \\ &= 2\underline{i} - 3\underline{j} + \underline{k} - \frac{1}{\sqrt{1+4+4}}(-2-6-2) \times \frac{1}{3}(-\underline{i} + 2\underline{j} - 2\underline{k}) \\ &= 2\underline{i} - 3\underline{j} + \underline{k} + \frac{10}{9}(-\underline{i} + 2\underline{j} - 2\underline{k}) \\ &= \frac{8}{9}\underline{i} - \frac{7}{9}\underline{j} - \frac{11}{9}\underline{k} \\ &= \frac{1}{9}(8\underline{i} - 7\underline{j} - 11\underline{k}) \end{aligned}$$

The answer is A.

Question 13

$$\begin{aligned} \text{distance apart} &= \left| \underline{r}_B - \underline{r}_A \right| \\ &= \left| 2\underline{i} + (-2t-1)\underline{j} \right| \\ &= \sqrt{4 + (-2t-1)^2} \end{aligned}$$

Solve $\sqrt{4 + (-2t-1)^2} < 2.5$ for t .
 $-1.25 < t < 0.25$

but $t \geq 0$

so period of time is closest to 0.25 hour.

The answer is A.

Question 14

$$\begin{aligned} \underline{r}(t) &= 3\sin(t)\underline{i} + \sqrt{a}\cos(t)\underline{j} \\ x &= 3\sin(t) & y &= \sqrt{a}\cos(t) \\ \left(\frac{x}{3}\right)^2 &= \sin^2(t) & \left(\frac{y}{\sqrt{a}}\right)^2 &= \cos^2(t) \\ \frac{x^2}{9} + \frac{y^2}{a} &= 1 \end{aligned}$$

Since the path is circular then $a = 9$.

$$\underline{v}(t) = 3\cos(t)\underline{i} - 3\sin(t)\underline{j}$$

$$\underline{v}\left(\frac{\pi}{2}\right) = -3\underline{j}$$

The direction of motion at $t = \frac{\pi}{2}$ is $-3\underline{j}$.

The answer is B.

Question 15

$$\underline{p} = m \underline{v} \quad (\text{formula sheet})$$

$$\underline{p} = 4(3\underline{i} - 5\underline{j})$$

$$= 12\underline{i} - 20\underline{j}$$

Initial momentum is $12\underline{i} - 20\underline{j}$

change in momentum = final momentum - initial momentum

$$-20\underline{i} + 44\underline{j} = \text{final momentum} - (12\underline{i} - 20\underline{j})$$

$$\text{final momentum} = -20\underline{i} + 44\underline{j} + 12\underline{i} - 20\underline{j}$$

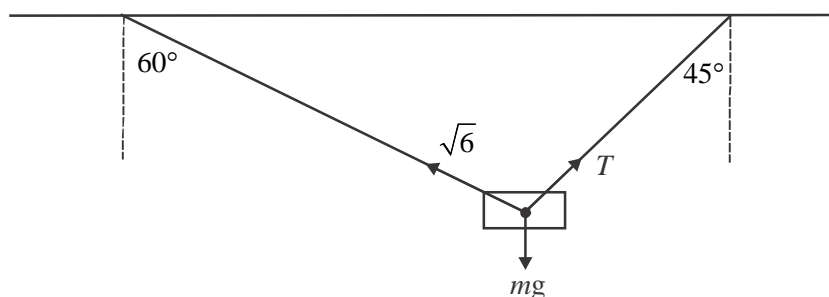
$$= -8\underline{i} + 24\underline{j}$$

$$\text{final velocity} = -2\underline{i} + 6\underline{j} \quad (\text{since } m = 4 \text{ kg})$$

The answer is A.

Question 16

Mark in the forces.



Resolving horizontally:

$$\sqrt{6} \sin(60^\circ) = T \sin(45^\circ)$$

$$\sqrt{6} \times \frac{\sqrt{3}}{2} = T \times \frac{1}{\sqrt{2}}$$

$$T = \frac{\sqrt{18}}{2} \times \sqrt{2}$$

$$T = 3$$

Resolving vertically:

$$\sqrt{6} \cos(60^\circ) + T \cos(45^\circ) = mg$$

$$\frac{\sqrt{6}}{2} + \frac{3}{\sqrt{2}} = mg$$

$$\frac{\sqrt{6} + 3\sqrt{2}}{2} = mg$$

$$m = \frac{\sqrt{2}(\sqrt{3} + 3)}{2g}$$

The answer is E.

Question 17

$$\begin{aligned}
 F &= ma \\
 &= 5(2+4x) \\
 &= 10+20x
 \end{aligned}$$

To find a :

$$\begin{aligned}
 v &= 1+2x \\
 a &= v \frac{dv}{dx} && \text{(formula sheet)}
 \end{aligned}$$

$$\begin{aligned}
 &= (1+2x) \times 2 \\
 &= 2+4x
 \end{aligned}$$

OR

$$\begin{aligned}
 v &= 1+2x \\
 \frac{1}{2}v^2 &= \frac{1}{2}(1+2x)^2 \\
 &= \frac{1}{2}(1+4x+4x^2)
 \end{aligned}$$

$$\begin{aligned}
 \frac{d}{dx} \left(\frac{1}{2}v^2 \right) &= \frac{1}{2}(4+8x) \\
 &= 2+4x
 \end{aligned}$$

$$\text{Since } a = \frac{d}{dx} \left(\frac{1}{2}v^2 \right) \quad \text{(formula sheet)}$$

$$a = 2+4x$$

The answer is C.

Question 18Let X represent the weights of the population of the sea birds.

$$E(X) = 5.2, \quad \text{sd}(X) = 0.4$$

For the sample taken, $n = 7$.

$$\begin{aligned}
 E(\bar{X}) &= 5.2, && \text{sd}(\bar{X}) = \frac{\sigma}{\sqrt{n}} \\
 & && = \frac{0.4}{\sqrt{7}} \\
 & && = 0.1511\dots
 \end{aligned}$$

$$\Pr(\bar{X} < 5) = 0.09293\dots$$

The closest answer is 0.0929.

The answer is C.

Question 19

For the particular sample, $\bar{x} = \frac{68.1 + 72.3}{2}$

$$= 70.2$$

$$\text{confidence interval} = \left(\bar{x} - z \frac{s}{\sqrt{n}}, \bar{x} + z \frac{s}{\sqrt{n}} \right) \quad (\text{formula sheet})$$

confidence interval given = (68.1, 72.3)

$$\text{So,} \quad \bar{x} + z \frac{s}{\sqrt{n}} = 72.3$$

$$\text{i.e.} \quad 70.2 + 1.96 \times \frac{15}{\sqrt{n}} = 72.3$$

$$n = 196$$

$$\text{Alternately,} \quad \bar{x} - z \frac{s}{\sqrt{n}} = 68.1$$

$$70.2 - 1.96 \times \frac{15}{\sqrt{n}} = 68.1$$

$$n = 196$$

The answer is D.

Question 20

Let A represent the annual rainfall in Alban.

Let B represent the annual rainfall in Beachtown.

$$\Pr(B > 2A) = \Pr(B - 2A > 0)$$

$$\begin{aligned} E(B - 2A) &= E(B) - 2E(A) \\ &= 640 - 2 \times 310 \\ &= 20 \end{aligned}$$

$$\begin{aligned} \text{Var}(B - 2A) &= \text{Var}(B) + (-2)^2 \text{Var}(A) \\ &= 625 + 4 \times 100 \\ &= 1025 \end{aligned}$$

$$\begin{aligned} \text{sd}(B - 2A) &= \sqrt{1025} \\ &= 32.0156... \end{aligned}$$

$$B - 2A \sim N(20, 1025)$$

$$\text{So} \quad \Pr(B - 2A > 0) = 0.73391...$$

The closest answer is 0.7339.

The answer is A.

SECTION B

Question 1 (10 marks)

a. $f(x) = 5 \arccos((x-1)^2)$

$$f'(x) = \frac{5 \times -1}{\sqrt{1 - ((x-1)^2)^2}} \times 2(x-1) \times 1 \quad (\text{chain rule})$$

$$= \frac{-10(x-1)}{\sqrt{1 - (x-1)^4}}$$

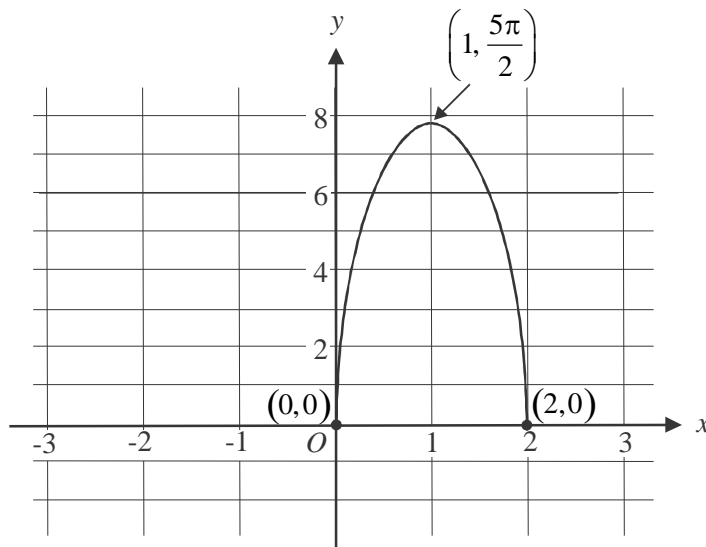
$$= \frac{-10x + 10}{\sqrt{1 - (x-1)^4}}$$

So $a = -10$, $b = 1$ and $c = 4$.

(1 mark)

(1 mark)

b.



(1 mark) – correct shape

(1 mark) – correct stationary point

(1 mark) – correct endpoints

c. i. length = $\int_0^1 \sqrt{1 + \frac{100(x-1)^2}{(1-(x-1)^4)^2}} dx$ using part a.

(1 mark)

ii. length = 8.02124...
= 8.02 units (correct to 2 decimal places)

(1 mark)

iii. $\text{volume} = \pi \int_0^{\frac{5\pi}{2}} x^2 dy$ (1 mark)

Now $y = 5 \arccos((x-1)^2), \quad x \in [0,1]$

$$\frac{y}{5} = \arccos((x-1)^2)$$

$$\cos\left(\frac{y}{5}\right) = (x-1)^2$$

$$x-1 = \pm \sqrt{\cos\left(\frac{y}{5}\right)}$$

$$x = 1 \pm \sqrt{\cos\left(\frac{y}{5}\right)}$$

but $x \in [0,1]$

$$\text{so } x = 1 - \sqrt{\cos\left(\frac{y}{5}\right)}$$

$$\text{So volume} = \pi \int_0^{\frac{5\pi}{2}} \left(1 - \sqrt{\cos\left(\frac{y}{5}\right)}\right)^2 dy$$

(1 mark)

iv. $\text{volume} = 2.74128\dots$
 $= 2.74$ cubic units (correct to 2 decimal places)

(1 mark)

Question 2 (9 marks)

a. $|z - \sqrt{2} - \sqrt{2}i| = \sqrt{2}$
 $|x + yi - \sqrt{2} - \sqrt{2}i| = \sqrt{2}$
 $\sqrt{(x - \sqrt{2})^2 + (y - \sqrt{2})^2} = \sqrt{2}$
 $(x - \sqrt{2})^2 + (y - \sqrt{2})^2 = 2$

(1 mark)

b. $z\bar{z} = 2$
 $(x + yi)(x - yi) = 2$
 $x^2 - xyi + xyi + y^2 = 2$
 $x^2 + y^2 = 2$

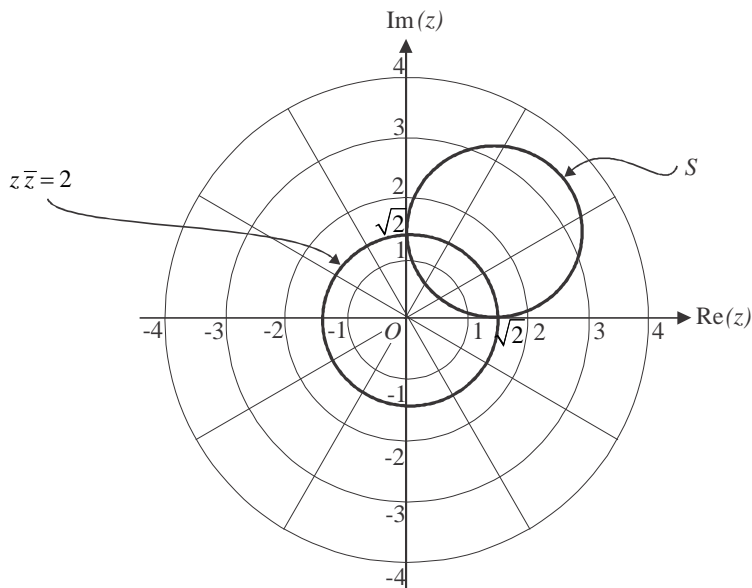
(1 mark)

Using CAS, solve $(x - \sqrt{2})^2 + (y - \sqrt{2})^2 = 2$ and $x^2 + y^2 = 2$ simultaneously.

The two points of intersection are $(0, \sqrt{2})$ and $(\sqrt{2}, 0)$.

(1 mark)

c.

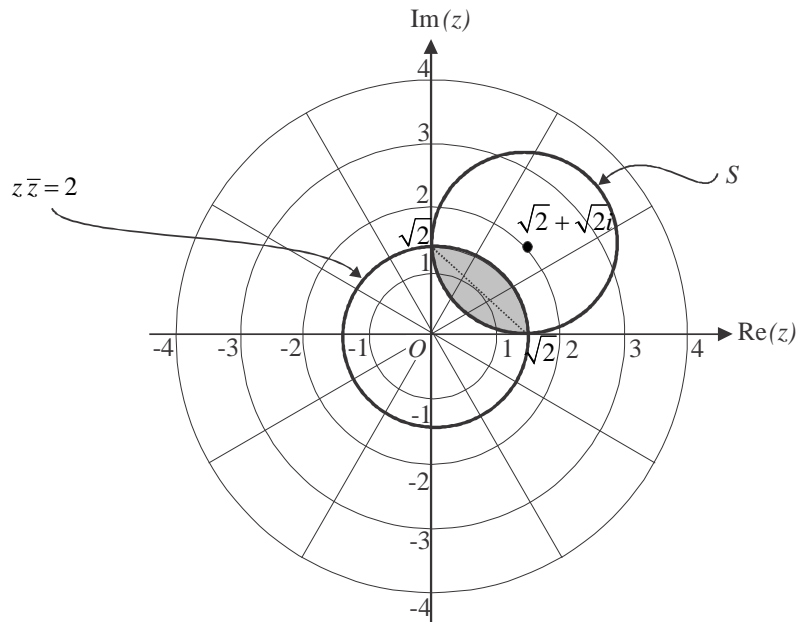
**(1 mark)** for S**(1 mark)** for $z\bar{z} = 2$

d. Using the graph from part c., the minimum value of $\text{Arg}(z)$ for which the ray intersects **both** graphs is zero and the maximum value is $\frac{\pi}{2}$.

$$\text{So } \alpha \in \left[0, \frac{1}{2}\right].$$

(1 mark) – left endpoint**(1 mark)** – right endpoint

- e. The area required is shaded in the diagram below.
Note that the centre of the circle S lies at $\sqrt{2} + \sqrt{2}i$.



area required = $2 \times$ area of minor segment bounded by S

and the line segment passing through the points $(0, \sqrt{2})$ and $(\sqrt{2}, 0)$

$$= 2 \times \left(\frac{1}{4} \times \pi r^2 - \frac{1}{2} \times bc \sin \left(\frac{\pi}{2} \right) \right) \quad \text{(1 mark)}$$

$$= 2 \times \left(\frac{1}{4} \times \pi \times (\sqrt{2})^2 - \frac{1}{2} \times \sqrt{2} \times \sqrt{2} \times 1 \right)$$

$$= 2 \times \left(\frac{\pi}{2} - 1 \right)$$

$$= \pi - 2 \text{ square units}$$

(1 mark)

Question 3 (9 marks)

$$\begin{aligned} \text{a.} \quad \frac{dx}{dt} &= \frac{dx_{\text{inflow}}}{dl} \times \frac{dl_{\text{inflow}}}{dt} - \frac{dx_{\text{outflow}}}{dl} \times \frac{dl_{\text{outflow}}}{dt} \\ &= 0 \times 6 - \frac{x}{100+4t} \times 2 \end{aligned}$$

$$\text{So } \frac{dx}{dt} = \frac{-x}{50+2t}$$

(1 mark)

$$\begin{aligned} \text{b.} \quad \frac{dx}{dt} &= \frac{-x}{50+2t} \\ \int \frac{-1}{x} dx &= \int \frac{1}{50+2t} dt \quad (\text{separation of variables}) \end{aligned}$$

(1 mark)

$$-\log_e(x) + c_1 = \frac{1}{2} \log_e(50+2t) + c_2 \quad x > 0, \quad t \geq 0$$

$$\log_e(x) - c_1 = -\frac{1}{2} \log_e(50+2t) - c_2$$

$$\log_e(x) = \log_e(50+2t)^{-\frac{1}{2}} + c, \quad c = c_1 - c_2$$

When $t=0, x=10$

$$\log_e(10) = \log_e(50)^{-\frac{1}{2}} + c$$

$$c = \log_e(10) - \log_e(50)^{-\frac{1}{2}}$$

$$c = \log_e(50\sqrt{2})$$

(1 mark)

$$\begin{aligned} \text{So,} \quad \log_e(x) &= \log_e \frac{1}{\sqrt{50+2t}} + \log_e(50\sqrt{2}) \\ &= \log_e \frac{50\sqrt{2}}{\sqrt{2(25+t)}} \\ &= \log_e \frac{50}{\sqrt{25+t}} \end{aligned}$$

$$\text{So,} \quad x = \frac{50}{\sqrt{25+t}}$$

(1 mark)

c. The solution given is $s = 10 - \frac{t}{10}$.

So
$$\frac{ds}{dt} = -\frac{1}{10}$$

The differential equation given is

$$\begin{aligned} \frac{ds}{dt} &= \frac{-s}{100-t} \\ RS &= \left(-10 + \frac{t}{10}\right) \div (100-t) \\ &= \frac{-100+t}{10} \times \frac{1}{100-t} \\ &= \frac{-1}{10} \end{aligned}$$

So the solution given satisfies the d.e. given. **(1 mark)**

Check the initial conditions:

In $s = 10 - \frac{t}{10}$,

when $t = 0$, $s = 10$

So the solution given satisfies the initial conditions given. **(1 mark)**

d. Initially there is 400 litres of sugar solution in the tank.
Each minute, 4 litres is lost. So it takes $400 \div 4 = 100$ minutes. **(1 mark)**

e. Concentration of sugar in the sugar solution is $\frac{s}{400-4t}$. **(1 mark)**

Since $s = 10 - \frac{t}{10}$,

then $s = \frac{100-t}{10}$

so the concentration becomes

$$\begin{aligned} &\frac{100-t}{10} \times \frac{1}{400-4t} \\ &= \frac{100-t}{10} \times \frac{1}{4(100-t)} \\ &= \frac{1}{40} \end{aligned}$$

which is constant. **(1 mark)**

Question 4 (11 marks)

- a. i.** Solve $15t \cos(60^\circ) = 6$ for t .
 $t = 0.8$ seconds **(1 mark)**

- ii.** To hit the tiny object, the stone must be 6 m horizontally from the release point and 2.5 m vertically above the release point at the same time.

$$\text{Solve } 15t \sin(60^\circ) - 4.9t^2 = 2.5 \text{ for } t.$$

$$t = 0.2089\dots$$

$$\text{or } t = 2.4421\dots$$

Neither of these values of t equal 0.8 hence the stone cannot be at the point (6, 2.5) and hence cannot hit the tiny object.

(1 mark)

- iii.** From part **i.**, the stone is 6 metres from the release point at $t = 0.8$ seconds. At this time its height above the release point is given by

$$15 \times 0.8 \sin(60^\circ) - 4.9 \times 0.8^2$$

$$= 7.2563\dots$$

(1 mark)

$$\text{Now } 7.2563\dots - 2.5 = 4.7563\dots$$

The stone passes 4.76 m (correct to two decimal places) vertically above the tiny object.

(1 mark)

- iv.** $\underline{r}(t) = 15t \cos(60^\circ) \underline{i} + (15t \sin(60^\circ) - 4.9t^2) \underline{j}$

$$\underline{v}(t) = \frac{15}{2} \underline{i} + \left(\frac{15\sqrt{3}}{2} - 9.8t \right) \underline{j}$$

(1 mark)

The stone is vertically above the tiny object when $t = 0.8$ (from part **a.**).

$$|\underline{v}(0.8)| = \sqrt{\left(\frac{15}{2}\right)^2 + \left(\frac{15\sqrt{3}}{2} - 9.8 \times 0.8\right)^2}$$

$$= 9.0981\dots$$

$$\text{speed} = 9.1 \text{ ms}^{-1}$$

(1 mark)

- b.** The position of the tiny object relative to the release point is (6, 2.5).
 Solve $15t \cos(\alpha) = 6$ and $15t \sin(\alpha) - 4.9t^2 = 2.5$ simultaneously for t and α . **(1 mark)**
 Some solutions must be rejected because $t \geq 0$ and $0 < \alpha < 90^\circ$.

$$t = 0.4651\dots$$

$$\text{and } \alpha = 0.5355\dots$$

$$= 30.6820\dots^\circ$$

$$\text{or } t = 2.8520\dots$$

$$\text{and } \alpha = 1.4300\dots$$

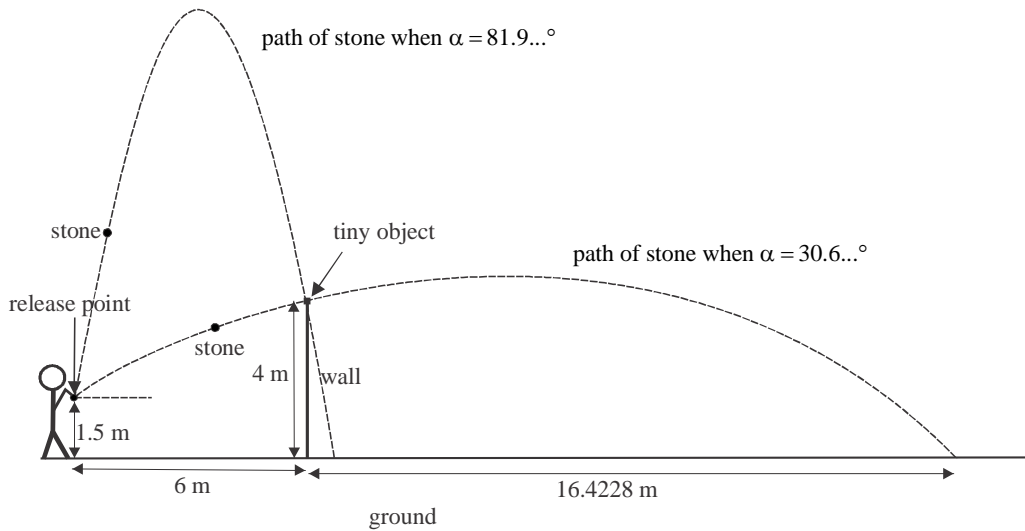
$$= 81.9377\dots^\circ$$

So the two possible angles of projection are 30.7° and 81.9° (both correct to one decimal place).

(1 mark) for 30.7°

(1 mark) for 81.9°

- c. Method 1 – using a diagram
 From part b. $\alpha = 30.6820\dots^\circ$ or $\alpha = 81.9377\dots^\circ$.
 Draw a diagram.



(1 mark)

If the distance of the stone on the ground from the base of the wall on the opposite side of the wall from where it was thrown is 16.4228 metres, then the angle of projection must have been 30.7° , (correct to 1 decimal place).

(1 mark)

Method 2 – using result to part b.

From part b. $\alpha = 30.6820\dots^\circ$ or $\alpha = 81.9377\dots^\circ$

If $\alpha = 30.6820\dots^\circ$ then solve

$$15t \sin(30.6820\dots^\circ) - 4.9t^2 = -1.5 \quad \text{for } t$$

$$t = 1.7381\dots \quad t \geq 0$$

Horizontal distance from projection

$$= 15t \cos(30.6820\dots^\circ)$$

$$= 22.4228\dots$$

(1 mark)

So distance from base of opposite side of wall is $22.4228 - 6 = 16.4228$ m.

The angle of projection must have been 30.7° (correct to 1 decimal place). (1 mark)

If you opted to test $\alpha = 81.9377\dots^\circ$ then solve $15t \sin(81.9377\dots^\circ) - 4.9t^2 = -1.5$ for t .

$$t = 3.1288\dots$$

Horizontal distance from point of projection

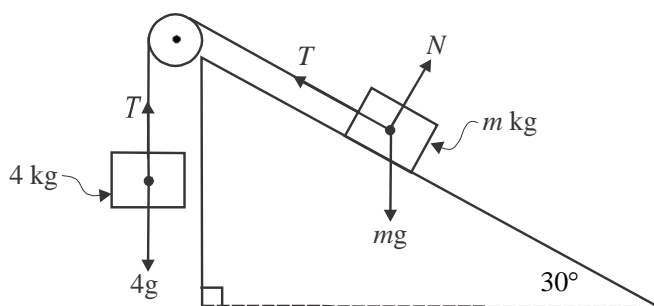
$$= 15t \cos(81.9377\dots^\circ)$$

$$= 6.5821\dots$$

So distance from base of opposite side of wall is $6.5821 - 6 = 0.5821$ so angle of projection can't have been 81.9° and must have been 30.7° .

Question 5 (11 marks)

a.



(1 mark) for forces around the m kg mass
 (1 mark) for forces around the 4 kg mass

b.

Resolving:

Around the 4 kg mass:

$$T = 4g$$

$$\text{So } \frac{mg}{2} = 4g$$

$$m = 8$$

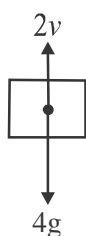
Around the m kg mass

$$T = mg \sin(30^\circ)$$

$$T = \frac{mg}{2}$$

(1 mark)

c.



The mass accelerates downwards so the equation of motion is

$$4g - 2v = 4a$$

$$a = \frac{4g - 2v}{4}$$

$$a = g - \frac{v}{2}$$

as required

(1 mark)

d.

$$a = g - \frac{v}{2} \quad \text{from part c.}$$

$$\frac{dv}{dt} = g - \frac{v}{2}$$

$$= \frac{2g - v}{2}$$

$$\frac{dt}{dv} = \frac{2}{2g - v}$$

$$t = \int \frac{2}{2g - v} dv$$

(1 mark)

Method 1 – using CAS

Using the boundary conditions

$$t = 0, v = 0,$$

$$t = 2 \log_e \left(\frac{2g}{2g - v} \right)$$

(1 mark)

Method 2 – by hand

$$t = -2 \log_e (2g - v) + c$$

When $t = 0$, $v = 0$,

$$0 = -2 \log_e (2g) + c$$

$$c = 2 \log_e (2g)$$

So $t = -2 \log_e (2g - v) + 2 \log_e (2g)$

$$t = 2 \log_e \left(\frac{2g}{2g - v} \right) \quad (1 \text{ mark})$$

e.
$$t = 2 \log_e \left(\frac{2g}{2g-v} \right) \quad (\text{from part d.})$$

$$e^{\frac{t}{2}} = \frac{2g}{2g-v}$$

$$2g-v = 2ge^{-\frac{t}{2}}$$

$$v = 2g - 2ge^{-\frac{t}{2}}$$

as $t \rightarrow \infty$, $e^{-\frac{t}{2}} \rightarrow 0$, $v \rightarrow 2g$.

The limiting (terminal velocity) is $2g \text{ ms}^{-1}$.

(1 mark)

f.
$$v = 2g - 2ge^{-\frac{t}{2}} \quad (\text{from part e.})$$

$$\frac{dx}{dt} = 2g - 2ge^{-\frac{t}{2}}$$

$$x = \int_0^3 (2g - 2ge^{-\frac{t}{2}}) dt$$

$$= 28.3467\dots$$

The liquid is 28.3 metres deep (correct to 1 decimal place).

(1 mark)

(1 mark)

g.
$$a = g - \frac{v}{2} \quad (\text{from part c.})$$

$$v \frac{dv}{dx} = g - \frac{v}{2}$$

$$= \frac{2g-v}{2}$$

$$\frac{dv}{dx} = \frac{2g-v}{2v}$$

$$\frac{dx}{dv} = \frac{2v}{2g-v}$$

$$x = \int_0^g \frac{2v}{2g-v} dv$$

$$= 7.5713\dots$$

The 4 kg mass is 7.6 metres below the surface (correct to 1 decimal place) when

$v = g \text{ ms}^{-1}$.

(1 mark)

(1 mark)

Question 6 (10 marks)

- a. Let S represent the distribution of weights of small muffins.
Let L represent the distribution of weights of large muffins.

$$L = 2S \quad (\text{i.e. } L \text{ is a multiple of } S)$$

Since $S \sim N(108, 2^2)$

$$L \sim N(2 \times 108, 2^2 \times 2^2)$$

i.e. $L \sim N(216, 16)$

$$\begin{aligned} \text{mean of } L \text{ is } 216 \quad \text{sd}(L) &= \sqrt{16} \\ &= 4 \end{aligned}$$

(1 mark) – mean

(1 mark) – sd

- b. Let B represent the distribution of weights of boxes of 25 small muffins.

$$B = S_1 + S_2 + S_3 + \dots + S_{25}$$

$$B \sim N(25 \times 108, 25 \times 2^2)$$

i.e. $B \sim N(2700, 100)$

Mean weight of a box is 2700 grams.

(1 mark)

Standard deviation of the weight of a box is $\sqrt{100} = 10$ grams.

(1 mark)

- c. $\Pr(B < 2680) = 0.02275\dots$

$$= 0.023 \quad (\text{correct to 3 decimal places})$$

(1 mark)

- d. $H_0 : \mu = 450$

$$H_1 : \mu < 450$$

(1 mark)

- e. p value = $\Pr(\bar{X} < 440 | \mu = 450)$

(1 mark)

$$= \Pr\left(Z < \frac{440 - 450}{20/\sqrt{30}}\right)$$

$$= \Pr(Z < -2.7386\dots)$$

$$= 0.003085\dots$$

$$= 0.0031 \quad (\text{correct to 4 decimal places})$$

$$\begin{aligned} \text{sd}(\bar{X}) &= \frac{\sigma}{\sqrt{n}} \\ &= \frac{20}{\sqrt{30}} \end{aligned}$$

(1 mark)

- f. Since $p < 0.05$ (i.e. $p = 0.0031$ from part e.) we should reject H_0 .

(1 mark)

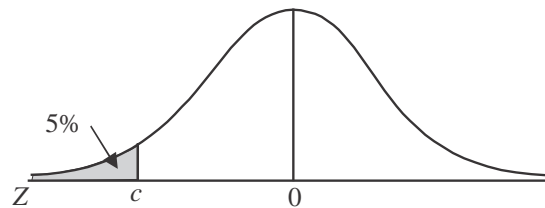
- g. Find c such that $\Pr(Z < c) = 0.05$

$$c = -1.6448\dots$$

$$\text{Solve } \frac{\bar{X} - 450}{20/\sqrt{30}} < -1.6448\dots$$

$$\bar{X} < 443.9938\dots$$

$$\text{So } \bar{X} = 443.9$$



(1 mark)

Note that the largest number (correct to 1 decimal place) that is less than $443.9938\dots$ is 443.9 . Rounding $443.9938\dots$ to one decimal place, does not answer the question.

(ie $\Pr(\bar{X} < 444.0) > 0.05$ but $\Pr(\bar{X} < 443.9) < 0.05$)