VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2018

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER									Letter

SPECIALIST MATHEMATICS
Written examination 1

Tuesday 5 June 2018

Reading time: 2.00 pm to 2.15 pm (15 minutes)
Writing time: 2.15 pm to 3.15 pm (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

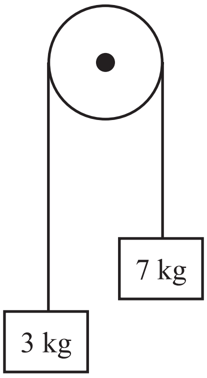
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (3 marks)

A light inextensible string hangs over a frictionless pulley connecting masses of 3 kg and 7 kg, as shown below.



a. Draw all of the forces acting on the two masses on the diagram above. 1 mark

b. Calculate the tension in the string. 2 marks

Question 2 (3 marks)

Let $\underset{\sim}{a} = 3\underset{\sim}{i} - 2\underset{\sim}{j} + m\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$, where $m \in R$.

Find the value(s) of m such that the magnitude of the vector resolute of $\underset{\sim}{a}$ parallel to $\underset{\sim}{b}$ is equal to $\sqrt{14}$.

Question 3 (3 marks)

Find $\sin(t)$ given that $t = \arccos\left(\frac{12}{13}\right) + \arctan\left(\frac{3}{4}\right)$.

Question 4 (4 marks)

Throughout this question, use an integer multiple of standard deviations in calculations.

The standard deviation of all scores on a particular test is 21.0

a. From the results of a random sample of n students, a 95% confidence interval for the mean score for all students was calculated to be (44.7, 51.7).

Calculate the mean score and the size of this random sample. 2 marks

b. Determine the size of another random sample for which the endpoints of the 95% confidence interval for the population mean of the particular test would be 1.0 either side of the sample mean. 2 marks

Question 5 (4 marks)

Evaluate $\displaystyle\int_{1}^{2\sqrt{3}-1}\left(\frac{1}{x^2+2x+5}\right)dx$.

Question 6 (4 marks)

Given that $y = (x - 1)e^{2x}$ is a solution to the differential equation $a\dfrac{d^2y}{dx^2} + b\dfrac{dy}{dx} = y$, find the values of a and b, where a and b are real constants.

Question 7 (4 marks)

a. Find $\frac{d}{dx}\left((1-x^2)^{\frac{1}{2}}\right)$. 2 marks

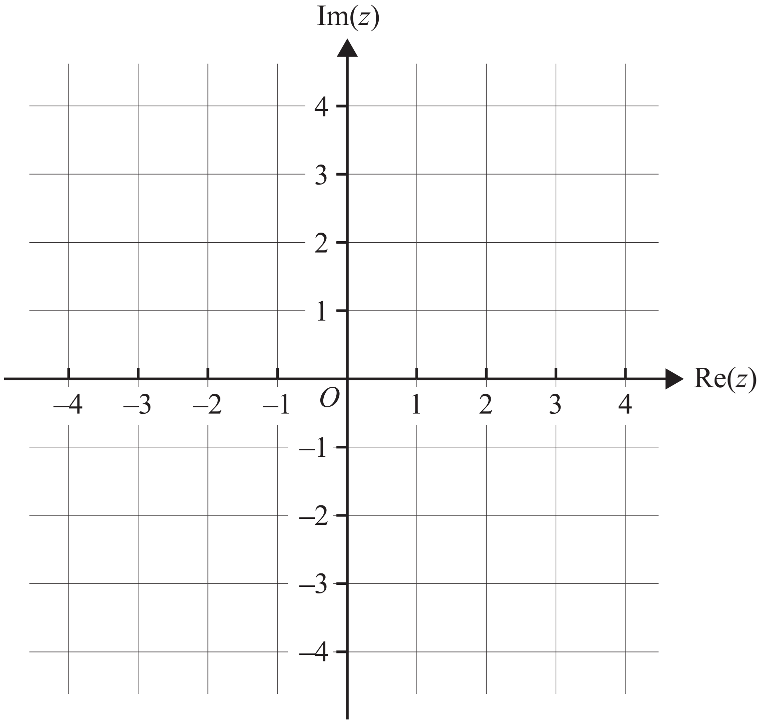
b. Hence, find the length of the curve specified by $y=\sqrt{1-x^2}$ from $x=\frac{1}{2}$ to $x=\frac{\sqrt{3}}{2}$. Give your answer in the form $k\pi$, $k \in R$. 2 marks

TURN OVER

Question 8 (6 marks)

A circle in the complex plane is given by the relation $|z - 1 - i| = 2$, $z \in C$.

a. Sketch the circle on the Argand diagram below. 1 mark



b. **i.** Write the equation of the circle in the form $(x - a)^2 + (y - b)^2 = c$ and show that the gradient of a tangent to the circle can be expressed as $\frac{dy}{dx} = \frac{1-x}{y-1}$. 2 marks

ii. Find the gradient of the tangent to the circle where $x = 2$ in the first quadrant of the complex plane. 1 mark

c. Find the equations of all rays that are perpendicular to the circle in the form $\text{Arg}(z) = \alpha$. 2 marks

DO NOT WRITE IN THIS AREA

Question 9 (9 marks)

a. **i.** Given that $\cot(2\theta) = a$, show that $\tan^2(\theta) + 2a\tan(\theta) - 1 = 0$. 2 marks

ii. Show that $\tan(\theta) = -a \pm \sqrt{a^2 + 1}$. 1 mark

iii. Hence, show that $\tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}$, given that $\cot(2\theta) = \sqrt{3}$, where $\theta \in (0, \pi)$. 1 mark

b. Find the gradient of the tangent to the curve $y = \tan(\theta)$ at $\theta = \frac{\pi}{12}$. 2 marks

Question 9 – continued

c. A solid of revolution is formed by rotating the region between the graph of $y = \tan(\theta)$, the horizontal axis, and the lines $\theta = \frac{\pi}{12}$ and $\theta = \frac{\pi}{3}$ about the horizontal axis.

Find the volume of the solid of revolution. 3 marks

END OF QUESTION AND ANSWER BOOK

DO NOT WRITE IN THIS AREA

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SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\operatorname{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \operatorname{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}\left(\theta_1 - \theta_2\right)$
$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$ $\mathrm{E}(aX + bY) = a\mathrm{E}(X) + b\mathrm{E}(Y)$ $\operatorname{var}(aX + b) = a^2\operatorname{var}(X)$
for independent random variables X and Y	$\operatorname{var}(aX + bY) = a^2\operatorname{var}(X) + b^2\operatorname{var}(Y)$
approximate confidence interval for μ	$\left(\overline{x} - z\frac{s}{\sqrt{n}},\ \overline{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\overline{X}$	mean $\mathrm{E}\left(\overline{X}\right) = \mu$ variance $\operatorname{var}\left(\overline{X}\right) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \log_e\lvert x\rvert + c$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
	$\int (ax+b)^{-1}\,dx = \frac{1}{a}\log_e\lvert ax+b\rvert + c$
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2}\sqrt{1+\left(f'(x)\right)^2}\,dx$ or $\int_{t_1}^{t_2}\sqrt{\left(x'(t)\right)^2+\left(y'(t)\right)^2}\,dt$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert = \sqrt{x^2+y^2+z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2\cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

END OF FORMULA SHEET