



---

Trial Examination 2018

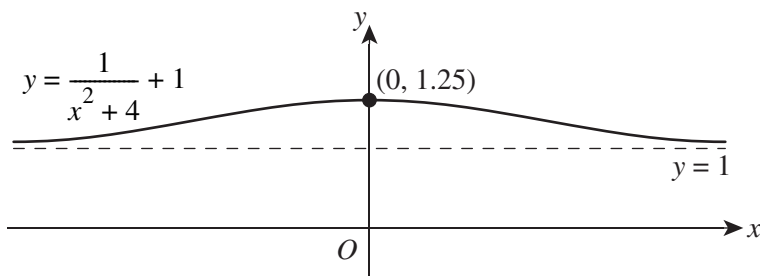
# **VCE Specialist Mathematics Units 3&4**

Written Examination 1

**Suggested Solutions**

---

Neap Trial Exams are licensed to be photocopied or placed on the school intranet and used only within the confines of the school purchasing them, for the purpose of examining that school's students only. They may not be otherwise reproduced or distributed. The copyright of Neap Trial Exams remains with Neap. No Neap Trial Exam or any part thereof is to be issued or passed on by any person to any party inclusive of other schools, non-practising teachers, coaching colleges, tutors, parents, students, publishing agencies or websites without the express written consent of Neap.

**Question 1** (3 marks)

correct shape (concavity and asymptotic behaviour) A1  
 y-coordinate and stationary point is  $(0, 1.25)$  A1  
 horizontal asymptote is  $y = 1$  A1

**Question 2** (3 marks)

The parametric equations are  $x = 2 - t^2$  and  $y = 4t$ .

Use  $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$  to obtain  $\frac{dy}{dx} = -\frac{2}{t}$ . M1

Let the gradient of the normal be  $m_N$ .

When  $t = 1$ ,  $\frac{dy}{dx} = -2$  and so,  $m_N = \frac{1}{2}$ . M1

When  $t = 1$ ,  $x = 1$  and  $y = 4$ .

So,  $y = \frac{1}{2}x + \frac{7}{2}$ . A1

**Question 3** (3 marks)

Let  $X$  represent the weight of the lemons.

$X \sim N(58, 9^2)$

$\bar{X} \sim N\left(58, \frac{9^2}{36}\right)$ , that is,  $E(\bar{X}) = 58$  and  $sd(\bar{X}) = \frac{9}{6}$  ( $= 1.5$ ) A1

$\Pr(\bar{X} > 61) = \Pr(\bar{X} > 58 + 2 \times 1.5)$  M1

$= 0.025$  A1

**Question 4** (3 marks)

The equations of motion are  $T = 3ma$  and  $2mg - T = 2ma$ . A1

$3mg - \frac{3T}{2} = 3ma \Rightarrow 3mg - \frac{3T}{2} = T$  M1

Solving for  $T$  gives  $T = \frac{6mg}{5}$  (N). A1

**Question 5** (5 marks)

It is given that  $x^3 + xy^2 - y^3 = 2$ .

Using implicit differentiation obtains  $3x^2 + y^2 + 2xy\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$ .

M1 A1

$$\frac{dy}{dx} = 0 \Rightarrow 3x^2 + y^2 = 0$$

A1

So,  $x = y = 0$ .

A1

Formal verification (that is the substitution of  $x = 0$  and  $y = 0$  into  $x^3 + xy^2 - y^3 = 2$ ) shows that  $(0, 0)$  is not on  $C$ .

M1

Hence, there is no point on  $C$  at which  $\frac{dy}{dx} = 0$ .

**Question 6** (5 marks)

The equation to solve is  $z^3 = -2 + 2i$ .

$$z^3 = 2\sqrt{2}\text{cis}\left(\frac{3\pi}{4} + 2k\pi\right), k = 0, 1, 2$$

M1

$$z = \sqrt{2}\text{cis}\left(\frac{\pi}{4} + \frac{2k\pi}{3}\right), k = 0, 1, 2$$

A1

$$z_1 = \sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$$

A1

Adding or subtracting  $\frac{2\pi}{3}$  to obtain the other two solutions:

$$z_2 = \sqrt{2}\text{cis}\left(\frac{11\pi}{12}\right), z_3 = \sqrt{2}\text{cis}\left(-\frac{5\pi}{12}\right)$$

A2

**Question 7** (3 marks)

The question asks to prove that  $\underline{p} \cdot \underline{q} = |\underline{p}|^2$ .

$$\angle OPQ = 90^\circ \text{ and so } \overrightarrow{PO} \cdot \overrightarrow{PQ} = |\overrightarrow{PO}| |\overrightarrow{PQ}| \cos(\angle OPQ) = 0$$

M1

$$\overrightarrow{PO} = -\underline{p} \text{ and } \overrightarrow{PQ} = \underline{q} - \underline{p}, \text{ so } -\underline{p} \cdot (\underline{q} - \underline{p}) = 0.$$

A1

$$-\underline{p} \cdot \underline{q} + \underline{p} \cdot \underline{p} = 0$$

$$\underline{p} \cdot \underline{q} = \underline{p} \cdot \underline{p} \text{ (dot product is distributive)}$$

A1

$$\text{So } \underline{p} \cdot \underline{q} = |\underline{p}|^2.$$

**Question 8** (5 marks)

- a. It is given that  $a = 2x - \frac{1}{6}x^2$ ,  $0 \leq x \leq 18$ .

**Method 1:**

$$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) \text{ and so } \frac{d}{dx}\left(\frac{1}{2}v^2\right) = 2x - \frac{1}{6}x^2$$

$$\int \frac{d}{dx}\left(\frac{1}{2}v^2\right) dx = \int \left(2x - \frac{1}{6}x^2\right) dx$$

M1

$$\frac{1}{2}v^2 = x^2 - \frac{1}{18}x^3 + c$$

A1

When  $x = 0$ ,  $v = 0$  and so  $c = 0$ .

$$\text{So } v^2 = 2x^2 - \frac{1}{9}x^3.$$

A1

**Method 2:**

$$a = v\frac{dv}{dx} \text{ and so } v\frac{dv}{dx} = 2x - \frac{1}{6}x^2$$

$$\int v dv = \int \left(2x - \frac{1}{6}x^2\right) dx$$

M1

$$\frac{1}{2}v^2 = x^2 - \frac{1}{18}x^3 + c$$

A1

When  $x = 0$ ,  $v = 0$  and so  $c = 0$ .

$$\text{So } v^2 = 2x^2 - \frac{1}{9}x^3.$$

A1

- b.  $v = 0 \Rightarrow 2x^2 - \frac{1}{9}x^3 = 0$

M1

$$x^2\left(2 - \frac{1}{9}x\right) = 0$$

$$x = 18 \text{ (m) } (x > 0)$$

A1

**Question 9** (5 marks)

a.  $\sin^2(\theta) = \frac{1}{4}(1 - \cos(2\theta))$

$$\sin^4(\theta) = \frac{1}{4}(1 - \cos(2\theta))^2 \quad \text{M1}$$

$$= \frac{1}{4}(1 - 2\cos(2\theta) + \cos^2(2\theta)) \quad \text{A1}$$

$$= \frac{1}{4}\left(1 - 2\cos(2\theta) + \frac{1 + \cos(4\theta)}{2}\right) \text{ (or equivalent)} \quad \text{A1}$$

Leading to  $\sin^4(\theta) = \frac{1}{8}(3 - 4\cos(2\theta) + \cos(4\theta))$ .

b.  $\int_0^{\frac{\pi}{4}} \sin^4(\theta) d\theta = \int_0^{\frac{\pi}{4}} \frac{1}{8}(3 - 4\cos(2\theta) + \cos(4\theta)) d\theta$  (from **part a.**)

$$= \frac{1}{8} \left[ 3\theta - 2\sin(2\theta) + \frac{\sin(4\theta)}{4} \right]_0^{\frac{\pi}{4}} \quad \text{M1}$$

$$= \frac{1}{8} \left( \frac{3\pi}{4} - 2\sin\left(\frac{\pi}{2}\right) + \frac{\sin(\pi)}{4} - \left( 0 - 2\sin(0) + \frac{\sin(0)}{4} \right) \right)$$

$$= \frac{3\pi - 8}{32} \quad \text{A1}$$

**Question 10** (5 marks)

Solve  $\arccos(x) + \arccos(2x) = \frac{\pi}{2}$  for  $x$  where  $x \geq 0$ .

**Method 1:**

Apply  $\cos$  to both sides of the equation  $\arccos(2x) = \frac{\pi}{2} - \arccos(x)$  to obtain:

$$2x = \cos\left(\frac{\pi}{2} - \arccos(x)\right) \quad \text{A1}$$

Use of  $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$  on the RHS to obtain:

$$2x = \cos\left(\frac{\pi}{2}\right)\cos(\arccos(x)) + \sin\left(\frac{\pi}{2}\right)\sin(\arccos(x)) \quad \text{M1}$$

$$\text{So, the equation becomes } 2x = \sqrt{1 - x^2}. \quad \text{A1}$$

$$\text{Squaring both sides gives: } 4x^2 = 1 - x^2. \quad \text{M1}$$

Solving this equation gives:

$$x = \frac{1}{\sqrt{5}} \quad (x \geq 0) \quad \text{A1}$$

**Method 2:**

Apply  $\cos$  to both sides of the equation to obtain:

$$\cos(\arccos(x) + \arccos(2x)) = \cos\left(\frac{\pi}{2}\right) \quad \text{A1}$$

Use of  $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$  on the LHS to obtain:

$$\begin{aligned} \cos(\arccos(x) + \arccos(2x)) &= \cos(\arccos(x))\cos(\arccos(2x)) - \sin(\arccos(x))\sin(\arccos(2x)) \\ &= 2x^2 - \left(\sqrt{1 - x^2}\right)\left(\sqrt{1 - 4x^2}\right) \end{aligned} \quad \text{M1}$$

$$\text{So, the equation becomes } 2x^2 - \left(\sqrt{1 - x^2}\right)\left(\sqrt{1 - 4x^2}\right) = 0. \quad \text{A1}$$

$$\text{Rearranging and squaring both sides gives: } 4x^4 = 4x^4 - 5x^2 + 1. \quad \text{M1}$$

Solving this equation gives:

$$x = \frac{1}{\sqrt{5}} \quad (x \geq 0) \quad \text{A1}$$