The Mathematical Association of Victoria

Trial Examination 2018

SPECIALIST MATHEMATICS

Trial Written Examination 2 - SOLUTIONS

SECTION A: Multiple Choice

Question	Answer	Question	Answer
1	E	**11**	B
2	D	**12**	C
3	A	**13**	D
4	A	**14**	E
5	B	**15**	B
6	B	**16**	A
7	C	**17**	C
8	A	**18**	D
9	D	**19**	E
10	C	**20**	E

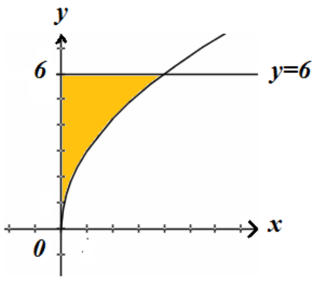
Question 1 **Answer E**

$$y = \frac{a}{\pi}\arctan(bx - 3)$$

if $f(x) = \arctan(x)$ where $dom_f = R, ran_f = \left(-\frac{a}{2}, \frac{a}{2}\right)$

Since $y = \frac{a}{\pi}f(bx - 3)$ it has a dilation of factor $\frac{a}{\pi}$ from x-axis from the $f(x) = \arctan(x)$ function which results in range of $(-\frac{a}{2}, \frac{a}{2})$. Further dilations from y-axis and translations in the direction of the x-axis result in domain of R

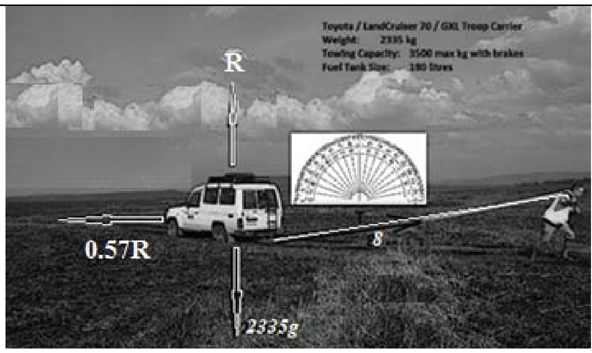
Question 2 **Answer D**

	$x = \frac{y^2}{9} = f(y)$ $V = \pi \int_0^6 f(y)^2\, dy = \pi \int_0^6 [(\frac{y^2}{9})]^2\, dy$ The volume of the solid generated is $\frac{96\pi}{5}$ cubic units

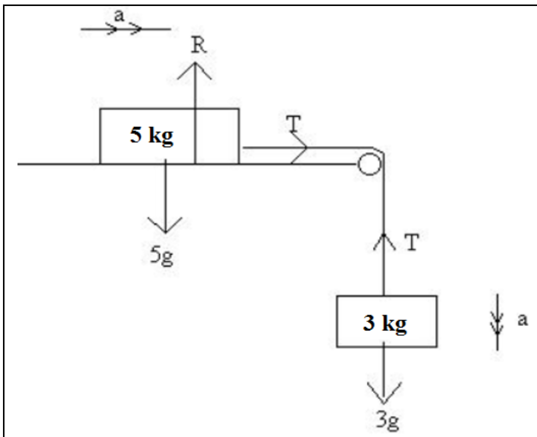
Question 3 **Answer A**

$y = e^{kx}$ $\frac{dy}{dx} = ke^{kx}$ $\frac{d^2y}{dx^2} = k^2e^{kx}$ $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$	$k^2e^{kx} - 4ke^{kx} + 3e^{kx} = 0$ $e^{kx}(k^2 - 4k + 3) = 0$ $e^{kx}(k-3)(k-1) = 0$ $k = 1\ or\ k = 3$

Question 4 **Answer A**

$\sum F$=0 when stationary $R + T\sin(8°) = 2335g$ (1) $T\cos(8°) = 0.57R$ (2) $T = 12195\ Newtons$ $T > 12195\ Newtons$	

Question 5 **Answer B**

 	Using Newton's Second Law on 5kg body (horizontally): $T = 5a$ (1) Using Newton's Second Law on 3kg body (vertically): $3g - T = 3a$ (2) Solving (1) and (2) simultaneously: $a = \frac{3g}{8}$ $T = \frac{15g}{8}$ Therefore the acceleration is 3.68 ms^{-2}

Question 6 **Answer B**

$$\dot{\boldsymbol{r}}(t) = \frac{20}{8}\cos(\frac{t}{8})\,\boldsymbol{i} + \frac{12}{8}\sin\left(\frac{t}{8}\right)\boldsymbol{j} = 2.5\cos(\frac{t}{8})\,\boldsymbol{i} + 1.5\sin\left(\frac{t}{8}\right)\boldsymbol{j}$$

Magnitude of the velocity vector = speed $s = |\dot{\boldsymbol{r}}(t)| = \sqrt{6.25cos^2\left(\frac{t}{8}\right) + 2.25sin^2\left(\frac{t}{8}\right)}$

Question 7 **Answer C**

$\frac{d^2y}{dx^2} = -9x^2$ $\frac{dy}{dx} = \int -9x^2\, dx = -3x^3 + C_1$ when $x = 1, \frac{dy}{dx} = 1 \therefore C_1 = 4$ $y = \int -3x^3 + 4\, dx$	$y = -\frac{3}{4}x^4 + 4x + C_2$ when $x = 1, y = -\frac{3}{4} \therefore C_2 = -4$ $y = -\frac{3}{4}x^4 + 4x - 4$

Question 8 **Answer A**

$v = \int \sqrt{t+1}dt$ $v = 2(t+1)^{0.5} + C$ At $t = 8, v = 9$ $9 = 2\sqrt{(8+1)} + C$ $C = 3$	$v = 2\sqrt{(t+1)} + 3$ Find t when $v = 13$ $13 = 2\sqrt{(t+1)} + 3$ $t = 24\ s$

Question 9 **Answer D**

$x(t) = 2t^3 + 6t$
$y(t) = 6\sin(t) - 3t$

$x'(t) = 6t^2 + t$
$y'(t) = 6\cos(t) - 3$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = (6\cos(t) - 3) \times \frac{1}{(6t^2+t)}$$

$$\frac{dy}{dx} = \frac{\cos(t) - \frac{1}{2}}{t^2 + 1}$$

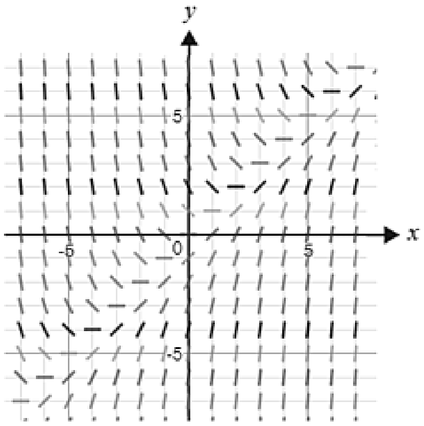
Question 10 **Answer C**

$\frac{dy}{dx} = xy + 1$ and $x_0 = 0$ and $y_0 = 1$ using $h = 0.1$, the value of y_3

Let $g(x, y) = \frac{dy}{dx}$ then $y_{i+1} = y_i + h(g(x_i, y_i)$

i	x_i	y_i	$y_i + h(g(x_i, y_i)$
0	0.0	1	
1	0.1	1.1	$1 + (0.1)[(0)(1) + 1]$
2	0.2	1.211	$1.1 + (0.1)[(0.1)(1.1) + 1]$
3	0.3	1.335	$1.211 + (0.1)[(0.2)(1.211) + 1]$

Question 11 **Answer B**



$$\frac{dy}{dx} = x - y$$

Question 12 **Answer C**

$\sin^{-1}(\cos^2(x)) = \frac{\pi}{6}, -\pi < x < \pi$

$\cos^2(x) = \frac{1}{2}, \cos(x) = \pm\frac{1}{\sqrt{2}}$

$x = -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$

Question 13 **Answer D**

$(z^2 - 2zi - 1)(z^2 + 2i)(z^2 + 2zi + 2) = 0$

$z = -1 + i$, $z = 1 - i$, $z = i$, $z = i$, $z = -i - \sqrt{3}\,i$, $z = -i + \sqrt{3}\,i$

Five distinct roots

Question 14 **Answer E**

$2x - 3y = 1, \quad y = \frac{2}{3}x - \frac{1}{3}$

// to vector $a = 3i + 2j$

$\perp$ vector $b = 2i - 3j$ as ***a.b*** = 0 or just use gradients

Question 15 **Answer B**

$$\frac{\tilde{z}}{i} = a + ib$$
$$z = ia - b$$
$$= -b + ia$$
$$z = -b - ai$$

Question 16 **Answer A**

$$z + \tilde{z} = 1$$
$$(x + iy) + (x - iy) = 1$$
$$x = \frac{1}{2}$$

Question 17 **Answer C**

E(X) = E(Y) = $\frac{21}{6}$, Var(X) = Var(Y) = $\frac{35}{12}$

Var(2X– Y) = 4 Var(X) + Var(Y) = 5Var(X) = $\frac{175}{12}$

Sd(2X – Y) = $\frac{5\sqrt{21}}{6}$

Question 18 **Answer D**

$H_0 : \mu = 12$

$H_1 : \mu < 12$

n = 50, $\bar{x}$ = 11.8, sd =0.5

p = 0.0023

Question 19 **Answer E**

$m = a\text{cis}(\theta_1)$, $n = 3\text{cis}(\theta_2)$, $mn = \frac{1}{2}\text{ cis}\left(-\frac{7\pi}{12}\right)$

$3a = \frac{1}{2}$, $a = \frac{1}{6}$

$\theta_1 + \theta_2 = -\frac{7\pi}{12}$

Checking

$$\frac{2\pi}{3} + \frac{3\pi}{4} = \frac{17\pi}{12}$$

$$= -\frac{7\pi}{12}$$

Question 20 **Answer E**

$a = 2i + j - k$ and $b = i - j + 2k$

Let $c = qi + rj + sk$

Then $2q + r - s = 0$

$q - r + 2s = 0$

$$q = -\frac{s}{3}$$

$$r = \frac{5s}{3}$$

Conditions satisfied by $c = i - 5j - 3k$

SECTION B – Extended Response Questions

Question 1 (10 marks)

Observing a new fashion trend that takes off in the city of Trendigo, which has a maximum population of 100000 people, the rate of the spread is modelled by the differential equation:

$$\frac{dN}{dt} = \frac{0.5N(100000 - N)}{100000}$$

Where N is the number of people adopting the trend and t is the number of weeks since the trend began.

a. State an integral that when evaluated, gives t, the number of weeks since the trend began in terms of N, the number of people adopting the trend. 1 mark

separation of variables $\int \frac{100000}{N(100000-N)} dN = \int 0.5\, dt$
$t = \int \frac{200000}{N(100000-N)} dN$ (1 mark)

b. Hence or otherwise, give N, the number of people adopting the trend in terms of t, the number of weeks since the trend began, given that 1000 people started this trend.

Give your answer in the form: $N(t) = \frac{100000}{1+ae^{-bt}}$, where $a, b \in R^+$ 3 marks

$\int \frac{1}{N} + \frac{1}{(100000 - N)} dN = 0.5t + C$	$\ln\left(\frac{99N}{100000 - N}\right) = 0.5t$
$\ln\lvert N\rvert - \ln\lvert 100000 - N\rvert = 0.5t + C$	$-\ln\left(\frac{99N}{100000 - N}\right) = -0.5t$
$\ln\left\lvert\frac{N}{100000-N}\right\rvert = 0.5t + C$	$\ln\left(\frac{100000 - N}{99N}\right) = -0.5t$
$when\ t = 0, N = 1000$	$\frac{100000-N}{99N} = e^{-0.5t}$ (1 mark)
$\ln\left\lvert\frac{1000}{99000}\right\rvert = 0.5(0) + C$	$100000 - N = 99Ne^{-0.5t}$
$\ln\left\lvert\frac{1}{99}\right\rvert = C$	$100000 = 99Ne^{-0.5t} + N$
$\ln\left\lvert\frac{N}{100000-N}\right\rvert - \ln\left\lvert\frac{1}{99}\right\rvert = 0.5t$ (1 mark)	$100000 = N(99e^{-0.5t} + 1)$
$\ln\left\lvert\frac{99N}{100000 - N}\right\rvert = 0.5t$	$N(t) = \frac{100000}{99e^{-0.5t}+1}$ (1 mark)
remove modulus sign since	
$N \in [1000,100000]$	

Question 1 (continued)

c. Determine the estimated time to the nearest day at which the trend is spreading at the greatest rate in Trendigo. 2 marks

Since there is a positive gradient on graph for t>=0 with a non-stationary point of inflection
hence solve
$N''(t) = 0$ to find when the gradient is increasing at the maximum rate (1 mark)

$$\frac{d^2N}{dt^2} = \frac{100000-2N}{200000}$$

$$\frac{d^2N}{dt^2} = 0 \Rightarrow N = 50000$$

Max rate of growth at t=9.19024 weeks
which is approximately 64 days (1 mark)

Define $f(x)=\frac{100000}{1+99\cdot e^{-0.5\cdot x}}|x\geq 0$ Done

Define $df(x)=\frac{d}{dx}(f(x))$ Done

$df(x)$ $\left\{\frac{4.95E6\cdot e^{\frac{x}{2}}}{\left(e^{\frac{x}{2}}+99\right)^2}, x>0.\right.$

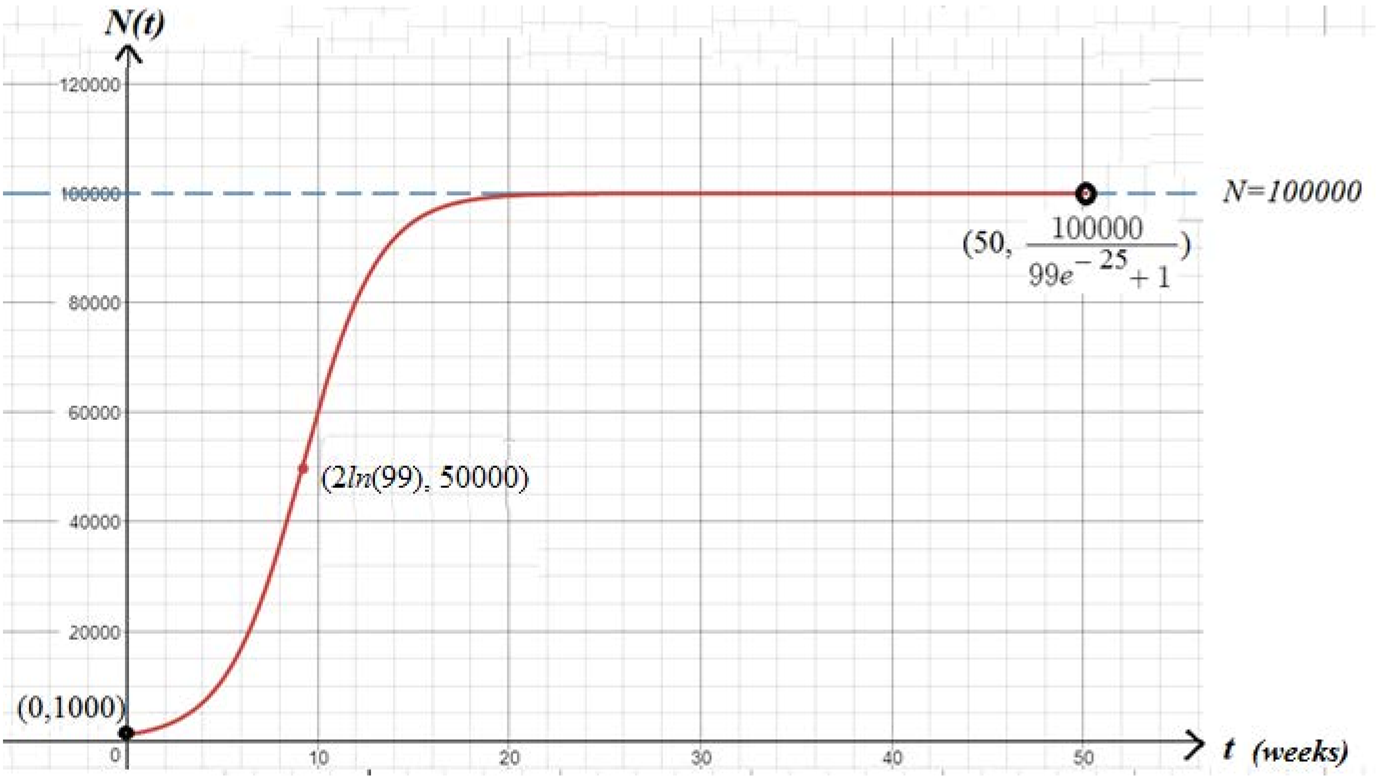
Define $ddf(x)=\frac{d}{dx}(df(x))$ Done

$ddf(x)$ $\left\{\frac{-2.475E6\cdot e^{\frac{x}{2}}\cdot\left(e^{\frac{x}{2}}-99\right)}{\left(e^{\frac{x}{2}}+99\right)^3}, x>0.\right.$

solve$(ddf(x)=0,x)$ $x=9.19024$

$(x=9.1902397002692)\cdot 7$ $7\cdot x=64.3317$

d. Sketch the graph of $N(t)$ for the first 50 weeks, indicating endpoints, points of inflection and any asymptotes where they exist. 3 marks



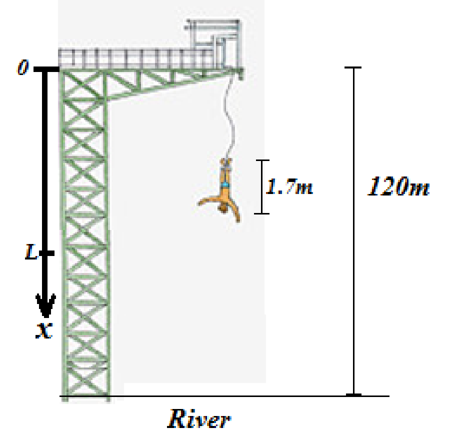
Question 2 (12 marks)

A bungee jumper of height 1.7m falls from rest, from the top of a very high platform, which is 120 m above the surface of a deep river. The bungee jumper's feet are tied to an elastic cord, that when un-stretched is of length L m. The displacement of the jumper's feet, measured ***downwards*** from the jumping point of the platform is x m.

For the first part of the fall the acceleration of the jumper is given by the equation:

$$\ddot{x} = g - rv, \text{ where } 0 \le x \le L$$

Where r is a positive constant related to the air resistance, and depends on the weather conditions, and v is the velocity of the jumper at any time.



a. Show that the displacement x is given by: 2 marks

$$x = \frac{g}{r^2}\ln\left(\frac{g}{g-rv}\right) - \frac{v}{r}$$

using $\ddot{x} = v\frac{dv}{dx}$ $x = -\frac{g}{r^2}\ln\left(\frac{g-rv}{g}\right) - \frac{v}{r}$ $\frac{dx}{dv} = -\frac{g}{r^2}(-\frac{r}{g})\left(\frac{g}{g-rv}\right) - \frac{1}{r}$ $= \frac{v}{g-rv}$	$\frac{dv}{dx} = \frac{g-rv}{v}$ $v\frac{dv}{dx} = v(\frac{g-rv}{v})$ $v\frac{dv}{dx} = g - rv$ therefore shown as $v\frac{dv}{dx} = \ddot{x}$

b. Given acceleration due to gravity and the air resistance factor on the day of the jump is $r = 0.2$, find the length, L, of the cord such that the jumper's velocity is 30 m/s when $x = L$. Give your answer to the nearest metre. 1 mark

Substitute the values given into the equation $x = \frac{g}{r^2}\log_e\left(\frac{g}{g-rv}\right) - \frac{v}{r}$	$L = \frac{9.8}{(0.2)^2}\log_e\left(\frac{9.8}{9.8-(0.2)(30)}\right) - \frac{30}{0.2} = 82$ m

c. Determine the time taken to 2 decimal places for the bungee jumper to reach a velocity of 30 m/s from rest. 2 marks

First part travels straight down $\ddot{x} = g - rv$ $\frac{dv}{dt} = 9.8 - 0.2v$ (1 mark) $\int \frac{1}{9.8-0.2v}\,dv = \int 1\,dt$ $-5\ln\lvert 49 - v\rvert = t + C$	Since at $t = 0, v = 0, C = -5\,ln\lvert 49\rvert$ When $v = 30\ m/s$, $t = 5\,ln(49) - 5\ln(19)$ $t = 4.74\ s$ (1 mark)

Question 2 (continued)

In the second stage of the fall, the displacement of the jumper's feet is determined by the elasticity of the bungee rope and the atmospheric conditions and is given by the equation:

$$x_2 = e^{-rt}(35\sin(t) - 8\cos(t)) + 90$$

Where $r = 0.2$ and t is the time in seconds after the jumper's feet first pass $x = L$.

d. Determine whether or not the jumper's head goes into the water. 2 marks

Solve for first time

$$x_2{}'(t) = 0$$

$$x_2 = 115.574$$

1 mark

Add height of jumper 1.7m

115.574+1.7=117.274m

Doesn't get wet as platform is 120 m high

1 mark

e. Determine the closest possible distance the bungee jumper's head rebounds toward the platform to the nearest metre. 2 marks

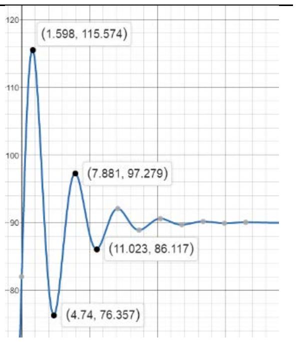
$x_2 = e^{-rt}(35\sin(t) - 8\cos(t)) + 90$

Next Turning point $x'(t) = 0$

$x_2 = 76.357\text{m}$ 1 mark

76.357+1.7m=78 metre is largest possible rebound distance, 120-78 is closest possible to platform 42 m

1 mark



f. For what values of r (to 3 decimal places will the bungee jumper's head get wet? 3 marks

$x_2(t) \geq 118.3$

$$\frac{dx_2}{dt} = -re^{-rt}(35\sin(t) - 8\cos(t)) + e^{-rt}(35\cos(t) + 8\sin(t))$$

$$\frac{dx_2}{dt} = 0$$

$$\tan(t) = \frac{8r+35}{35r-8}$$

$$t = \arctan\left(\frac{8r+35}{35r-8}\right) + k\pi \ \ where \ \ k \in N$$

$$t = \begin{cases} \arctan\left(\frac{8r+35}{35r-8}\right), r > \frac{8}{35} \\ \arctan\left(\frac{8r+35}{35r-8}\right) + \pi, r < \frac{8}{35} \\ \frac{\pi}{2}, r = \frac{8}{35} \end{cases}$$

Sub each t into $x_2(t)$ making $x_2(r)$ and solve for $x_2(r) \geq 118.3$

$$r \in (0, 0.138)$$

Alternative method by guessing and checking using a bisection method, noting that r is a positive constant

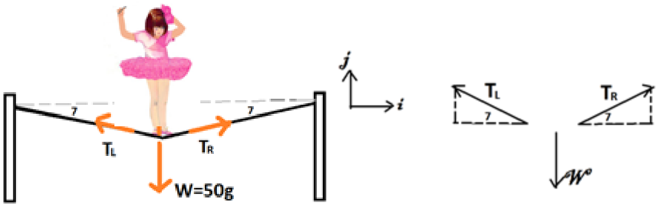
$$x_2(t) = e^{-rt}(35\sin(t) - 8\cos(t)) + 90$$

r	Max range ≥118.3	Gets wet
0	125	yes
0.2	115.574	no
0.1	120.152	yes
0.15	117.735	no
0.125	118.909	yes
0.1375	118.314	yes
0.138	118.29	no

$$r \in (0, 0.138)$$

Question 3 (10 marks)

The weight of a stationary circus tightrope walker of mass 50kg standing midway between the supporting poles causes a tightrope wire to sag by 7.0 degrees from the horizontal.

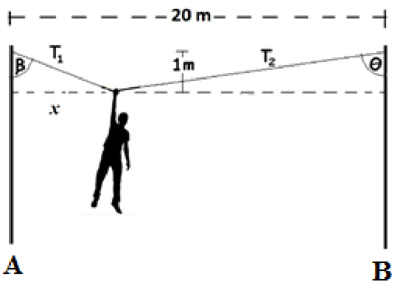


a. Ignoring the weight of the wire, show all the forces acting in this system. (1 mark)

b. For uniform tensile strength in the wire, find the tension force to 2 decimal places (1 mark)

$\sum F = 0$ Horizontally: $-T_L\cos(7^o) + T_R\cos(7^o) = 0$ $\lvert T_L\rvert = \lvert T_R\rvert = \lvert T\rvert$	Vertically: $2T sin(7^o) - 50g = 0$ $T = \frac{50g}{2\sin(7^o)} = 2010.35N$ (1 mark)

A man falls off the tightrope and manages to grab the tightrope, and is left hanging by one arm. He exerts a constant weight force of W newtons on the inextensible tightrope wire, where the wire is of negligible weight.



The poles A and B are spaced 20 metres apart, he fell at x m from pole A, at which point the vertical displacement of the tightrope from the horizontal is 1 metre below the top of the poles.

The angle made between the tightrope and the pole A at the distance of x metres is β, the angle made between the tightrope and pole B is θ.

c. Find each of the tension forces T_1 and T_2 in Newtons, in terms of W and x when the man is x m from pole A. 3 marks

$\tan(\beta) = \frac{x}{1}$, $\sin(\beta) = \frac{x}{\sqrt{(x^2+1)}}$, $\cos(\beta) = \frac{1}{\sqrt{(x^2+1)}}$ $\tan(\theta) = \frac{(20-x)}{1}$, $\sin(\theta) = \frac{(20-x)}{\sqrt{(20-x)^2+1}}$ $\cos(\theta) = \frac{1}{\sqrt{(20-x)^2+1}}$ 1 mark Horizontal components: $\sum F = 0$ $T_1\sin(\beta) - T_2\sin(\theta) = 0$ Equation (1) Vertical components: $\sum F - W = 0$ $T_1\cos(\beta) + T_2\cos(\theta) = W$ Equation (2) Equation 1: $T_1 = \frac{T_2\sin(\theta)}{\sin(\beta)} = \frac{T_2\left(\frac{(20-x)}{\sqrt{(20-x)^2+1}}\right)}{\left(\frac{x}{\sqrt{(x^2+1)}}\right)}$	sub T_1 into equation 2 $\frac{T_2\left(\frac{(20-x)}{\sqrt{(20-x)^2+1}}\right)}{\left(\frac{x}{\sqrt{(x^2+1)}}\right)}\left(\frac{1}{\sqrt{(x^2+1)}}\right) + T_2\left(\frac{1}{\sqrt{(20-x)^2+1}}\right) = W$ 1 mark $\frac{(20-x)}{x\sqrt{(20-x)^2+1}}T_2 + T_2\left(\frac{x}{x\sqrt{(20-x)^2+1}}\right) = W$ $\frac{20}{x\sqrt{(20-x)^2+1}}T_2 = W$ $T_1 = \left(\frac{(20-x)}{\sqrt{(20-x)^2+1}}\right)\left(\frac{\sqrt{(x^2+1)}}{x}\right)\left(\frac{x\sqrt{(20-x)^2+1}}{20}W\right)$ $T_2 = \frac{x\sqrt{(20-x)^2+1}}{20}W$ $T_1 = \frac{(20-x)\sqrt{(x^2+1)}}{20}W$ 1 mark

Question 3 (continued)

d. f the vertical displacement of the rope remains at 1 m from the top of the poles, and the man moves from $x = 5$ m relative to pole A along the rope towards pole B, at a rate where the angle β changes at $\frac{d\beta}{dt} = 1°/\text{minute}$. How long will it take the man to get to $x = 7$ m to the nearest minute? 1 mark

At $x_1 = 5, \beta_1 = 78.69°$ At $x_2 = 7, \beta_2 = 81.86°$	$\dfrac{\beta_2 - \beta_1}{\frac{d\beta}{dt}} = 3.17$ Approximately 3 minutes

e. Show that the rate of change of x is given by $\frac{dx}{dt} = \frac{\pi}{180}(\sec(\beta))^2$ metres/minute, when the man is moving from $x = 5$ m relative to pole A, towards pole B. 1 mark

$x = \tan(\beta)$ $\dfrac{dx}{d\beta} = (\sec(\beta))^2$ $\dfrac{d\beta}{dt} = 1°/\text{minute}=\frac{\pi}{180}\ \text{radians/minute}$	$\dfrac{dx}{dt} = \dfrac{d\beta}{dt} \times \dfrac{dx}{d\beta} = \dfrac{\pi}{180}(\sec(\beta))^2$

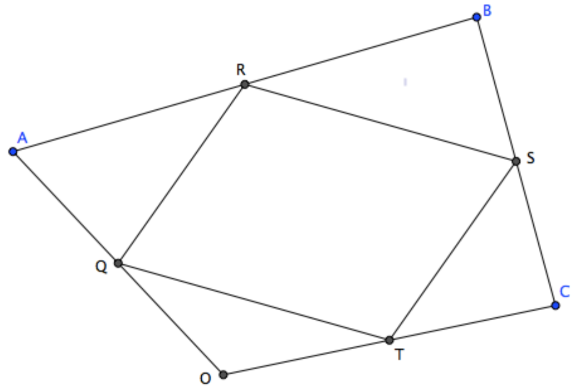
f. Hence or otherwise, determine the rate of change for angle θ in degrees (to two decimal places) per minute, when the man is at $x = 7$m relative to pole **A**. 3 marks

Let $y = 20 - x$ $\dfrac{dy}{dx} = -1$ $\therefore \frac{dy}{dt} = \frac{dx}{dt} \times \frac{dy}{dx} = -\frac{\pi}{180}(\sec(\beta))^2$ (1 mark) $\tan(\theta) = (y)$ $\theta = \tan^{-1}(y)$	$\dfrac{d\theta}{dt} = \dfrac{dy}{dt} \times \dfrac{d\theta}{dy} = -\dfrac{\pi}{180}(\sec(\beta))^2 \times \dfrac{1}{(y^2+1)}$ (1 mark) $\dfrac{d\theta}{dt}_{(\beta=81.8698°, x=7, y=13)}$ $= -\dfrac{\pi}{180}(\sec(81.8698°))^2 \times \dfrac{1}{(13^2+1)} \times \dfrac{180}{\pi}$ $= -0.29°/minute$ (1 mark)

Question 4 (13 marks)

OABC is a quadrilateral with $a = \vec{OA}$, $b = \vec{OB}$, and $c = \vec{OC}$

Let Q, R, S and T be the midpoints of $\vec{OA}$, $\vec{AB}$, $\vec{BC}$ and $\vec{OC}$ respectively



a **i)** Find $\vec{AB}$ and $\vec{BC}$ in terms of a, b and c 1 mark

$\vec{AB} = -a + b \qquad \vec{BC} = -b + c$

ii) Find $\vec{QR}$ and $\vec{ST}$ in terms of a, b and c 2 marks

$\vec{QR} = \frac{1}{2}b \qquad \vec{ST} = -\frac{1}{2}b$

iii) Find $\vec{RS}$ and $\vec{TQ}$ in terms of a, b and c 2 marks

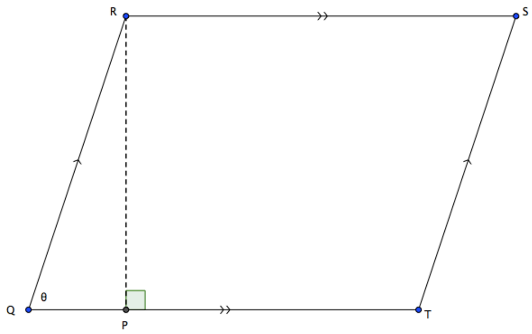
$\vec{RS} = -\frac{1}{2}(a-c) \qquad \vec{TQ} = \frac{1}{2}(a-c)$

Question 4 (continued)

iv) Hence show that *QRST* is a parallelogram. 1 mark

Opposite sides of the quadrilateral are equal in length and parallel.
Hence *QRST* is a parallelogram

Consider the parallelogram *QRST*



If $a = 2i + j, b = i - 3j + 2k$ and $c = 5i - 4j + 3k$

b) Write down $\vec{QR}$ and $\vec{QT}$ 1 mark

$\vec{QR} = \frac{1}{2}(i - 3j + 2k)$, $\vec{QT} = -\vec{TQ} = \frac{1}{2}(c - a) = \frac{1}{2}(3i - 5j + 3k)$

c) Find $\angle RQT$ in degrees correct to 1 decimal place. 2 marks

$|\vec{QR}| = \frac{\sqrt{14}}{2}$, $|\vec{QT}| = \frac{\sqrt{43}}{2}$, $\vec{QR} . \vec{QT} = 6$

$\cos(\angle RQT) = \frac{6}{\sqrt{14}.\sqrt{43}} \approx 0.2445$

$\angle RQT = 85.7°$

d) Find the height $|\vec{RP}|$ of the parallelogram in terms of $|\overline{QR}|$ and $\angle RQT$. 1 mark

$|\vec{RP}| = |\overline{QR}|\sin(\angle RQT)$

Question 4 (continued)

e) Hence find the area of the parallelogram *QRST* correct to two decimal places. 2 marks

Area = $\frac{1}{2}| \, |QT|. \, |\overrightarrow{RP}|$

$= \frac{1}{2} . \frac{\sqrt{43}}{2} . \frac{\sqrt{14}}{2} . \sin\left(85.7^\circ\right)$

= 3.06 sq units

Question 5 (7 marks)

The complex numbers z_1 and z_2 are given by

$z_1 = m + 2i$ and $z_2 = 1 - 2i$

a) Find $\frac{z_1}{z_2}$ in the form $a + bi$ where $a, b \in R$. 1 mark

$$\frac{z_1}{z_2} = \frac{m-4}{5} + \frac{2(m+1)}{5}i$$

b) Given $\left|\frac{z_1}{z_2}\right| = 2$, find m 1 mark

$$\frac{z_1}{z_2} = \frac{m-4}{5} + \frac{2(m+1)}{5}i$$

$$\left(\frac{m-4}{5}\right)^2 + \left(\frac{2(m+1)}{5}\right)^2 = 4$$

$$m = \pm 4$$

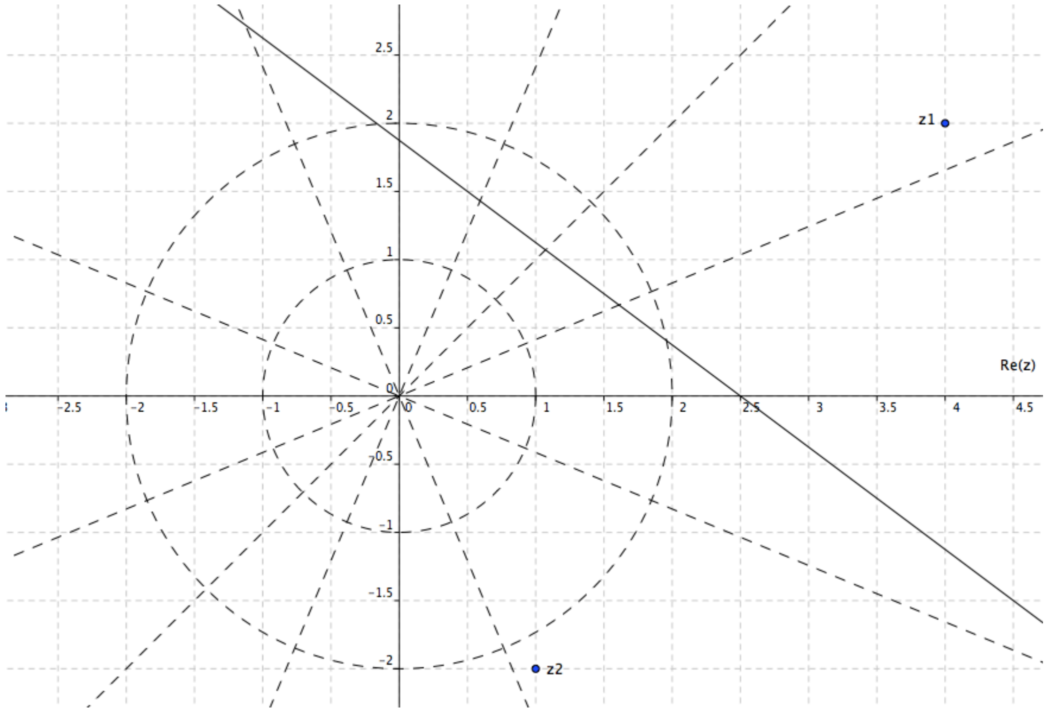
c) Let $m = 4$. Find the cartesian equation of the locus of the of the points given by $\{z: |z - z_1| = |z - z_2|\}$ 2 marks

$$(x - 4)^2 + (2 - y)^2 = (x - 1)^2 + (y + 2)^2$$

$$y = -\frac{3x}{4} + \frac{15}{8}$$

Question 5 (continued)

d) Plot z_1 and z_2 on the Argand diagram below and sketch $\{z: |z - z_1| = |z - z_2|\}$ on the same diagram. 3 marks



Question 6 (9 marks)

The average height of males in a 1st Year university course was thought to be 174 cm with a standard deviation of 6.9 cm. Assume that heights are normally distributed.

a) Find the probability, correct to 4 decimal places, that a randomly selected students will be taller that 185.0 cm. 1 mark

X~N(174.0, 6.9^2) Pr(X > 185.0) = 0.0554

b) Given $\frac{a+b}{2} = 174$, $a < b$, find *a* and *b* correct to one decimal place such that the probability that a student's height is between *a* cm and *b* cm tall is 0.90. 2 marks

Pr(a < X < b) = 0.90 Pr(a < X) = 0.05, *a* = 162.7 cm Pr(b > X) = 0.05, *b* = 185.4 cm

The university measured the heights of 25 female students and found that the mean of the sample was 163.2 cm. The standard deviation of the height of females in this population is 6.1 cm

c) Write down the point estimate for the mean height. 1 mark

The point estimate of the mean is 163.2 cm

d) Find the 95% confidence level for the mean height of the female students. Give answers correct to two decimal places. 2 marks

(160.81, 165.59)

One of the lecturers feels sure the current male students are taller than in previous years. He measures the heights of 50 male students and finds that the mean height of this group students is 177.04

f) **i)** State H_0 and H_1 1 mark

$H_0 : \mu = 174$ $H_1 : \mu > 174$

ii) Calculate the *p*-value 1 mark

p = 0.0009

iii) If the lecturer uses a 1% level of significance, would the lecturer be justified with his assertion? 1 mark

No as the *p*-value of 0.0011 is greater than 0.01. The null hypothesis is not rejected.

END OF SOLUTIONS