The Mathematical Association of Victoria

Trial Exam 2018

SPECIALIST MATHEMATICS

Written Examination 1

STUDENT NAME _____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 18 pages
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

THIS PAGE IS BLANK

Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2} where $g = 9.8$.

Question 1 (3 marks)

The angle between the vectors $2\underset{\sim}{\text{i}} - m\underset{\sim}{\text{j}} - 3\sqrt{2}\underset{\sim}{\text{k}}$, where $m \in R$, and $\underset{\sim}{\text{i}} - \sqrt{2}\underset{\sim}{\text{k}}$ is $\cos^{-1}\left(\frac{2}{5}\right)$.

Find all possible values of m.

Question 2 (3 marks)

The volume of milk in a 2L carton varies normally with a mean of 2000 ml and a standard deviation of 15 ml. The volume of milk in a 1L carton varies normally with a mean of 1000 ml and a standard deviation of 4 ml.

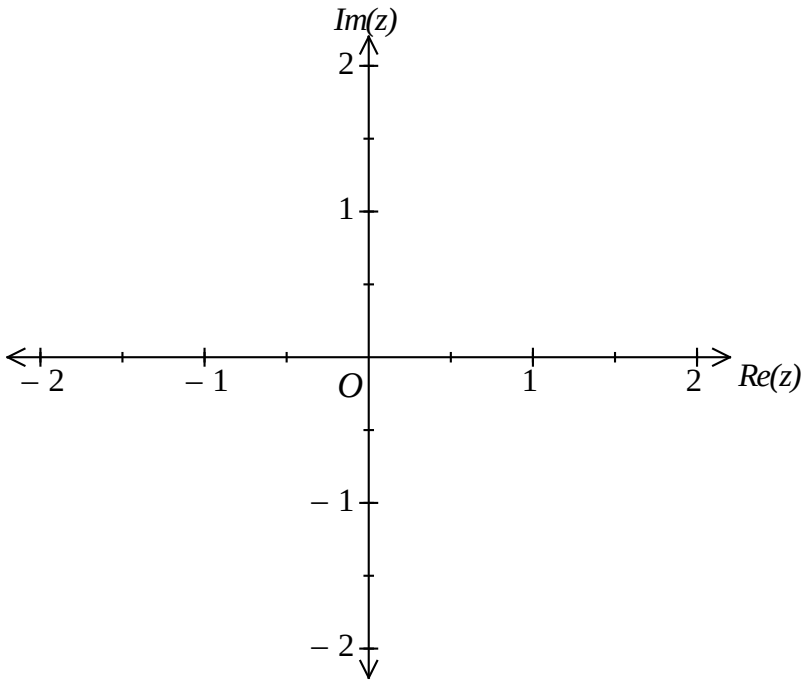
a. Find the variance of the difference between the volume of a 2L carton of milk and the volume of two 1L cartons of milk. 1 mark

b. A quality control officer suspects that the mean volume of milk in a 1L carton of milk is different to 1000 ml. She collects thirty-six 1L cartons and calculates a C% confidence interval for the mean volume of milk in a 1L carton to be (994.7, 997.3).

The value of C is given by $\Pr(-z^* < Z < z^*) = \frac{C}{100}$ where Z is the standard normal random variable. Find the value of z^*. 2 marks

Question 3 (5 marks)

a. Sketch the graph of $\left|z - \bar{z} + 1\right| = 2$, $z \in C$. Label all axes intercepts with their corresponding value of z. 2 marks



Working space

TURN OVER

b. Consider $z = \dfrac{1+i}{(-\sqrt{3}+i)^3}$, $z \in C$.

Find the principal argument of z in the form $k\pi$, $k \in R$. 3 marks

Question 4 (3 marks)

Let $\text{cosec}(2\theta) = -\frac{13}{12}$ where $\pi < 2\theta < \frac{3\pi}{2}$.

Find the value of $\sin(\theta)$.

TURN OVER

Find an anti-derivative of $\frac{2x^2+1}{4x^2-4x+3}$.

Question 6 (4 marks)

a. A body of mass 5 kg is moving in a straight line with an acceleration given by $-e^{-2x}$ ms^{-2} where x metres is the distance of the body from a fixed origin at time t seconds. The body is initially at the origin and travelling with a speed of 1 ms^{-1}.

Find, in kg ms^{-1}, the magnitude of the momentum of the body when it is 2 m to the right of the origin. 2 marks

TURN OVER

b. Find, in metres, the distance of the body from the origin after 3 seconds. 2 marks

CONTINUES OVER THE PAGE

TURN OVER

Question 7 (5 marks)

Water is poured into a tank whose shape is that of the volume generated when the region bounded by the *y*-axis and the graph of $y = 3\arctan(x) - \frac{\pi}{2}$ over the interval $x \in \left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$ is rotated about the *y*-axis. All lengths are measured in metres.

Water is poured into the tank at a constant rate of k m^3 per minute.

a. Write down a definite integral which gives the volume, V m^3, of water in the tank when the depth of water is *h* metres. 1 mark

b. Calculate the value of V when the tank is full.

Give the answer in the form $\frac{a-b}{2}$ where $a, b \in R^+$. 2 marks

c. Find in terms of k the rate with respect to time at which the depth of water in the tank is increasing when $h = \frac{\pi}{4}$. 2 marks

Question 8 (7 marks)

a. State the maximal domain and the range of $f(x) = \arccos\left(2\sqrt{x}\right)$. 2 marks

b. Find the value of $f'\left(\frac{1}{8}\right)$. 2 marks

TURN OVER

c. Find the gradient of the tangent to the graph of $\arccos\left(2\sqrt{x}\right)-\frac{1}{3}\arcsin(y)=\frac{\pi}{3}$ at the point where $x=\frac{1}{8}$. 3 marks

Question 9 (6 marks)

a. Find an anti-derivative of $\sin\left(\frac{\pi}{3}t\right)\cos\left(\frac{\pi}{6}t\right)$. 2 marks

TURN OVER

b. The velocity of a particle moving relative to an origin at time t seconds is given by

$$\underset{\sim}{v}(t) = \frac{3\sqrt{t}}{2 + t\sqrt{t}}\underset{\sim}{i} + \sin\left(\frac{\pi}{3}t\right)\cos\left(\frac{\pi}{6}t\right)\underset{\sim}{j}, \quad t \geq 0$$

where the components are measured in metres per second.
The particle is initially at the origin. Find the position of the particle after 8 seconds. 4 marks

END OF QUESTION AND ANSWER BOOK