VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2018

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

SOLUTION

STUDENT NUMBER | | | | | | | | | Letter | |

SPECIALIST MATHEMATICS
Written examination 2

Wednesday 6 June 2018

Reading time: 10.00 am to 10.15 am (15 minutes)
Writing time: 10.15 am to 12.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	7	7	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 29 pages
- Formula sheet
- Answer sheet for multiple-choice questions

Instructions
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1

Let $f(x) = \text{cosec}(x)$. The graph of f is transformed by:

- a dilation by a factor of 3 from the x-axis, followed by
- a translation of 1 unit horizontally to the right, followed by
- a dilation by a factor of $\frac{1}{2}$ from the y-axis.

The rule of the transformed graph is

A. $g(x) = 2\text{cosec}(3x + 1)$

B. $g(x) = 3\text{cosec}(2x - 1)$

C. $g(x) = 3\text{cosec}(2(x-1))$

D. $g(x) = 2\text{cosec}\left(\frac{x}{3} - 1\right)$

E. $g(x) = 3\text{cosec}\left(\frac{x-1}{2}\right)$

$y = 3\,\text{cosec}(x)$

$y = 3\,\text{cosec}(x-1)$

$y = 3\,\text{cosec}(2(x-1))$

Question 2

Let $f(x) = \frac{\sqrt{x+1}}{x}$ and $g(x) = \tan^2(x)$, where $0 < x < \frac{\pi}{2}$.

$\cos x > 0$

$f(g(x))$ is equal to

A. $\sin(x)\sec^2(x)$

B. $\sec(x)\tan^2(x)$

C. $\cos(x)\cot^2(x)$

D. $\cos(x)\text{cosec}^2(x)$

E. $\text{cosec}(x)\cos^2(x)$

CAS

$f(g(x)) = \frac{\cos x}{\sin^2 x}$

DO NOT WRITE IN THIS AREA

Question 3

The implied domain of the function with rule $f(x) = \dfrac{3x}{\frac{\pi}{2} - \arccos(2-x)}$ is

A. $[1, 3]$

B. $[-1, 1]$

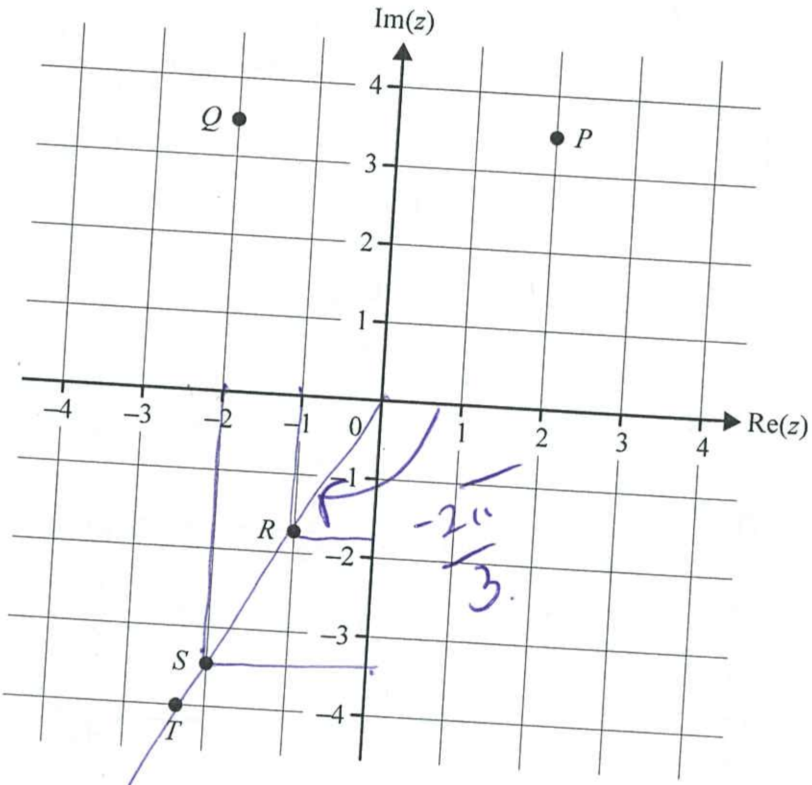
C. $[0, 1) \cup (1, 2]$

D. $[-1, 0) \cup (0, 1]$

E. $[1, 2) \cup (2, 3]$

$x \neq 2$ $\quad 1 \leq x \leq 3$

Question 4



On the Argand diagram shown above, $4\operatorname{cis}\left(-\frac{2\pi}{3}\right)$ is represented by the point

A. P

B. Q

C. R

D. S

E. T

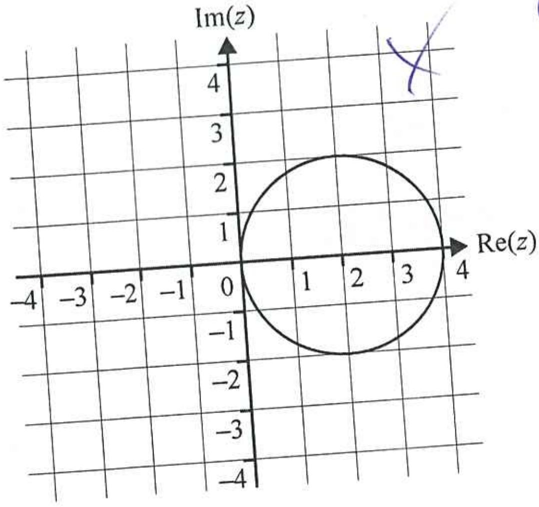
$C(-2, 0)$ $r = 2$

$(x+2)^2 + y^2 = 4$

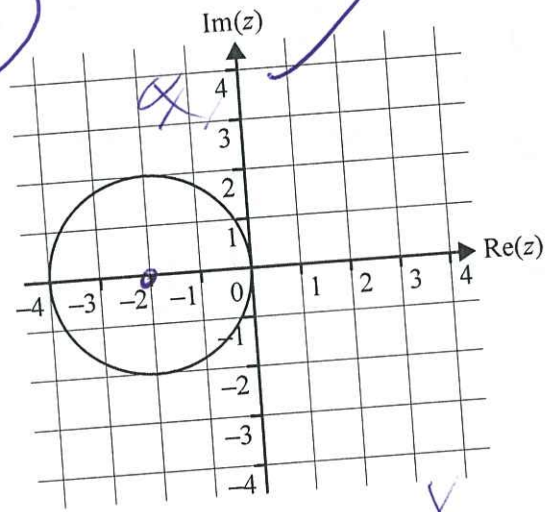
Question 5

Which one of the following graphs shows the set of points in the complex plane specified by the relation $\{z : (z+2)(\overline{z}+2) = 4,\ z \in C\}$?

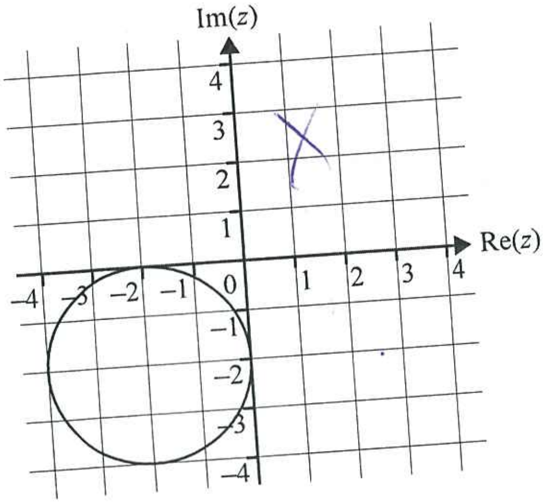
A.



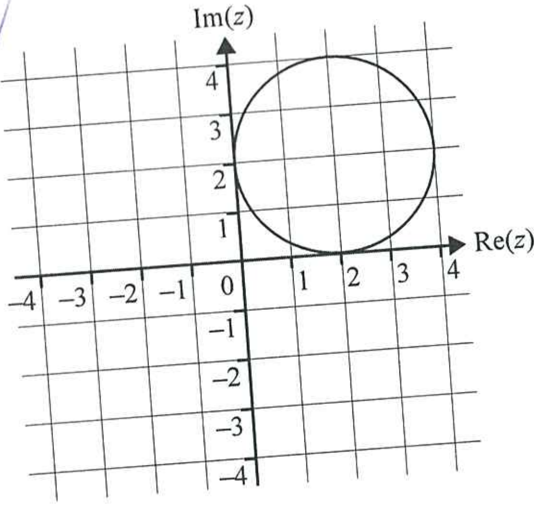
B.



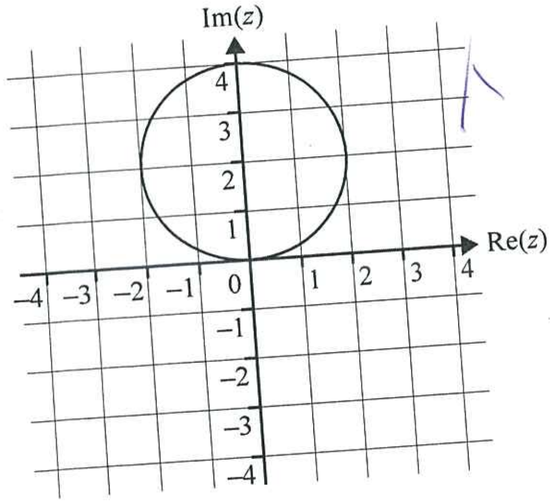
C.



D.



E.



Question 6

Given that $(z - 3i)$ is a factor of $P(z) = z^3 + 2z^2 + 9z + 18$, which one of the following statements is **false**?

A. $P(3i) = 0$

B. $P(-3i) = 0$

C. $P(z)$ has three linear factors over C

D. $P(z)$ has no real roots

E. $P(z)$ has two complex conjugate roots

$(z^2+9)(z^2 \quad + \quad)$

$z = \pm 3i, \; z = -2$

Question 7

The gradient of the line that is **perpendicular** to the graph of the relation $3y^2 - 5xy - x^2 = 1$ at the point $(1, 2)$ is

A. $-\frac{1}{12}$

B. $\frac{12}{7}$

C. 21

D. $-\frac{7}{12}$

E. $-\frac{7}{13}$

Question 8

Using a suitable substitution, $\int_1^2 \left(\frac{3}{2+(4x+1)^2}\right) dx$ can be expressed as

A. $\frac{3}{4}\int_1^2 \left(\frac{1}{2+u^2}\right) du$

B. $\frac{3}{4}\int_5^9 \left(\frac{1}{2+u^2}\right) du$

C. $3\int_5^9 \left(\frac{1}{2+u^2}\right) du$

D. $3\int_1^2 \left(\frac{1}{2+u^2}\right) du$

E. $-12\int_9^5 \left(\frac{1}{2+u^2}\right) du$

$u = 4x+1$

$\frac{du}{dx} = 4$

$\frac{1}{4}\,du = dx$

$x = 1, \; u = 5$

$x = 2 \quad u = 9$

$\frac{1}{4}\int_5^9 \frac{3}{2+u^2}\,du$

$= \frac{3}{4}\int_5^9 \frac{1}{2+u^2}\,du$

$\frac{\sin 10x}{10}$

$1.0544 = x -$

Question 9

$\int(1-\cos(10x))\,dx$ is equivalent to

$x - \sin x \cos x$

A. $\int\left(\sin^2(5x)\right)dx$ X

B. $\frac{1}{2}\int\left(\sin^2(20x)\right)dx$ X

$\cos 2x = 2\cos^2 x - 1$

C. $\int\left(\cos^2(5x)\right)dx$ X

D. $2\int\left(\cos^2(10x)\right)dx$ X

$\frac{\cos 2x + 1}{2} = \cos^2 x$

E. $2\int\left(\sin^2(5x)\right)dx$

Tried and er

$\cos 2x = 1 - 2\sin^2 x$

$\int_0^1 \cdots \cos(10x) = 1 - 2\sin^2 5x$

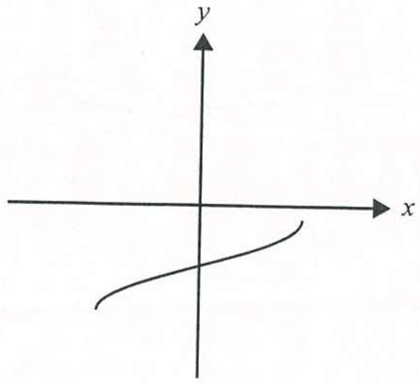
$1 - \cos(10x) = 1 - 1 + 2\sin^2 5x.$

$= 2\sin^2 5x$

$\cos^2(5x) = \frac{\cos 10x + 1}{2.}$

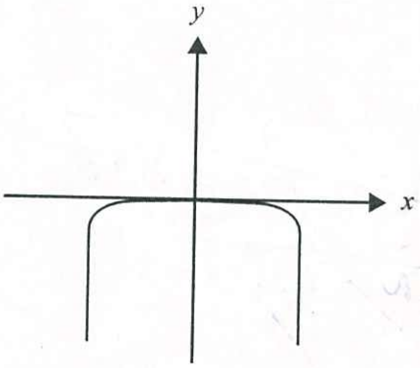
Question 10

The graph of an **antiderivative** of a function g is shown below.

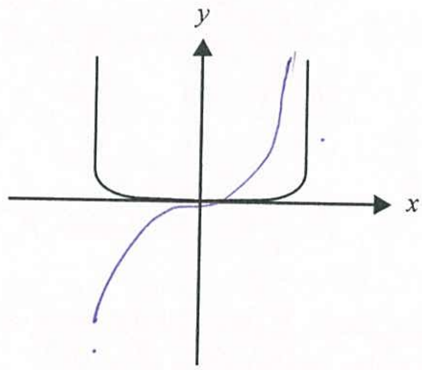


Which one of the following could best represent the graph of g?

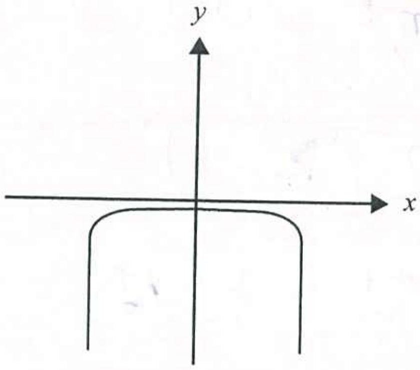
A.



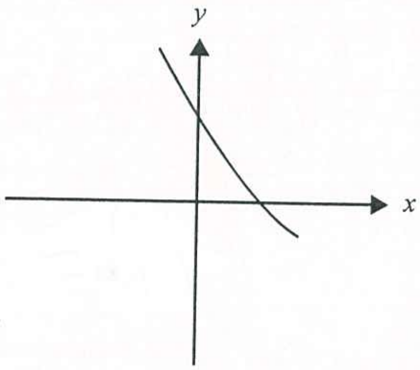
B.



C.

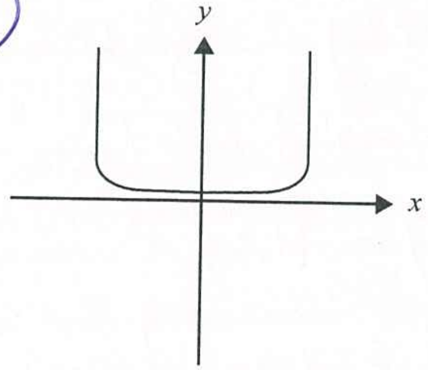


D.



CAS

E.



angle

Question 11

Let $\underset{\sim}{a} = 2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 6\underset{\sim}{k}$.

The acute angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is closest to

A. 11°

B. 75°

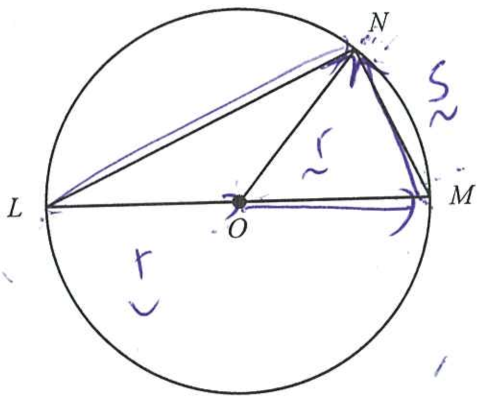
C. 79°

D. 86°

E. 88°

$$\frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|} = \frac{4}{3 \cdot 7}$$

Question 12



In the diagram above, LOM is a diameter of the circle with centre O.
N is a point on the circumference of the circle.
If $\underset{\sim}{r} = \overrightarrow{ON}$ and $\underset{\sim}{s} = \overrightarrow{MN}$, then $\overrightarrow{LN}$ is equal to

A. $2\underset{\sim}{r} - 2\underset{\sim}{s}$

B. $\underset{\sim}{r} - 2\underset{\sim}{s}$

C. $\underset{\sim}{r} + 2\underset{\sim}{s}$

$\Rightarrow$ D. $2\underset{\sim}{r} + \underset{\sim}{s}$

E. $2\underset{\sim}{r} - \underset{\sim}{s}$

$\overrightarrow{LN} = 2\underset{\sim}{r} + \underset{\sim}{s}$

Question 13

Let $\underset{\sim}{i}$ and $\underset{\sim}{j}$ be unit vectors in the east and north directions respectively.
At time t, $t \geq 0$, the position of particle A is given by $\underset{\sim}{r}_A = \left(t^2 - 5t + 6\right)\underset{\sim}{i} + (5t - 8)\underset{\sim}{j}$ and the position of particle B is given by $\underset{\sim}{r}_B = (3 - t)\underset{\sim}{i} + \left(t^2 - t\right)\underset{\sim}{j}$.
Particle A will be directly east of particle B when t equals

A. 1

B. 2

C. 1 and 2

D. 2 and 4

E. 4

$\underset{\sim}{r}_A(2) = 0 + 2\underset{\sim}{j}$

$\underset{\sim}{r}_B(2) = 1 + 2\underset{\sim}{j}$

$r_B(4) = -1 + 12j$

$r_A(4) = 2\underset{\sim}{i} + 12\underset{\sim}{j}$

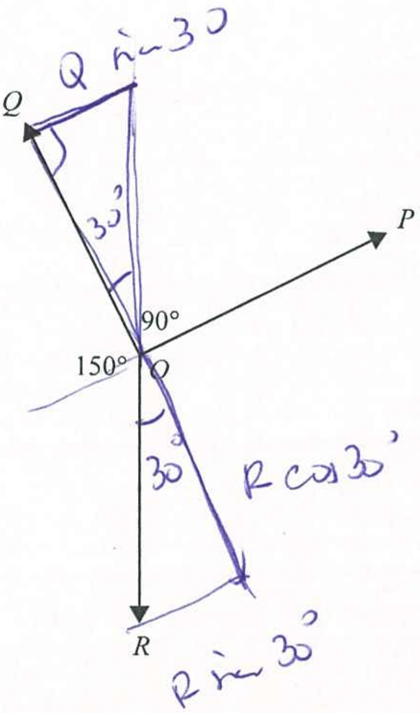
$r_A(2) = 0, 2$

$r_A(4) = (2, 12)$

$r_B(2) = (1, 2)$

$r_B(4) = (-1, 12)$

Question 14



The diagram above shows a particle at O in equilibrium in a plane under the action of three forces of magnitudes P, Q and R.

Which one of the following statements is **false**?

A. $R = Q\sin(60°)$

B. $Q = R\sin(60°)$

C. $P = R\sin(30°)$

D. $Q\cos(60°) = P\cos(30°)$

E. $P\cos(60°) + Q\cos(30°) = R$

Question 15

An 80 kg person stands in an elevator that is accelerating downwards at 1.2 ms^{-2}.

The reaction force of the elevator floor on the person, in newtons, is

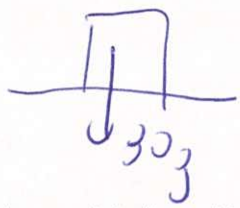
A. 688

B. 704

C. 784

D. 880

E. 896



$80(9.8 - 1.2)$

Question 16

A body of mass 2 kg is moving in a straight line with constant velocity when an external force of 8 N is applied in the direction of motion for t seconds.

If the body experiences a change in momentum of 40 kg ms^{-1}, then t is

A. 3

B. 4

C. 5

D. 6

E. 7

$a = \frac{40}{2t}$

$a = \frac{20}{t}$

$2a = \cancel{40}\ 8$

$\frac{2 \cdot 20}{t} = 8$

$40 = 8t$ $t = 5$

SECTION A – continued

TURN OVER

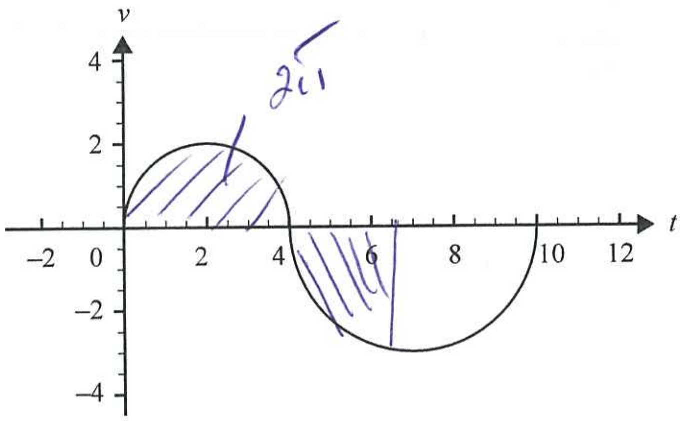
Question 17

An object travels in a straight line relative to an origin O.

At time t seconds its velocity, v metres per second, is given by

$$v(t) = \begin{cases} \sqrt{4-(t-2)^2}, & 0 \le t \le 4 \\ -\sqrt{9-(t-7)^2}, & 4 < t \le 10 \end{cases}$$

The graph of $v(t)$ is shown below.



The object will be back at its initial position when t is closest to

A. 4.0
B. 6.5
C. 6.7
D. 6.9
E. 7.0

not a good question!
only approximately equals!

Question 18

The heights of all six-year-old children in a given population are normally distributed. The mean height of a random sample of 144 six-year-old children from this population is found to be 115 cm.

If a 95% confidence interval for the mean height of all six-year-old children is calculated to be (113.8, 116.2) cm, the standard deviation used in this calculation is closest to

A. 1.20
B. 7.35
C. 15.09
D. 54.02
E. 88.13

$1.2 = 1.96 \cdot \frac{\sigma}{\sqrt{144}}$

$\sigma = 7.35$

$E(A) = 115$ $\quad$ $VAR(9A) = 9 \times 49 = 441$

$VAR(A) = 49$ $\quad$ $E(9A) = 1035$

Question 19

A local supermarket sells apples in bags that have **negligible mass**. The stated mass of a bag of apples is 1 kg.

The mass of this particular type of apple is known to be normally distributed with a mean of 115 grams and a standard deviation of 7 grams. A particular bag contains nine randomly selected apples.

The probability that the nine apples in this bag have a total mass of less than 1 kg is

A. 0.0478

B. 0.1132

C. 0.4265

D. 0.5373

E. 0.9522

$Pr(9A \leq 1000) = Pr(9A < 1000)$

CAS $= 0.04779$

Question 20

A farm grows oranges and lemons. The oranges have a mean mass of 200 grams with a standard deviation of 5 grams and the lemons have a mean mass of 70 grams with a standard deviation of 3 grams.

Assuming masses for each type of fruit are normally distributed, what is the probability, correct to four decimal places, that a randomly selected orange will have at least three times the mass of a randomly selected lemon?

A. 0.0062

B. 0.0828

C. 0.1657

D. 0.8343

E. 0.9172

$O \sim N(200, 25)$

$L \sim N(70, 9)$

$Pr(O > 3L) = Pr(O - 3L > 0)$

$E(O - 3L) = -10$

$VAR(O - 3L) = 25 + 3 \times 9 = 52$

$O - 3L \sim N(-10, 52)$

$Pr(O - 3L > 0) = 0.082759$

END OF SECTION A
TURN OVER

DO NOT WRITE IN THIS AREA

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

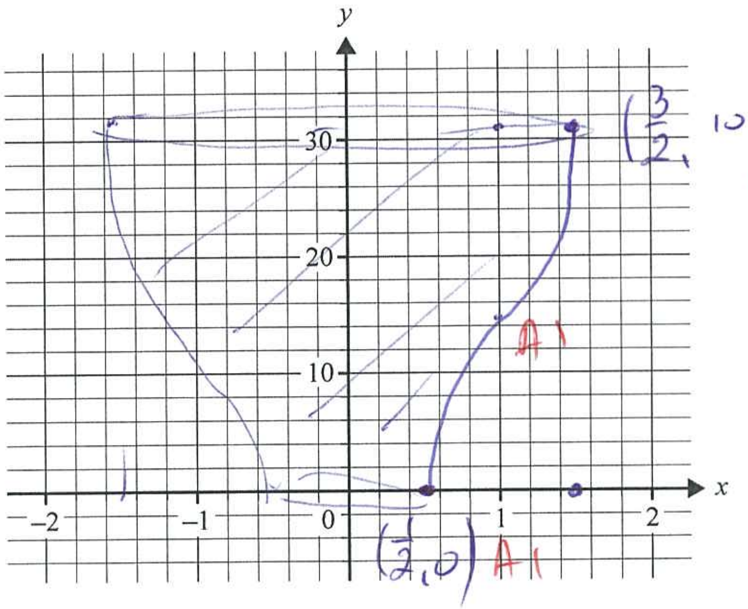
Question 1 (10 marks)

$-1 \le 2-2x \le 1$

$\frac{1}{2} \le x \le 3/2$

Consider the function f with rule $f(x) = 10 \arccos(2 - 2x)$.

a. Sketch the graph of f over its maximal domain on the set of axes below. Label the endpoints with their coordinates. 3 marks



$f\left(\frac{3}{2}\right) = 10 \arccos\left(2 - 2 \cdot \frac{3}{2}\right) = 10\pi$

$y = 10 \arccos(2-2x)$

$\frac{y}{10} = \arccos(2-2x)$

$\cos\left(\frac{y}{10}\right) = 2-2x$

$2x = 2 - \cos\left(\frac{y}{10}\right)$ | $x = 1 - \frac{1}{2}\cos\left(\frac{y}{10}\right)$

SECTION B – Question 1 – continued

DO NOT WRITE IN THIS AREA

A vase is to be modelled by rotating the graph of f about the y-axis to form a solid of revolution, where units of measurement are in centimetres.

b. **i.** Write down a definite integral in terms of y that gives the volume of the vase. 2 marks

$$V = \pi \int_0^{10\pi} x^2 \, dy$$

$$= \pi \int_0^{10\pi} \left(1 - \frac{1}{2}\cos\left(\frac{y}{10}\right)\right)^2 dy \quad \text{M1}$$

ii. Find the volume of the vase in cubic centimetres. 1 mark

$$\boxed{\frac{45\pi^2}{4} \text{ cm}^3} \quad \text{A1}$$

c. Water is poured into the vase at a rate of 20 cm^3 s^{-1}.

Find the rate, in centimetres per second, at which the depth of the water is changing when the depth is 5π cm. 3 marks

$$\frac{dV}{dt} = 20 \text{ cm}^3/\text{s}$$

$$\frac{dV}{dt} = \frac{dV}{dy} \times \frac{dy}{dt}$$

$$20 = \frac{dV}{dy} \times \frac{dy}{dt} \quad \text{M1}$$

$$\frac{dy}{dt} = \frac{20}{\frac{dV}{dy}}$$

$$\boxed{\frac{dy}{dt} = \frac{20}{\pi}} \quad \text{A1}$$

$$V = \pi \int_0^h \left(1 - \frac{1}{2}\cos\left(\frac{y}{10}\right)\right)^2 dy$$

$$\frac{dV}{dh} = \pi \left[1 - \frac{1}{2}\cos\left(\frac{h}{10}\right)\right]^2 - \pi\left(1 - \frac{1}{2}\right)$$

$$h = 5\pi \;;\; \frac{dV}{dh} = \pi\left(1 - \frac{1}{2}\cos\frac{\pi}{2}\right) = \pi \quad \text{A1}$$

d. The vase is placed on a table. A bee climbs from the bottom of the outside of the vase to the top of the vase.

What is the minimum distance the bee will need to travel? Give your answer in centimetres, correct to one decimal place. 1 mark

$$L = \int_{\frac{1}{2}}^{\frac{3}{2}} \sqrt{1 + (f'(x))^2}\, dx.$$

$$= \boxed{31.4} \text{ cm.}$$

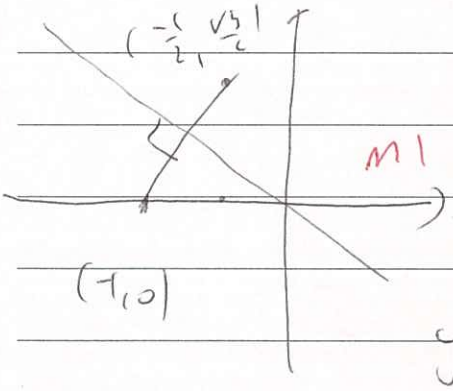
A1

DO NOT WRITE IN THIS AREA

Question 2 (11 marks)

In the complex plane, L is the line given by $|z+1| = \left|z + \frac{1}{2} - \frac{\sqrt{3}}{2}i\right|$.

a. Show that the cartesian equation of L is given by $y = -\frac{1}{\sqrt{3}}x$. 2 marks



$m = \dfrac{\frac{\sqrt{3}}{2} - 0}{-\frac{1}{2} + 1} = \sqrt{3}$

M1

$M\left(-\frac{3}{4}, \frac{\sqrt{3}}{4}\right)$

A1

$m_\perp = -\frac{1}{\sqrt{3}}$

$y - \frac{\sqrt{3}}{4} = -\frac{1}{\sqrt{3}}\left(x + \frac{3}{4}\right)$

$y = -\frac{1}{\sqrt{3}}x \cancel{- \frac{3}{4\sqrt{3}} + \frac{\sqrt{3}}{4}}$ $\quad y = -\frac{1}{\sqrt{3}}x$ ✓

b. Find the point(s) of intersection of L and the graph of the relation $z\bar{z} = 4$ in cartesian form. 2 marks

$z\bar{z} = x^2 + y^2$

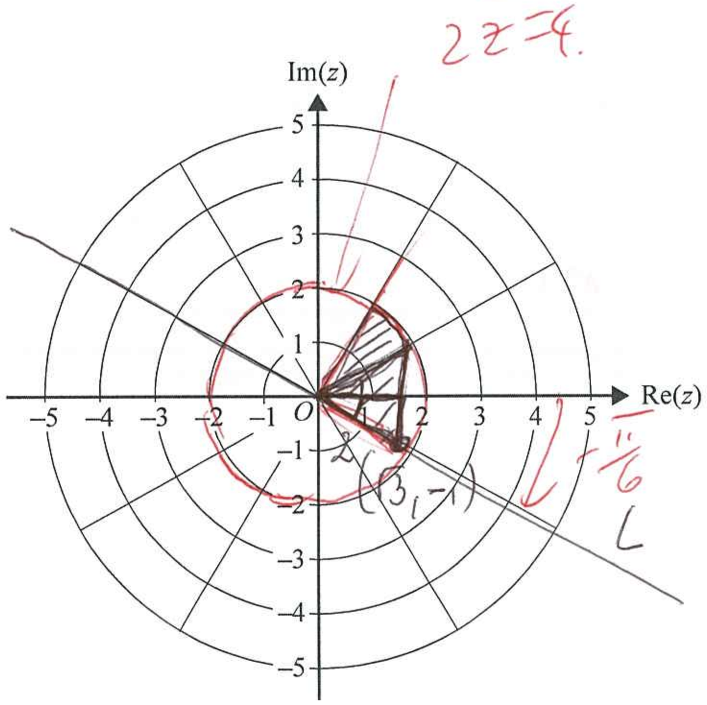
$\boxed{x^2 + y^2 = 4}$

$y = -\frac{1}{\sqrt{3}}x$

$\boxed{(-\sqrt{3}, 1)}$ and $\boxed{(\sqrt{3}, -1)}$

A1 A1

c. Sketch L and the graph of the relation $z\bar{z} = 4$ on the Argand diagram below. 2 marks



The part of the line L in the fourth quadrant can be expressed in the form $\text{Arg}(z) = \alpha$.

d. State the value of α. 1 mark

$$\boxed{\alpha = -\frac{\pi}{6}}$$ A1

e. Find the area enclosed by L and the graphs of the relations $z\bar{z} = 4$, $\text{Arg}(z) = \frac{\pi}{3}$ and $\text{Re}(z) = \sqrt{3}$. 2 marks

$$A = \frac{1}{2} \times 4 \times \sin\left(\frac{\pi}{3}\right) + \frac{1}{2} \times 2^2 \times \frac{\pi}{6}$$

$$= 2\,\frac{\sqrt{3}}{2} + 2 \times \frac{\pi}{6}$$

$$= \boxed{\sqrt{3} + \frac{\pi}{3}}$$

A1 A1

f. The straight line L can be written in the form $z = k\bar{z}$, where $k \in C$.

Find k in the form $r\text{cis}(\theta)$, where θ is the principal argument of k. 2 marks

$x + yi = k(x - yi)$ and $y = -\frac{1}{\sqrt{3}}x$

$x + yi = kx - kyi$

$yi + kyi = kx - x$

$y = \frac{x(k-1)}{(1+k)i}$

$\text{cSolve}\left(\frac{1-k}{1+k}i = -\frac{1}{\sqrt{3}}, k\right)$

$\boxed{k = \frac{1}{2} - \frac{\sqrt{3}}{2}i}$ A1

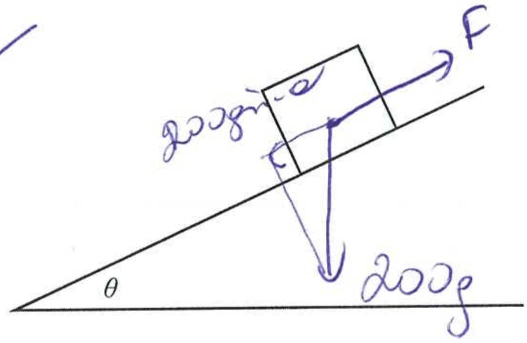
$\boxed{k = 1\ \text{cis}\left(\frac{-\pi}{3}\right)}$

Question 3 (10 marks) no friction

A 200 kg crate rests on a smooth plane inclined at θ to the horizontal. An external force of F newtons acts up the plane, parallel to the plane, to keep the crate in equilibrium.

a. On the diagram below, draw and label all forces acting on the crate. 1 mark

a ✓



b. Find F in terms of θ. 1 mark

$F = \boxed{200g\sin(\theta)}$ A1

The magnitude of the external force F is changed to 780 N and the plane is inclined at $\theta = 30°$.

c. **i.** Taking the direction down the plane to be positive, find the acceleration of the crate. 2 marks

$200a = 200g\sin(30°) - 780$ M1

$\boxed{a = 1\text{ ms}^{-2}}$ A1

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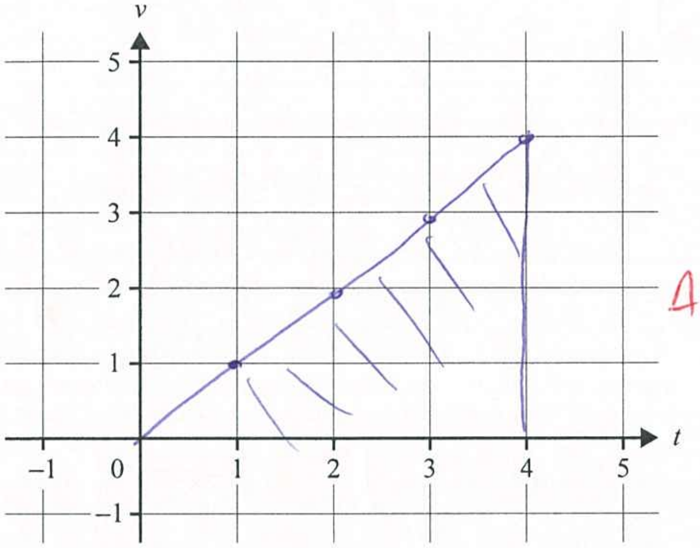
ii. On the axes below, sketch the velocity–time graph for the crate in the positive direction for the first four seconds of its motion. 1 mark



A1

iii. Calculate the distance the crate travels, in metres, in its first four seconds of motion. 1 mark

$d = \frac{1}{2} \times 4 \times 4 = 8\text{m}.$ A1

Starting from rest, the crate slides down a smooth plane inclined at α degrees to the horizontal. A force of 295 $\cos(\alpha)$ newtons, up the plane and parallel to the plane, acts on the crate.

d. If the momentum of the crate is 800 kg ms^{-1} after having travelled 10 m, find the acceleration, in ms^{-2}, of the crate. 2 marks

constant force.

$u = 0, \quad v = \frac{800}{200} = 4$ A1 $\quad s = 10\text{m}.$

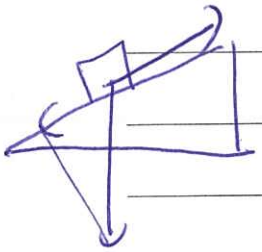
$2as = v^2$

$a = \frac{16}{2(10)} = \frac{16}{20} = \boxed{0.8}\ \text{ms}^{-2}$

A1

e. Find the angle of inclination, α, of the plane if the acceleration of the crate down the plane is $0.75\ \text{ms}^{-2}$. Give your answer in degrees, correct to one decimal place. 2 marks

$295\cos(\alpha)$



$200a = 200g\sin\alpha - 295\cos(\alpha)$ A1

$200g$

$200(0.75) = 200\times 9.8\sin\alpha - 295\cos\alpha$

$\boxed{\alpha = 12.9^\circ}$ A1

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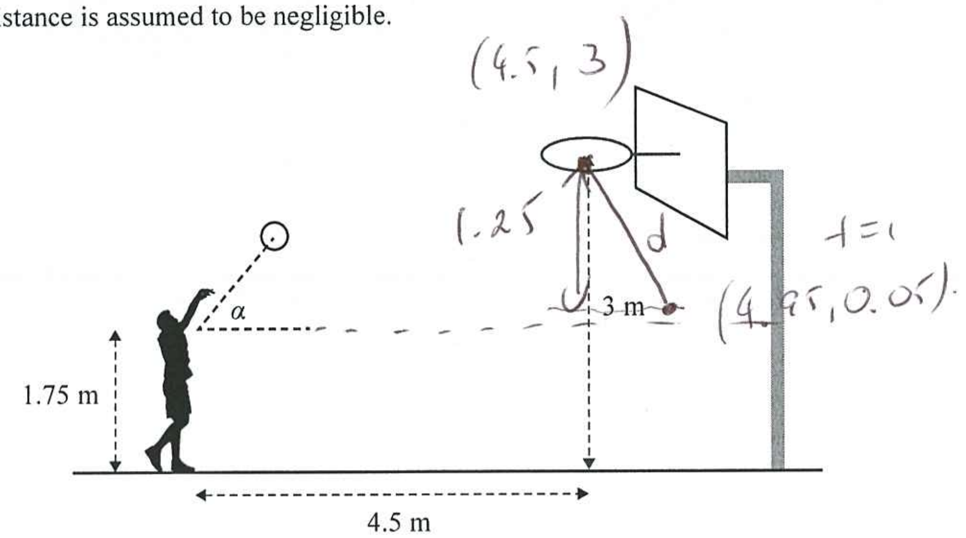
Question 4 (11 marks)

A basketball player aims to throw a basketball through a ring, the centre of which is at a horizontal distance of 4.5 m from the point of release of the ball and 3 m above floor level. The ball is released at a height of 1.75 m above floor level, at an angle of projection α to the horizontal and at a speed of V ms^{-1}. Air resistance is assumed to be negligible.



The position vector of the centre of the ball at any time, t seconds, for $t \geq 0$, relative to the point of release is given by $\underset{\sim}{r}(t) = Vt\cos(\alpha)\underset{\sim}{i} + \left(Vt\sin(\alpha) - 4.9t^2\right)\underset{\sim}{j}$, where $\underset{\sim}{i}$ is a unit vector in the horizontal direction of motion of the ball and $\underset{\sim}{j}$ is a unit vector vertically up. Displacement components are measured in metres.

a. For the player's first shot at goal, $V = 7$ ms^{-1} and $\alpha = 45°$.

i. Find the time, in seconds, taken for the ball to reach its maximum height. Give your answer in the form $\frac{a\sqrt{b}}{c}$, where a, b and c are positive integers. 2 marks

$\underset{\sim}{r}(t) = 7t\cos(45°)\underset{\sim}{i} + (7t\sin(45°) - 4.9t^2)\underset{\sim}{j}$

$= \frac{7}{\sqrt{2}}t\,\underset{\sim}{i} + \left(\frac{7t}{\sqrt{2}} - 4.9t^2\right)\underset{\sim}{j}$

$\dot{\underset{\sim}{r}}(t) = \frac{7}{\sqrt{2}}\underset{\sim}{i} + \left(\frac{7}{\sqrt{2}} - 9.8t\right)\underset{\sim}{j}$

CAS

solve$\left(\frac{7}{\sqrt{2}} - \frac{98t}{10} = 0, t\right)$ $\quad \frac{7}{\sqrt{2}} - 9.8t = 0.$

$\boxed{t = \frac{5\sqrt{2}}{14}}$ (0.505)

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ii. Find the maximum height, in metres, above floor level, reached by the centre of the ball. 2 marks

$t = \frac{5\sqrt{2}}{14}$

$\underset{\sim}{r}\left(\frac{5\sqrt{2}}{14}\right) = 1.25$ A1

$1.25 + 1.75 = \boxed{3\text{m}}$ A1

iii. Find the distance of the centre of the ball from the centre of the ring one second after release. Give your answer in metres, correct to two decimal places. 2 marks

$\underset{\sim}{r}(1) = \frac{7}{\sqrt{2}}\underset{\sim}{i} + 0.049747\ldots\underset{\sim}{j}$

$4.95\ldots$

$d = \sqrt{(4.95\ldots - 4.5)^2 + (1.25 - 0.04977\ldots)^2}$ M1

$= \boxed{1.28\text{m}}$ A1

b. For the player's second shot at goal, $V = 10$ ms^{-1}.

Find the possible angles of projection, α, for the centre of the ball to pass through the centre of the ring. Give your answers in degrees, correct to one decimal place. 3 marks

$\underset{\sim}{r}(t) = 10\cos\alpha\, t\,\underset{\sim}{i} + (10\sin\alpha\, t - 4.9t^2)\underset{\sim}{j}$

$10\cos\alpha\, t = 4.5 \quad (1) \Rightarrow t = \frac{4.5}{10\cos\alpha}$

$10\sin\alpha\, t - 4.9t^2 = 1.25 \quad (2)$

M1

$10\sin\alpha \cdot \frac{4.5}{\cos\alpha} - 4.9\left(\frac{4.5}{10\cos\alpha}\right)^2 = 1.25$

$4.5\tan\alpha - 4.9\left(\frac{4.5}{10}\right)^2\sec^2(\alpha) = 1.25$

$4.5\tan\alpha - 4.9\left(\frac{4.5}{10}\right)^2(1 + \tan^2\alpha) = 1.25$

$\boxed{\alpha = 29.7^\circ}$ or $\boxed{\alpha = 75.8^\circ}$

A1 A1

SECTION B – Question 4 – continued

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c. For the player's third shot at goal, the angle of projection is $\alpha = 60°$.

Find the speed V required for the centre of the ball to pass through the centre of the ring. Give your answer in metres per second, correct to one decimal place. 2 marks

$\underset{\sim}{r}(t) = V \cos 60° \, t \, \underset{\sim}{i} + \left(V \sin 60° \, t - 4.9t^2\right) \underset{\sim}{j}$

$\underset{\sim}{r}(t) = \frac{1}{2} V t \, \underset{\sim}{i} + \left(\frac{\sqrt{3}}{2} V t - 4.9t^2\right) \underset{\sim}{j}$

$\frac{1}{2} V t = 4.5 \qquad t = \frac{9}{V}$

$\frac{\sqrt{3}}{2} V t - 4.9t^2 = 1.25$ M1

$\frac{\sqrt{3}}{2} V \left(\frac{9}{V}\right) - 4.9\left(\frac{9}{V}\right)^2 = 1.25$

$\boxed{V = 7.8 \text{ ms}^{-1}}$ A1

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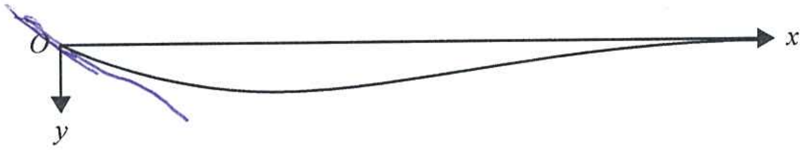
Question 5 (9 marks)

A horizontal beam is supported at its endpoints, which are 2 m apart. The deflection y metres of the beam measured downwards at a distance x metres from the support at the origin O is given by the differential equation $80\frac{d^2y}{dx^2} = 3x - 4$.



a. Given that both the inclination, $\frac{dy}{dx}$, and the deflection, y, of the beam from the horizontal at $x = 2$ are zero, use the differential equation above to show that $80y = \frac{1}{2}x^3 - 2x^2 + 2x$. 2 marks

$y(0)=0 \qquad \frac{dy}{dx}(0)=0.$

$\frac{dy}{dx} = \int \frac{3x-4}{80}\,dx$

~~or deflate.~~ $80\frac{dy}{dx} = \int 3x-4\,dx$

$80\frac{dy}{dx} = \frac{3x^2}{2} - 4x + c_1$ A1

$0 = 3\cdot 2 - 4\cdot 2 + c_1$

$c_1 = 2$

$80y = \frac{3x^3}{3\cdot 2} - \frac{4x^2}{2} + 2x + c_2$

$80y = \frac{x^3}{2} - 2x^2 + 2x + c_2$ A1 $\quad 0 = 4 - 8 + 4 + c_2$

$c_2 = 0$

$\therefore 80y = \frac{1}{2}x^3 - 2x^2 + 2x$

b. Find the angle of inclination of the beam to the horizontal at the origin O. Give your answer as a positive acute angle in degrees, correct to one decimal place. 2 marks

$x=0, \quad \frac{dy}{dx} = \frac{2}{80} = \frac{1}{40}$ A1

$\tan\theta = \frac{1}{40} \qquad \theta = \tan^{-1}\left(\frac{1}{40}\right)$

$\boxed{\theta = 1.4^\circ}$ A1

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c. Find the value of x, in metres, where the maximum deflection occurs, and find the maximum deflection, in metres. 3 marks

$\frac{dy}{dx} = 0$ M1

$x = \frac{2}{3}$ m A1

$y\left(\frac{2}{3}\right) = \frac{1}{135}$ m A1

d. Find the maximum angle of inclination of the beam to the horizontal in the part of the beam where $x \geq 1$. Give your answer as a positive acute angle in degrees, correct to one decimal place. 2 marks

$x = \frac{4}{3}$ inflection point

A1 $\frac{d^2y}{dx^2} = 0$

$\frac{dy}{dx} = -0.008333$

$\boxed{\text{angle} = 0.5^\circ}$ A1

Question 6 (5 marks)

A coffee machine dispenses coffee concentrate and hot water into a 200 mL cup to produce a long black coffee. The volume of coffee concentrate dispensed varies normally with a mean of 40 mL and a standard deviation of 1.6 mL.

Independent of the volume of coffee concentrate, the volume of water dispensed varies normally with a mean of 150 mL and a standard deviation of 6.3 mL.

a. State the mean and the standard deviation, in millilitres, of the total volume of liquid dispensed to make a long black coffee. 2 marks

$C \sim N(40, 1.6^2) \quad W \sim N(150, 6.3^2)$

$E(C+W) = 190$

$Var(C+W) : 1.6^2 + 6.3^2 = 42.25$

mean $= 190$ A1

$\text{sd. dev} = \sqrt{42.25} = \boxed{6.5}$ A1

b. Find the probability that a long black coffee dispensed by the machine overflows a 200 mL cup. Give your answer correct to three decimal places. 1 mark

$\Pr(X > 200) = \boxed{0.062}$ A1

c. Suppose that the standard deviation of the volume of water dispensed by the machine can be adjusted, but that the mean volume of water dispensed and the standard deviation of the volume of coffee concentrate dispensed cannot be adjusted.

Find the standard deviation of the volume of water dispensed that is needed for there to be only a 1% chance of a long black coffee overflowing a 200 mL cup. Give your answer in millilitres, correct to two decimal places. 2 marks

invNorm $(0.99, 0, 1) = 2.32635 \ldots$

$z = \frac{x - \mu}{\sigma}$

M1

$2.32635 = \frac{200 - 190}{\sigma}$

$\sigma = 4.29858 = \sqrt{1.6^2 + s^2}$

$\boxed{s = 3.99}$ A1

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Question 7 (4 marks)

According to medical records, the blood pressure of the general population of males aged 35 to 45 years is normally distributed with a mean of 128 and a standard deviation of 14. Researchers suggested that male teachers had higher blood pressures than the general population of males.

To investigate this, a random sample of 49 male teachers from this age group was obtained and found to have a mean blood pressure of 133.

a. State **two** hypotheses and perform a statistical test at the 5% level to determine if male teachers belonging to the 35 to 45 years age group have higher blood pressures than the general population of males. Clearly state your conclusion with a reason. 3 marks

$H_0 = 128$

$H_1 > 128$ A1

$p = 0.00621 < 0.05$
A1

Reject H_0 and conclude that there is a good reason to believe that male teachers have higher blood pressure than the general population of males. A1

b. Find a 90% confidence interval for the mean blood pressure of all male teachers aged 35 to 45 years using a standard deviation of 14. Give your answers correct to the nearest integer. 1 mark

$[130, 136]$ A1

END OF QUESTION AND ANSWER BOOK