SPECIALIST MATHEMATICS

Units 3&4 – Written Examination 1

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2017 Trial Examination

SOLUTIONS

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Question 1 (2 marks)

Method 1

$\sin(2\alpha) = \cos\left(2\alpha - \frac{\pi}{2}\right)$ 1 mark

$= 2\cos^2(\alpha - \frac{\pi}{4}) - 1$

$= 2 \times \left(\frac{4\sqrt{2}}{7}\right)^2 - 1$

$= \frac{15}{49}$ 1 mark

Method 2

$\cos\left(\alpha - \frac{\pi}{4}\right) = \cos(\alpha)\cos\left(\frac{\pi}{4}\right) + \sin(\alpha)\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\cos(\alpha) + \frac{1}{\sqrt{2}}\sin(\alpha) = \frac{4\sqrt{2}}{7}$

$\cos(\alpha) + \sin(\alpha) = \frac{8}{7}$ 1 mark

$(\cos(\alpha) + \sin(\alpha))^2 = \cos^2(\alpha) + 2\sin(\alpha)\cos(\alpha) + \sin^2(\alpha) = \frac{64}{49}$

$1 + \sin(2\alpha) = \frac{64}{49}$

$\sin(2\alpha) = \frac{15}{49}$ 1 mark

Question 2 (5 marks)

a. $P(z) = z^5 + 4z^4 + 5z^3 + 8z^2 + 32z + 40$

$= (z^3 + 8)(z^2 + 4z + 5)$

$\therefore$ $z^3 + 8$ is a factor of $P(z)$ 1 mark

b. Let $z^3 + 8 = 0$

Then

$z^3 = -8 = 8\,cis(\pi)$

$z = 2\,cis\,(\frac{\pi}{3}),\ \ 2\,cis\,(\pi),\ \ 2\,cis\,(-\frac{\pi}{3})$ 1 mark

$= 1 + \sqrt{3}i, -2, 1 - \sqrt{3}i$ 1 mark

Let $z^2 + 4z^2 + 5 = 0$

Then

$(z + 2)^2 + 1 = 0$ 1 mark

$\therefore\ z = -2 \pm i$ 1 mark

Question 3 (3 marks)

Let X be the random variable of the weight of any egg produced by Farm A and let Y be the random variable of the weight of any egg produced by Farm B. Let W be the weight of the 7 selected eggs.

a. $E(W) = 4E(X) + 3E(Y)$

$= 4 \times 70 + 3 \times 60$

$= 460\ g$ 1 mark

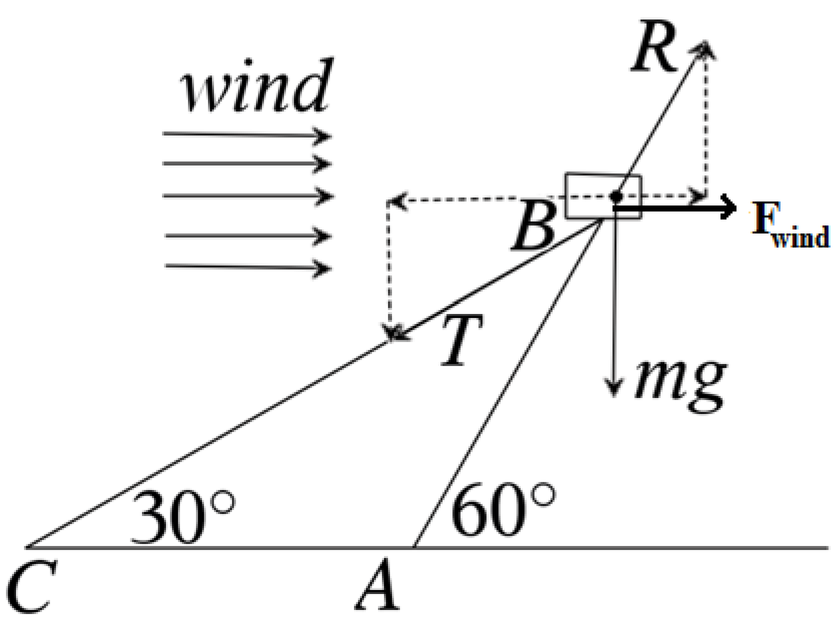
b. $sd(W) = \sqrt{4\,Var(X) + 3\,Var(Y)}$ 1 mark

$= \sqrt{4 \times 4 + 3 \times 3}$

$= 5\ g$ 1 mark

Question 4 (4 marks)

a. 1 mark



b. $R\sin(60°) = mg + T sin(30°)$
$R\cos(60°) + F_{wind} = T\cos(30°)$

Hence,

$\frac{\sqrt{3}}{2}R = 30g + \frac{1}{2}T$ (1) 1 mark

$\frac{1}{2}R + 10\sqrt{3}g = \frac{\sqrt{3}}{2}T$ (2) 1 mark

From Equation (2), $R = \sqrt{3}T - 20\sqrt{3}g$
Substitute into (1) and simplify

$$3T - 60g = 60g + T$$

Therefore

$$T = 60g, \quad R = 40\sqrt{3}g$$ 1 mark

Question 5 (4 marks)

a. Let $\theta = \angle BAC$.
Then

$$\cos(\theta) = \frac{\underset{\sim}{b} \cdot \underset{\sim}{c}}{|\underset{\sim}{b}||\underset{\sim}{c}|}$$

$$\sin(\theta) = \sqrt{1 - \cos^2(\theta)}$$

$$= \frac{1}{|\underset{\sim}{b}||\underset{\sim}{c}|}\sqrt{\left|\underset{\sim}{b}\right|^2 \left|\underset{\sim}{c}\right|^2 - \left(\underset{\sim}{b} \cdot \underset{\sim}{c}\right)^2}$$ 1 mark

The area of the triangle ABC

$$A_\Delta = \frac{1}{2}|AB||AC|\sin(\theta)$$

$$= \frac{1}{2}\left|\underset{\sim}{b}\right|\left|\underset{\sim}{c}\right|\sin(\theta)$$

$$= \frac{1}{2}\sqrt{\left|\underset{\sim}{b}\right|^2 \left|\underset{\sim}{c}\right|^2 - \left(\underset{\sim}{b} \cdot \underset{\sim}{c}\right)^2}$$ 1 mark

b. $\overrightarrow{AB} = \underset{\sim}{b} = 2\underset{\sim}{i} + 2\underset{\sim}{j}, \quad \overrightarrow{AC} = \underset{\sim}{c} = 4\underset{\sim}{i} - 2\underset{\sim}{j}$ 1 mark

$$\left|\underset{\sim}{b}\right|^2 = 8, \quad \left|\underset{\sim}{c}\right|^2 = 20, \quad \left(\underset{\sim}{b} \cdot \underset{\sim}{c}\right)^2 = (8-4)^2 = 16$$

Therefore, the required area

$$A_\Delta = \frac{1}{2}\sqrt{8 \times 20 - 16} = 6$$ 1 mark

Question 6 (4 marks)

Differentiate $y^4 + xy^2 - 6x^2 + 4 = 0$

$$4y^3\frac{dy}{dx} + y^2 + 2xy\frac{dy}{dx} - 12\,x = 0$$ 1 mark

Substitute $x = 1, y = 1,$

$$4\frac{dy}{dx} + 1 + 2\frac{dy}{dx} - 12 = 0$$

$$\frac{dy}{dx} = \frac{11}{6}$$ 1 mark

The gradient of the normal $m = -\frac{6}{11}$ 1 mark

Hence the equation of the normal

$$y - 1 = -\frac{6}{11}(x - 1)$$

Or

$$y = -\frac{6}{11}x + \frac{17}{11}$$ 1 mark

Question 7 (4 marks)

Separate variables

$$\frac{1}{y}dy = \frac{4x}{(2x+1)(2x+3)}dx$$ 1 mark

Integrate both sides

$$\int\frac{1}{y}dy = \int\frac{4x}{(2x+1)(2x+3)}dx$$

$$\ln|\mathrm{y}| = \int\left(\frac{-1}{2x+1} + \frac{3}{2x+3}\right)dx = \frac{1}{2}\ln\left|\frac{(2x+3)^3}{2x+1}\right| + C$$ 1 mark

$$y^2 = A\frac{(2x+3)^3}{2x+1} \quad \text{where } A = \pm e^{2c}$$ 1 mark

Substitute $x = -1,\ y = 1$

$$1 = A \times \frac{1^3}{-1} \Rightarrow A = -1$$

Therefore

$$y = \sqrt{-\frac{(2x+3)^3}{2x+1}}$$ 1 mark

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Question 8 (5 marks)

a. $\frac{dy}{dt} = 3 \times \frac{1-\sin(t)}{\cos(t)} \times \frac{-\sin(t)(1-\sin(t))-\cos(t)(-\cos(t))}{(1-\sin(t))^2}$

$= 3\sec(t)$

$= 3\sqrt{2}$ when $t = \frac{\pi}{4}$

$\frac{dx}{dt} = 3\sec(t)\tan(t) = 3\sqrt{2}$ when $t = \frac{\pi}{4}$

Therefore

$\frac{dy}{dx} = 1$ when $t = \frac{\pi}{4}$. 1 mark

when $t = \frac{\pi}{4}$, $x = 3\sqrt{2}$, $y = 3\ln\left(\frac{\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}\right) = 3\ln(\sqrt{2}+1)$ 1 mark

The tangent equation at $(3\sqrt{2}, 3\ln(\sqrt{2}+1))$:

$$y - 3\ln(\sqrt{2}+1) = x - 3\sqrt{2}$$

i.e.

$$y = x - 3\sqrt{2} + 3\ln(\sqrt{2}+1)$$ 1 mark

b. $L = \int_0^{\frac{\pi}{3}} \sqrt{(3\sec(t))^2 + (3\sec(t)\tan(t))^2}\,dt$ 1 mark

$= 3\int_0^{\frac{\pi}{3}} \sec^2(t)\,dt$

$= 3[\tan(t)]_0^{\frac{\pi}{3}} = 3\sqrt{3}$ 1 mark

Question 9 (4 marks)

$\frac{dV}{dt} = -9r^{\frac{4}{3}}$, $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$ 1 mark

$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} = -\frac{9r^{\frac{4}{3}}}{4\pi r^2} = -\frac{9}{4\pi r^{\frac{2}{3}}}$ 1 mark

$t = -\int_{27}^{8} \frac{4\pi r^{\frac{2}{3}}}{9}\,dr$ 1 mark

$= -\frac{4\pi}{9} \times \frac{3}{5} \times [r^{\frac{5}{3}}]_{27}^{8}$

$= \frac{844\pi}{15}$ 1 mark

Question 10 (4 marks)

$a = \frac{dv}{dt} = \frac{v(v^2+3)}{36}$

$v\frac{dv}{dx} = \frac{v(v^2+3)}{36}$ 1 mark

$x = \int_0^3 \frac{36}{v^2+3}\,dv$ 1 mark

$= \frac{36}{\sqrt{3}}[\arctan(\frac{v}{\sqrt{3}})]_0^3$ 1 mark

$= 4\sqrt{3}\pi$ 1 mark

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