Year 2017

VCE

Specialist Mathematics

Trial Examination 2

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Victorian Certificate of Education 2017

STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 36 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions and sign your name in the space provided.
- All written responses must be in English.

At the end of the examination

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The straight-line asymptotes of the graph of the function with the rule

$f(x) = \dfrac{x^3 + ax^2 - ax + x - a^2}{x^2 - a}$, where a is a positive real constant, are given by

A. $x = \sqrt{a}$ and $x = -\sqrt{a}$ only.

B. $x = \sqrt{a}$ and $y = x$ only.

C. $x = \sqrt{a}$ and $y = x + a$ only.

D. $x = \sqrt{a}$, $x = -\sqrt{a}$ and $y = x$ only.

E. $x = \sqrt{a}$, $x = -\sqrt{a}$ and $y = x + a$ only.

Question 2

If a is a positive real constant, then the domain and range of the function with rule

$f(x) = \dfrac{2}{\pi}\tan^{-1}\left(\dfrac{2x + a}{a}\right) + a$ are respectively

A. R and $\left(-\dfrac{\pi}{2} + a\,,\ \dfrac{\pi}{2} + a\right)$

B. R and $(-1 + a\,,\ 1 + a)$

C. $\left(-\dfrac{1}{2}, 0\right)$ and $\left(-\dfrac{\pi}{2} + a\,,\ \dfrac{\pi}{2} + a\right)$

D. $\left(-\dfrac{1}{2}, 0\right)$ and $(-1 + a\,,\ 1 + a)$

E. $\left[-\dfrac{1}{2}\,,\ 0\right]$ and $(-a\,,\ a)$

Question 3

The polynomial $P(z)$ has real coefficients. Three of the roots of the equation $P(z)=0$ are $z=a$, $z=a+ai$ and $z=a-i$ where a is a non-zero real constant. The minimum number of roots that the equation $P(z)=0$ could have is

A. 3

B. 4

C. 5

D. 6

E. 7

Question 4

If A, B, C, D, E and a are all non-zero real constants, then the algebraic fraction $\frac{x^2}{x^3-a^3}$ could be expressed in partial fractions as

A. $\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{(x-a)^3}$

B. $\frac{A}{x-a}+\frac{B}{x^2+ax+a^2}$

C. $\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x^2+ax+a^2}$

D. $\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{(x-a)^3}+\frac{Dx+E}{x^2+ax+a^2}$

E. $\frac{A}{x-a}+\frac{Bx+C}{x^2+ax+a^2}$

Question 5

A particle moves so that its position vector is given by $\underset{\sim}{r}(t) = a\sec(t)\underset{\sim}{i} + b\tan^2(t)\underset{\sim}{j}$

where *a* and *b* are non-zero real constants. The particle moves along part of

A. a straight line.

B. a parabola.

C. a circle.

D. a hyperbola.

E. an ellipse.

Question 6

Given the vectors $\underset{\sim}{a} = -\underset{\sim}{i} + y\underset{\sim}{j} - 3\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - 4\underset{\sim}{j} + 6\underset{\sim}{k}$. Which of the following is **false**?

A. If $y = 1$ the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are coplanar.

B. If $y = 2$ the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel

C. If $y > -5$ the angle between the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is obtuse.

D. If $y < -5$ the angle between the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is acute.

E. If $y = 8$ the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are equal in length.

Question 7

Given the complex number $z = \sqrt{\sqrt{2}+2} + \sqrt{2-\sqrt{2}}\, i$, then $\text{Arg}\left(\frac{1}{\bar{z}^{10}}\right)$ is equal to

A. $\left(\frac{8}{\pi}\right)^{10}$

B. $-\frac{10\pi}{8}$

C. $-\frac{3\pi}{4}$

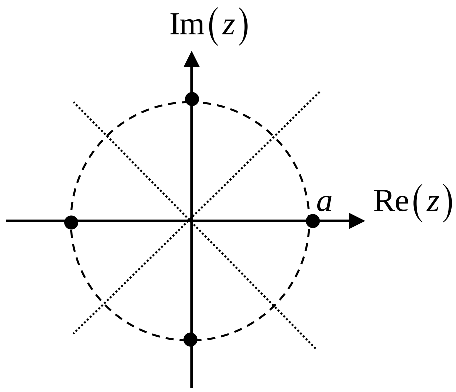
D. $\frac{3\pi}{4}$

E. $\frac{10\pi}{8}$

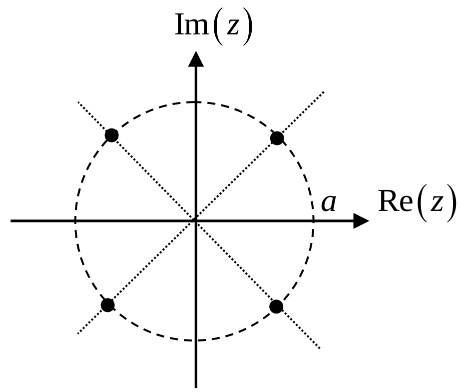
Question 8

Which one of the following diagrams represents the roots of the equation $z^4 - a^4 i = 0$ in the complex plane where a is a positive real constant.

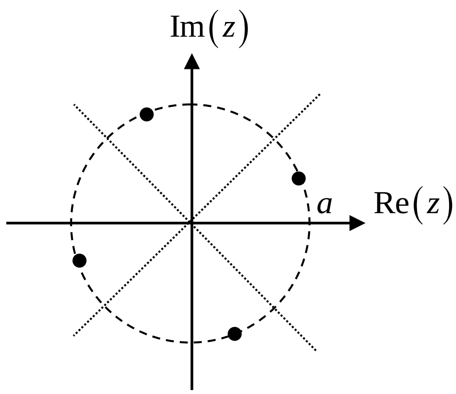
A.



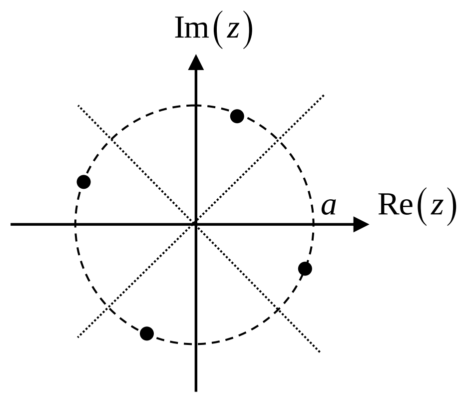
B.



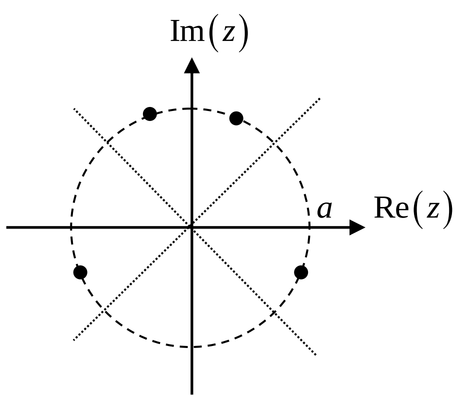
C.



D.



E.



Question 9

Two vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are such that $\underset{\sim}{u}$ is a unit vector, $\underset{\sim}{u}.\underset{\sim}{v}=\sqrt{3}$ and $|\underset{\sim}{v}|=2$.
Some students stated some observations regarding the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$.

Amanda stated that the angle between the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is 30^0.

Brianna stated that the scalar resolute of $\underset{\sim}{u}$ in the direction of $\underset{\sim}{v}$ is equal to $\dfrac{\sqrt{3}}{2}$.

Colin stated that the scalar resolute of $\underset{\sim}{v}$ in the direction of $\underset{\sim}{u}$ is equal to $\sqrt{3}$.

Dianne stated that $|\underset{\sim}{u}+\underset{\sim}{v}|=\sqrt{5+2\sqrt{3}}$.

Edward stated that $|\underset{\sim}{u}-\underset{\sim}{v}|=\sqrt{5-2\sqrt{3}}$.

Then

A. Only Amanda is correct.

B. Both Brianna and Colin are correct, the others are incorrect.

C. Both Dianne and Edward are correct, the others are incorrect.

D. Amanda, Brianna and Colin are correct, the others are incorrect.

E. All of Amanda, Brianna, Colin, Dianne and Edward are correct.

Question 10

When Euler's method, with a step size of $\dfrac{\pi}{8}$, is used to solve the differential equation $\dfrac{dy}{dx}=\sin^3(2x)$ with $x_0=0$ and $y_0=2$, the value of y_3 is equal to

A. $\dfrac{\sqrt{2}}{32}$

B. $2+\dfrac{\pi}{8}$

C. $2+\dfrac{\pi}{8}\left(\dfrac{\sqrt{2}}{2}+1\right)$

D. $2+\dfrac{\pi}{8}\left(\dfrac{\sqrt{2}}{4}+1\right)$

E. $\dfrac{1}{24}\left(35+5\sqrt{2}\right)$

Question 11

A moving particle has a velocity vector at a time *t*, given by $t\cos(t)\underset{\sim}{i}+t\sin(t)\underset{\sim}{j}$, for $t\geq 0$. Initially the particle is at the origin. The position vector is given by

A. $\left(\cos(t)+t\sin(t)-1\right)\underset{\sim}{i}+\left(\sin(t)-t\cos(t)\right)\underset{\sim}{j}$.

B. $\left(\cos(t)+t\sin(t)\right)\underset{\sim}{i}+\left(\sin(t)-t\cos(t)\right)\underset{\sim}{j}$

C. $t\sin(t)\underset{\sim}{i}-t\cos(t)\underset{\sim}{j}$.

D. $-t\sin(t)\underset{\sim}{i}+t\cos(t)\underset{\sim}{j}$.

E. $\frac{1}{2}t^2\cos(t)\underset{\sim}{i}+\frac{1}{2}t^2\sin(t)\underset{\sim}{j}$

Question 12

Two particles, *P* and *Q* have position vectors $\underset{\sim}{p}=(2t-2)\underset{\sim}{i}+\left(t^2-6t+9\right)\underset{\sim}{j}$ and $\underset{\sim}{q}=(3t-4)\underset{\sim}{i}+\left(t^2-4t+5\right)\underset{\sim}{j}$ respectively at a time *t* seconds, $t\geq 0$.
It follows that

A. both particles move on parabolic paths and *P* and *Q* are in the same position when $t=2$.

B. the particles paths intersect exactly once.

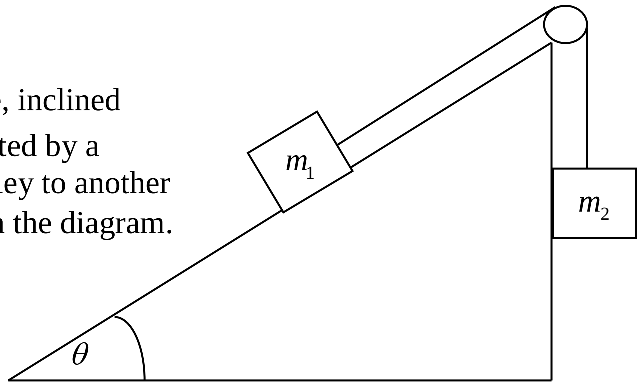
C. both particles move on straight line paths and *P* and *Q* are in the same position when $t=2$.

D. *P* and *Q* are travelling at the same speed when $t=3$.

E. *P* and *Q* are never in the same position.

Question 13

A particle of mass m_1 kg is on a smooth plane, inclined at an angle of θ to the horizontal. It is connected by a light string which passes around a smooth pulley to another mass of m_2 kg hanging vertically, as shown in the diagram.



Which of the following is **false?**

A. The tension in the string is equal to $\dfrac{m_1 m_2\left(1+\sin(\theta)\right)}{m_1+m_2}$ kg-wt.

B. If $m_2 > m_1\sin(\theta)$ the mass m_2 moves downwards with an acceleration $\dfrac{g\left(m_2 - m_1\sin(\theta)\right)}{m_1+m_2}$ ms^{-2}.

C. If $m_2 = m_1\sin(\theta)$ the masses remain at rest.

D. If $m_2 = 2m_1$ and $\theta = 30^0$ the tension in the string is $\dfrac{g}{2}$ newtons.

E. If $m_2 = 2m_1$ and $\theta = 30^0$ the mass m_2 moves downwards with an acceleration $\dfrac{g}{2}$ ms^{-2}.

Question 14

The velocity v ms^{-1} of a particle is given by $\cos\left(\sqrt{t}\right)$ at a time t seconds, where $t \ge 0$.

If $x = 2$ when $t = 1$, then the value of x when $t = 2$ can be found by evaluating

A. $\displaystyle\int_1^2 \cos\left(\sqrt{u}\right)du$

B. $\displaystyle\int_1^2 \cos\left(\sqrt{u}\right)du + 2$

C. $\displaystyle\int_1^2 \left(\cos\left(\sqrt{u}\right)+2\right)du$

D. $\displaystyle\int_1^2 \cos\left(\sqrt{u}\right)du - 2$

E. $\displaystyle\int_1^2 \left(\cos\left(\sqrt{u}\right)-2\right)du$

Question 15

A jogger runs so that at time *t*, his velocity is $v(t)$. Between the times of $t = a$ and $t = b$, the expression $\int_a^b |v(t)|\,dt$ represents the

A. displacement.

B. average displacement.

C. total distance travelled.

D. average speed.

E. average velocity.

Question 16

A body is moving in a straight line. Its velocity v ms^{-1} is given by $\frac{x^2}{\sqrt{t}}$ when it is *x* metres from the origin at a time *t* seconds. Given that $x = 1$ when $t = 4$, then the rule relating *x* to *t* is given by

A. $x = \frac{1}{5 - 2\sqrt{t}}$

B. $x = \frac{1}{2\sqrt{t} - 3}$

C. $x = \frac{5}{4} - \frac{1}{2\sqrt{t}}$

D. $x = \frac{2}{\sqrt{t}}$

E. $x = \frac{\sqrt{t}}{2}$

Question 17

The differential equation which best represents the direction field shown, is

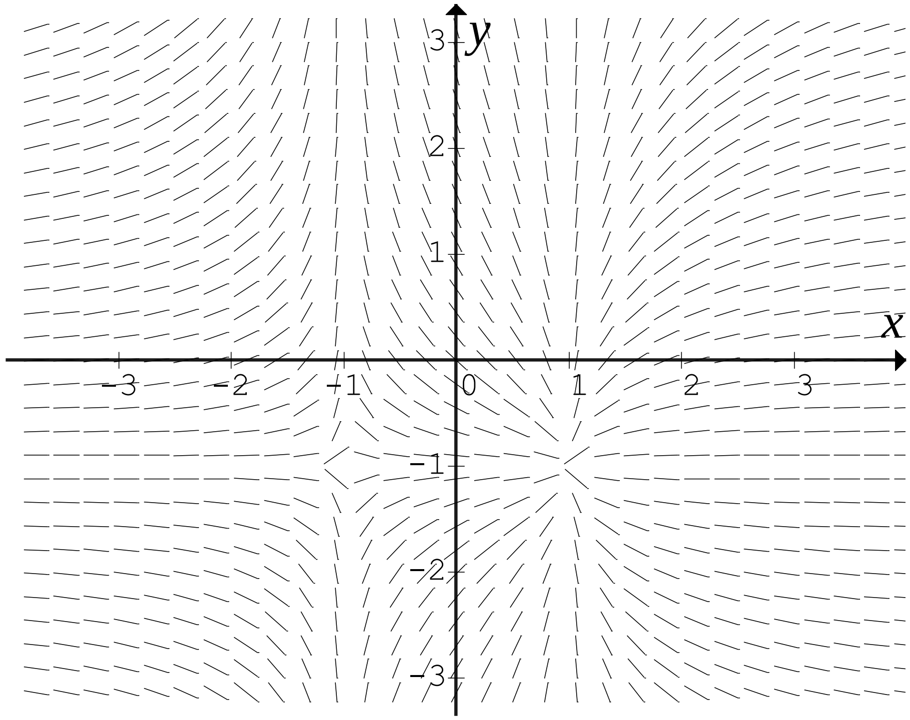
A. $\frac{dy}{dx}=\frac{y+1}{x-1}$

B. $\frac{dy}{dx}=\frac{y+1}{x+1}$

C. $\frac{dy}{dx}=\frac{y-1}{x^2-1}$

D. $\frac{dy}{dx}=\frac{y+1}{x^2-1}$

E. $\frac{dy}{dx}=(x+1)(y+1)$



Question 18

The weights of a tub of margarine are normally distributed, with a mean of 250 grams, and a standard deviation of 4 grams. The weights of a jar of vegemite are normally distributed, with a mean of 380 grams, and a standard deviation of 5 grams. The mean and standard deviation of two tubs of margarine and three jars of vegemite in grams respectively are

A. 4420, 17

B. 4420, 61

C. 1640, 17

D. 1640, $\sqrt{107}$

E. 1640, 107

Question 19

A school principal must decide whether or not to cancel the school sports day due to a threatening rain day. What are the Type I and Type II errors for the null hypothesis, that the weather will remain dry?

A. Type I error: Don't cancel the sports day, but it rains.
Type II error: The weather remains dry, but the sports day is needlessly cancelled.

B. Type I error: The weather remains dry, but the sports day is needlessly cancelled.
Type II error: Don't cancel the sports day, and it rains.

C. Type I error: Cancel the sports day, and it rains.
Type II error: Don't cancel the sports day and it rains.

D. Type I error: Don't cancel the sports day, and it rains.
Type II error: Don't cancel the sports day, and the weather remains dry.

E. Type I error: Don't cancel the sports day, but it rains.
Type II error: Cancel the sports day, and it rains.

Question 20

Suppose you do 25 independent tests of the form $H_0:\ \mu = \mu_0$ versus $H_1:\ \mu < \mu_0$ each at the 10% level of significance. The probability of committing a Type I error and incorrectly rejecting a true null hypothesis with at least one of the 25 tests, is closest to

A. 0.928

B. 0.723

C. 0.277

D. 0.1

E. 0.072

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (12 marks)

A flying drone follows the path described by the parametric equations $x = t - \sin\left(\frac{\pi t}{2}\right)$, $y = 3 - 2\cos\left(\frac{\pi t}{2}\right)$ for $0 \le t \le T$, where x is the distance horizontally forward and y is the height of the drone above ground level, at a time t seconds. All distances are measured in metres. At time $t = T$ the drone crashes into a vertical wall, 8 metres horizontally away from the point of release.

a. Show that $T = 7$.

1 mark

b. Find the initial height and the height at which the drone hits the wall.

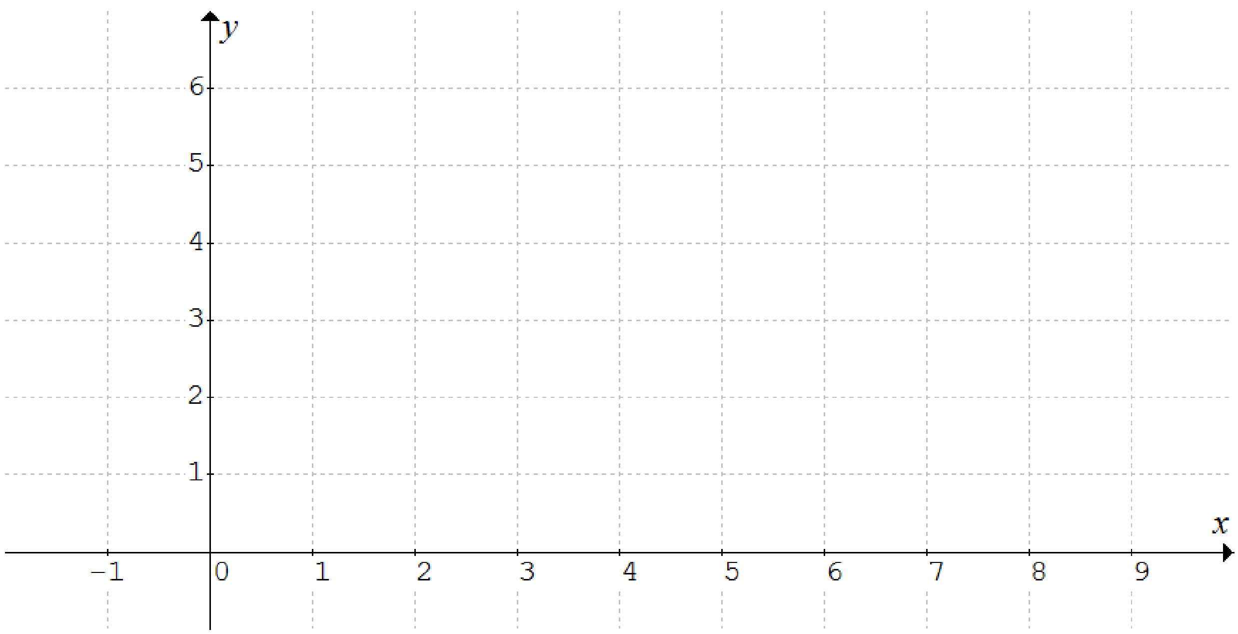
1 mark

c. Find the times and position (coordinates) for $0 < t \le T$ when the drone is flying horizontally.

3 marks

d. Sketch the path of the drone on the axis below.

2 marks



e. Find the speed in m/s at which the drone hits the wall.

2 marks

f. Find the acute angle, correct to the nearest tenth of a degree measured with the wall, at which the drone hits the wall.

1 mark

g.i. Write down a definite integral in terms of t, which gives the total distance in metres travelled by the drone, from the instant it is released, until it hits the wall.

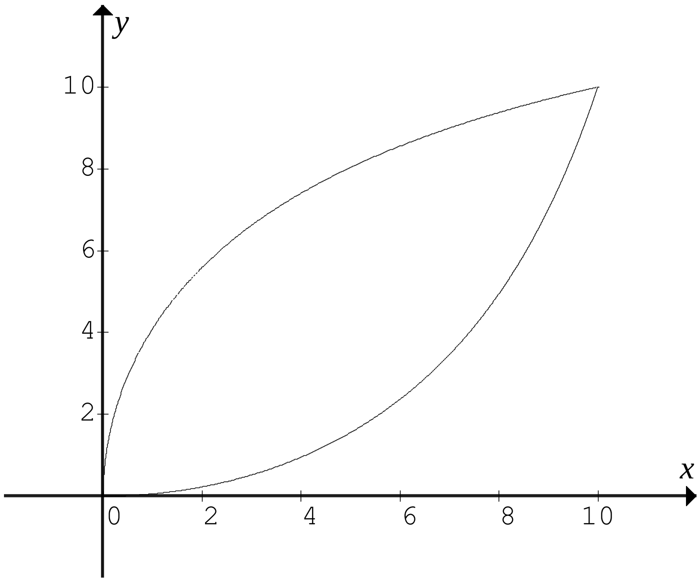
1 mark

ii. Find the distance travelled in metres, by the drone, from the time it is released until it hits the wall. Give your answer correct to two decimal places.

1 mark

Question 2 (9 marks)

The diagram below shows a picture of a garden leaf. The leaf can be modelled by two curves, an upper function $g(x)$ and a lower function $f(x)$. Both curves pass through the origin and the point $(10,10)$. All dimensions are measured in centimetres. The upper and lower curves are symmetrical about the line $y = x$.



a. The lower curve is a function of the form $f:[0,10] \to R$, $f(x) = 10\sec\left(\dfrac{\pi x}{n}\right) - 10$.

Show that $n = 30$.

1 mark

b. Show that the rule for the function g can be expressed as $g(x)=\frac{k}{\pi}\cos^{-1}\left(\frac{a}{x+a}\right)$ and show that $k=30$ and $a=10$.

1 mark

c.i Write down two equivalent, but different definite integrals in terms of *x*, which gives the area of the leaf in square centimetres.

1 mark

ii. Find the area of the leaf, in square centimetres. giving your answer correct to three decimal places.

1 mark

d. Find $f'(x)$.

1 mark

e. Show that $g'(x)=\dfrac{300}{\pi|x+10|\sqrt{x(x+20)}}$.

2 marks

f.i. Write down two equivalent, but different definite integrals in terms of *x*, which gives the total perimeter of the leaf in centimetres.

1 mark

ii. Find the total perimeter of the leaf, in centimetres, giving your answer correct to three decimal places.

1 mark

Question 3 (12 marks)

a. Consider the points $U\left(\sqrt{2}-1,\sqrt{2}\right)$, $V\left(-\sqrt{2}-1,\sqrt{2}\right)$ and $C\left(-1,0\right)$.

Find the vectors $\overrightarrow{CU}$ and $\overrightarrow{CV}$ and hence determine the angle between the vectors $\overrightarrow{CU}$ and $\overrightarrow{CV}$

2 marks

b. Let $u=\sqrt{2}-1+\sqrt{2}\,i$. If $|u|=\sqrt{b-a\sqrt{a}}$, find the values of a and b and find $\text{Arg}(u)$.

2 marks

c. Let $S = \{ z : |z+1| = 2,\ z \in C \}$. Find and describe the cartesian equation of *S*.

1 mark

d. Let $R = \{ z : \text{Arg}(z+1) = \frac{3\pi}{4},\ z \in C \}$. Find and describe the cartesian equation of *R*.

1 mark

e. Let $T = \{ z : |z| = |z+1-i|,\ z \in C \}$. Find and describe the cartesian equation of *T*.

1 mark

f. Find the point(s) of intersection between S and R.

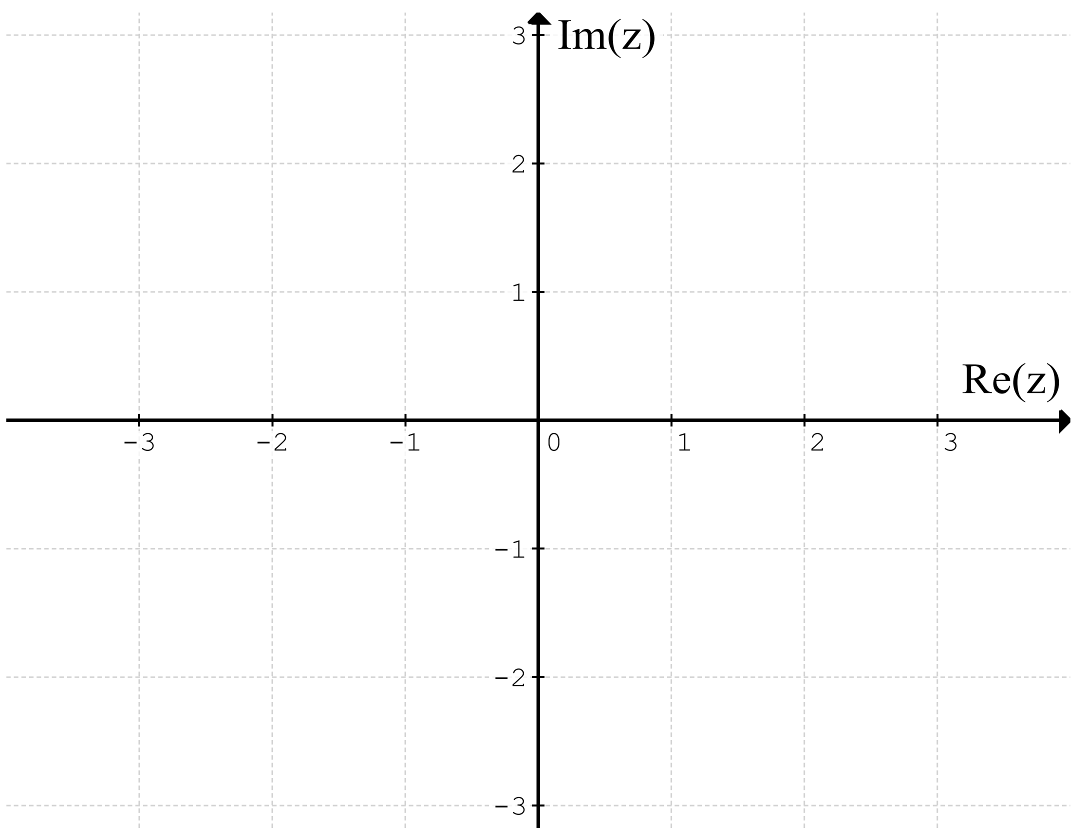
1 mark

g. Find the point(s) of intersection between S and T.

1 mark

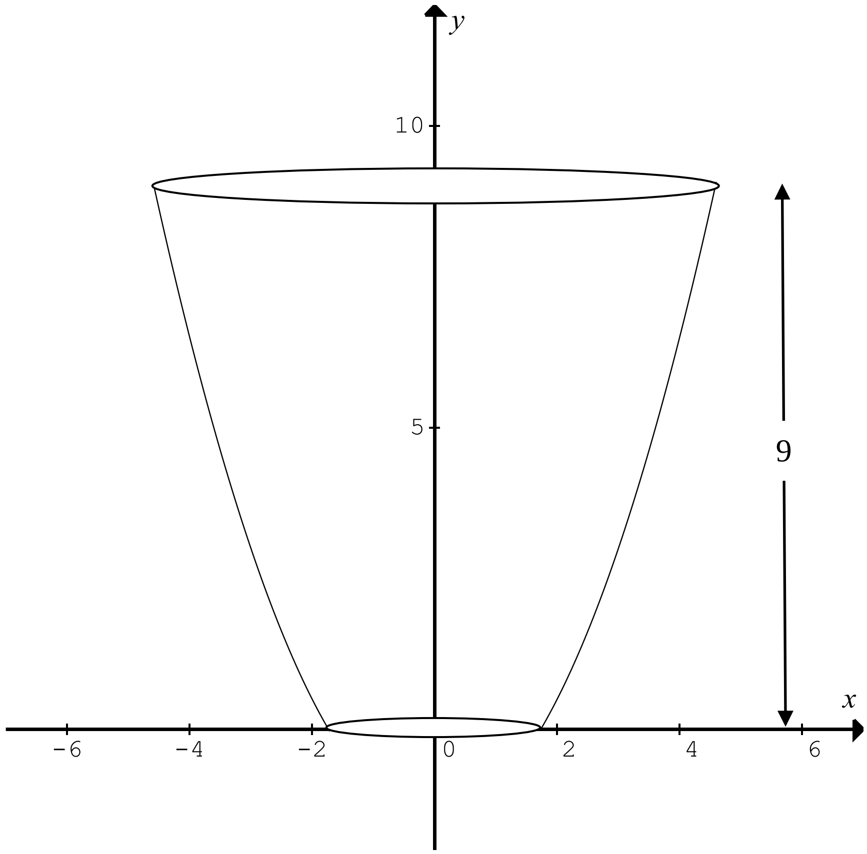
h. Clearly plot the point u and the sets R, S and T on the Argand diagram below.

3 marks



Question 4 (10 marks)

The diagram below shows a cup, with dimensions in cm. The sides of the cup are modelled by part of the curve $y = \frac{1}{2}\left(x^2 - 3\right)$. The cup is formed when the curve is rotated about the *y*-axis. The height of the cup is 9 cm.



a. Find the capacity of the cup in cubic centimetres.

1 mark

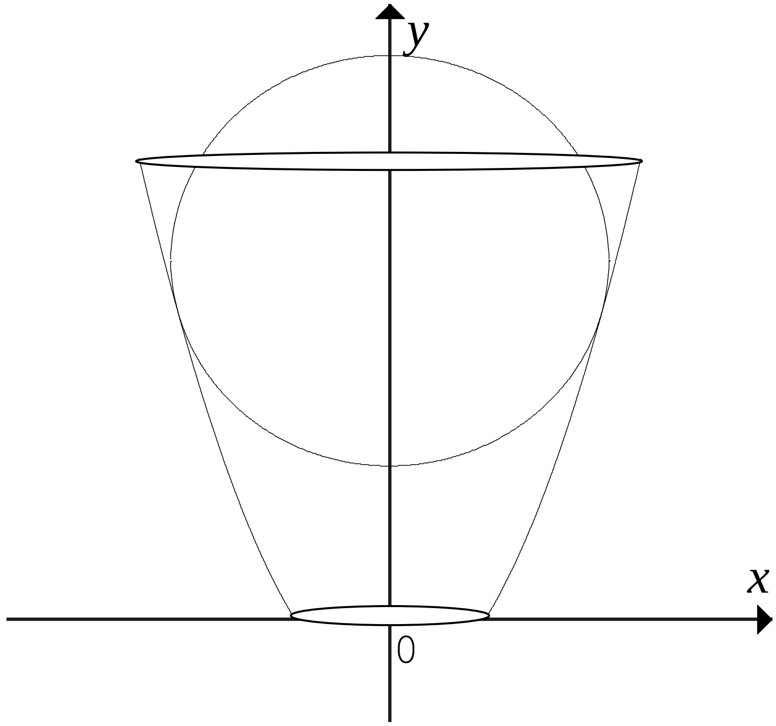
b. Find an expression for the volume in cubic centimetres when the cup is filled with water to a height of h cm, where $0 \le h \le 9$.

1 mark

c. When the cup is full, water leaks out at a rate of $3\sqrt{h}$ cm^3/sec, where *h* is the height of water remaining in the cup. How long in seconds would it take for the cup to empty?

3 marks

A large spherical ice block of radius 4 cm has been dropped into the cup.



d. Find the coordinates of the point of contact between the cup and the ice block.

3 marks

e. What is the volume of the cup (below the ice block) when the ice block is in the cup? Assume that the ice block has not melted.

2 marks

Question 5 (8 marks)

A boy throws a 4.5 kg bowling ball vertically upwards with an initial speed of 3.5 m/s from a point one metre above the ground. As the bowling ball rises it experiences a retardation force of $0.225v^2$ newtons, where v is its speed in m/s as it rises after a time t seconds.

a. Show that the equation of motion of the bowling ball is given by $\frac{dv}{dt} = \frac{-\left(196 + v^2\right)}{20}$

1 mark

b.i Using calculus show that $v = 14\tan\left(\tan^{-1}\left(\frac{1}{4}\right) - \frac{7t}{10}\right)$.

3 marks

ii. Hence find the time correct to three decimal places when the bowling ball reaches its maximum height. 1 mark

c.i Write down a definite integral which gives the distance D in metres that the bowling ball rises (above the point of release) .

2 marks

ii. Find this distance D, giving your answer correct to three decimal places.

1 mark

Question 6 (9 marks)

The time it takes Alan to walk to school is normally distributed with a mean of 30 minutes and a standard deviation of 5 minutes. When Alan walks to school on 30 days,

a.i. find the probability that his average time is less than 28 minutes. Give your answer correct to four decimal places.

1 mark

ii. Find a 95% confidence interval for the time it takes Alan to walk to school. Give your answers correct to two decimal places.

1 mark

The time it takes Belinda to walk to school is normally distributed with a mean of 25 minutes and a standard deviation of 4 minutes. Assume that the walking times of Alan and Belinda are independent.

b.i. If they leave at the same time, find the probability that Alan arrives at school before Belinda. Give your answer correct to four decimal places.

2 marks

ii. How much earlier should Alan leave before Belinda, so that he has a 90% chance of arriving before Belinda? Give your answer correct to two decimal places.

2 marks

Alan wants to get to school earlier, to talk to Belinda. He has discovered a new route, however he has to jump a fence and cross a busy road. Using the new route on 30 days, he finds his mean time to get to school is normally distributed with a mean of 28 minutes. Assume the standard deviation time of getting to school is still 5 minutes.

c.i. Write down suitable hypothesis H_0 and H_1 to test if his time in getting to school has decreased.

1 mark

ii. Find the *p* value for this test, correct to four decimal places.

1 mark

iii. State with reason, whether the sample times, support the idea of using the new route to get to school quicker. Test at the 5% level of significance.

1 mark

END OF EXAMINATION

EXTRA WORKING SPACE

END OF QUESTION AND ANSWER BOOKLET

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a pyramid	$\frac{1}{3}Ah$
area of triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\operatorname{cosec}^2(x)$
$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$	$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$	$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

Circular (trigonometric) functions - continued

Function	$\sin^{-1}$ (arcsin)	$\cos^{-1}$ (arcos)	$\tan^{-1}$ (arctan)
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$
$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$E(aX + b) = aE(X) + b$ $E(aX + bY) = aE(X) + bE(Y)$ $\text{Var}(aX + b) = a^2\,\text{Var}(X)$
for independent random variables X and Y	$\text{Var}(aX + bY) = a^2\,\text{Var}(X) + b^2\,\text{Var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $E(\bar{X}) = \mu$ variance $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r_1} \cdot \underset{\sim}{r_2} = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\lvert x\rvert+c$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$	$\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right)=\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right)=\frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right)=\frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$
	$\int (ax+b)^{-1}\,dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx}=f(x),\ x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf(x_n)$
acceleration	$a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2}\sqrt{1+\left(f'(x)\right)^2}\,dx$ or $\int_{t_1}^{t_2}\sqrt{\left(x'(t)\right)^2+\left(y'(t)\right)^2}\,dt$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

Figures										Letter
Words										

SIGNATURE ________________________________

SECTION A

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
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5	**A**	**B**	**C**	**D**	**E**
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7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
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11	**A**	**B**	**C**	**D**	**E**
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13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**