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2017 Specialist Mathematics Trial Exam 2 Solutions

SECTION A – Multiple-choice questions

1	2	3	4	5	6	7	8	9	10
D	E	A	C	D	A	E	D	D	A

11	12	13	14	15	16	17	18	19	20
E	E	B	E	A	B	C	D	A	D

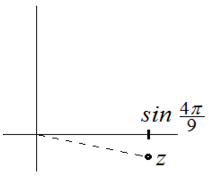
Q1 $y = x + \frac{10100}{(x-101)(x-100)}$ D

Q2 $\sin^{-1}(ax) + \frac{\pi}{2} = \frac{\pi}{2} - b$, $x = \frac{\sin(-b)}{a} = -\frac{\sin b}{a}$ E

Q3 $\cot u = x+1$, $\sec^2 u = y+2$, $1 + \tan^2 u = \sec^2 u$

.: $1 + \frac{1}{(x+1)^2} = y+2$, .: $(x+1)^2(y+1) = 1$ A

Q4 C

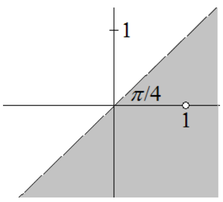


Q5 $z = (a+b)\left(\sqrt{2}cis\left(\frac{3\pi}{4}\right)+2\right) = (a+b)\sqrt{2}cis\left(\frac{\pi}{4}\right)$

$z^n = (a+b)^n\left(\sqrt{2}\right)^n cis\left(\frac{n\pi}{4}\right) \in R$ when n is a multiple of 4. D

Q6 Symmetry: $-\text{cosec}\left(\frac{x}{a}+\pi\right) = \text{cosec}\left(\frac{x}{a}\right)$ A

Q7 $\left|\frac{z-i}{z-1}\right| > 1$, $|z-i| > |z-1|$ for $z \neq 1$, .: the shaded region shown below. E



Q8 $x = \sin\left(\frac{1}{2}\right)$ or $x = \sqrt{3}$, but $\sin^{-1} x - \frac{1}{2}$ is undefined for $x = \sqrt{3}$ D

Q9 $x = \cos t - \sin t$ and $y = -\frac{1}{2}\cos 2t$

$\frac{dx}{dt} = -\sin t - \cos t$, $\frac{dy}{dt} = \sin 2t$

When $t = \frac{5\pi}{4}$, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sin 2t}{-\sin t - \cos t} = \frac{1}{\sqrt{2}}$ D

Q10 $x_0 = 1$, $y_0 = 0$, $f' = 0.5$

$x_1 = 1.1$, $y_1 \approx 0 + 0.1\times 0.5 \approx 0.05$, $f' \approx 0.4545$

$x_2 = 1.2$, $y_2 \approx 0.05 + 0.1\times 0.4545 \approx 0.095$ A

Q11 E

Q12 $(3\tilde{i} - \tilde{j}).(-\alpha\tilde{i} + 3\alpha\tilde{j} - \tilde{k}) = -6\alpha \neq 0$ when $\alpha \neq 0$ E

Q13 Vector resolute parallel to $\tilde{b}$ is:

$(3\tilde{i} - 2\tilde{j} + \tilde{k}) - (\tilde{i} - 2\tilde{j} + 3\tilde{k}) = 2\tilde{i} - 2\tilde{k}$ B

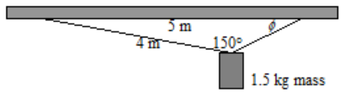
Q14 E

Q15 $f(|x|) = \begin{cases} f(-x) & \text{for } x < 0 \\ f(x) & \text{for } x \geq 0 \end{cases}$

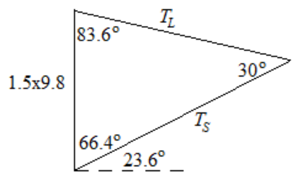
.: $\int_{-1}^{1} f(|x|)dx = 2 \times \int_{0}^{1} f(x)dx = -1$ A

Q16 $f(0) = \int_{\frac{\pi}{4}}^{0} \cot^2\left(\frac{\pi}{4}+x\right)dx + 2 \approx 1.8$ B

Q17 C



$\frac{\sin\phi}{4} = \frac{\sin 150°}{5}$, $\phi \approx 23.6°$



$\frac{\sin 66.4°}{T_L} = \frac{\sin 83.6°}{T_S}$, .: $\frac{T_S}{T_L} \approx 1.084$

Q18 $\text{Var}(X) = 0.050^2 = 0.0025$, $\text{Var}(Y) = 3^2 \times 0.0025 = 0.0225$

$\text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(Y_1) + \text{Var}(Y_2) = 2\times\text{Var}(X) + 2\times\text{Var}(Y)$

$= 2\times 0.0025 + 2\times 0.0225 = 0.05$

.: $\text{sd}(X_1 + X_2 + Y_1 + Y_2) = \sqrt{0.05} \approx 0.224$ D

Q19 $E(\bar{X}) \approx \mu = 168.0$, $\text{sd}(\bar{X}) = \frac{9}{\sqrt{9}} = 3$

$\Pr(\bar{X} = 175.0) = \Pr(174.5 < \bar{X} < 175.5) \approx 0.0089$ A

Q20 D

SECTION B

Q1a $\angle PTS = 180° - 60° = 120°$

Q1b $\angle POS = 2 \times 60° = 120°$

Q1c $\overrightarrow{PS} = \tilde{c} - \tilde{a}$

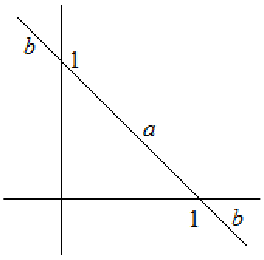
Q1d $|\overrightarrow{PS}|^2 = (\tilde{c} - \tilde{a}).(\tilde{c} - \tilde{a}) = \tilde{c}.\tilde{c} - 2\tilde{c}.\tilde{a} + \tilde{a}.\tilde{a}$
$= c^2 - 2|\tilde{c}|a\cos 120° + a^2 = b^2 + 2ab\cos 60° + a^2 = a^2 + ab + b^2$

Q1e Let $\overrightarrow{OP} = \tilde{p}$ and $\overrightarrow{OS} = \tilde{s}$, $|\overrightarrow{OP}| = p$ and $|\overrightarrow{OS}| = s$
$|\overrightarrow{PS}|^2 = |\tilde{s} - \tilde{p}|^2 = (\tilde{s} - \tilde{p}).(\tilde{s} - \tilde{p}) = \tilde{s}.\tilde{s} - 2\tilde{s}.\tilde{p} + \tilde{p}.\tilde{p}$
$= s^2 - 2sp\cos 120° + p^2 = r^2 + r^2 + r^2 = 3r^2$, .: $a^2 + ab + b^2 = 3r^2$

Q2a Midpoint between $z = 1$ and $z = i$, i.e. $z = \frac{1}{2}(1+i)$.

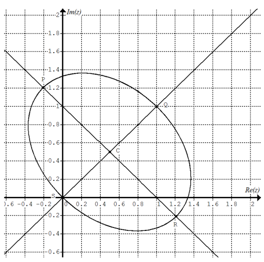
Q2b

Length of longest chord $= a + 2b = 2$



Q2c

$P: z = \left(\frac{1}{2} - \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right)i$
$Q: z = 1 + i$
$R: z = \left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2} - \frac{1}{\sqrt{2}}\right)i$
$S: z = 0$



Q2d $|(x-1) + yi| = 2 - |x + (y-1)i|$
Square both sides and simplify to $2|x + (y-1)i| = 2 + (x - y)$
Square both sides and simplify to $3(x+y)^2 = 4(x + xy + y)$

Q2e $\frac{d}{dx}3(x+y)^2 = \frac{d}{dx}4(x + xy + y)$
$6(x+y)\left(1 + \frac{dy}{dx}\right) = 4\left(1 + (x+1)\frac{dy}{dx} + y\right)$, $y = 2 - 3x$ when $\frac{dy}{dx} = 0$

Q2f $3(x+y)^2 = 4(x + xy + y)$
Let $y = 2 - 3x$, $3(x + 2 - 3x)^2 = 4(x + x(2-3x) + 2 - 3x)$
.: $6x^2 - 6x + 1 = 0$, $x = \frac{3 \pm \sqrt{3}}{6}$
$\text{Re}(z) = \frac{3+\sqrt{3}}{6}$ when $\text{Im}(z)$ is a minimum value.

Q3a $\tilde{v} = \int_0^t -9.8\tilde{j}\,dt + 20\cos\theta\,\tilde{i} + 20\sin\theta\,\tilde{j}$
.: $\tilde{v} = 20\cos\theta\,\tilde{i} + (20\sin\theta - 9.8t)\tilde{j}$
$\tilde{r} = (20\cos\theta)t\,\tilde{i} + ((20\sin\theta)t - 4.9t^2)\tilde{j}$

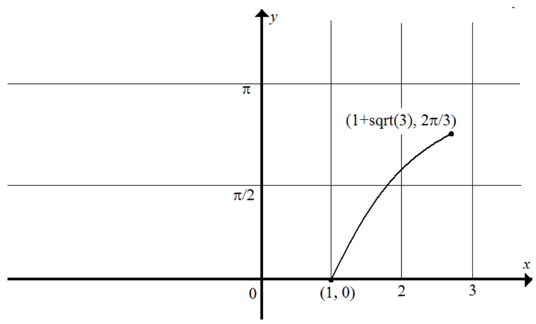
Q3b $x = (20\cos\theta)t$, $y = (20\sin\theta)t - 4.9t^2$
Eliminate t, $y = (\tan\theta)x - \frac{0.01225x^2}{\cos^2\theta}$

Q3c Let $x = 10$ and $y = 8$ ($y > 8$ to pass through the ring)
$\theta° = 46.7°$ or $81.9°$ (46.673 or 81.986)
Let $x = 10$ and $y = 10$ ($y < 10$ to pass through the ring)
$\theta° = 53.3°$ or $81.7°$ (53.342 or 81.658)
.: $\theta \in [46.7, 53.3]$ or $\theta \in [81.7, 81.9]$

Q3d The distance is longest when the projection angle is $81.7°$.
$y = (\tan 81.658°)x - \frac{0.01225x^2}{\cos^2 81.658°}$, .: $y = 6.8197x - 0.5820x^2$
$\frac{dy}{dx} = 6.8197 - 1.1640x$
Distance $= \int_0^{10}\sqrt{1 + (6.8197 - 1.1640x)^2}\,dx \approx 32.488 \approx 32.5$ m

Q3e Time taken: Consider the $\tilde{i}$ component,
$\Delta t = \frac{10}{20\cos 81.658°} \approx 3.446$ s, average speed $\approx \frac{32.488}{3.446} \approx 9.4$ m s^{-1}

Q4a



Q4b $y = 2\tan^{-1}(x-1)$, $x = 1 + \tan\frac{y}{2}$

$V = \int_0^{\frac{2\pi}{3}} \pi x^2\, dy = \int_0^{\frac{2\pi}{3}} \pi\left(1+\tan\frac{y}{2}\right)^2 dy$ m^3

Q4c $V = \int_0^h \pi\left(1+\tan\frac{y}{2}\right)^2 dy = 2\pi\left(\tan\frac{h}{2} - 2\log_e\left(\cos\frac{h}{2}\right)\right)$ m^3

Q4d

$V_{\max} = \int_0^{\frac{2\pi}{3}} \pi\left(1+\tan\frac{y}{2}\right)^2 dy \approx 19.593$ m^3

and given $\frac{dh}{dt} = 0.05$ m min^{-1}

$V = 2\pi\left(\tan\frac{h}{2} - 2\log_e\left(\cos\frac{h}{2}\right)\right)$, .: $\frac{dV}{dh} = 2\pi\left(\frac{1}{2}\sec^2\frac{h}{2} - \tan\frac{h}{2}\right)$

Let $2\pi\left(\tan\frac{h}{2} - 2\log_e\left(\cos\frac{h}{2}\right)\right) = \frac{1}{2}\times 19.593$, .: $h \approx 1.5015$ m

When $h \approx 1.5015$, $\frac{dV}{dt} = \frac{dV}{dh}\times\frac{dh}{dt} \approx 0.59$ m^3 per minute

Q4e Time taken $= \frac{\frac{2\pi}{3}}{0.05} = \frac{40\pi}{3}$ minutes

Q5a $a = \frac{F_{net}}{m} = \frac{12000 - 240\times 10 - 160\times 10}{2000+2000} = 2$ m s^{-2} forward

Q5b $T - 160\times 10 = 2000\times 2$, $T = 5600$ N

Q5c $400v = 12000$, $v = 30$ m s^{-1}

Q5d $4000\times 30 = 120000$ kg m s^{-1}

Q5ei $a = \frac{12000 - 240v - 160v}{4000}$, .: $a = 3 - \frac{v}{10}$, .: $v\frac{dv}{dx} = 3 - \frac{v}{10}$

.: $\frac{dv}{dx} = \frac{3}{v} - \frac{1}{10}$

Q5eii $\frac{dv}{dx} = \frac{30-v}{10v}$, $\frac{dx}{dv} = \frac{10v}{30-v}$

Distance $= \int_0^{10} \frac{10v}{30-v}\, dv = \int_0^{10}\left(\frac{300}{30-v} - 10\right) dv = 300\log_e\left(\frac{3}{2}\right) - 100$ m

Q6a Mean of $\bar{X} = E(\bar{X}) \approx \mu = 25.0$ mm

Q6b $sd(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{0.2}{\sqrt{5}} = 0.089$ mm

Q6c $\Pr(24.8 < \bar{X} < 25.2) \approx 0.975$

Q6d $\mu = 25.1$ mm

Q6e $\left(25.1 - 1.96\times\frac{0.2}{\sqrt{5}},\ 25.1 + 1.96\times\frac{0.2}{\sqrt{5}}\right)$,

i.e. $(24.925, 25.275)$ in mm

Q6eii $95\% \times 20 = 19$

Please inform mathline@itute.com re conceptual and/or mathematical errors