Year 12 *Trial Exam Paper*

2017

SPECIALIST MATHEMATICS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.
- Calculators are not permitted in this examination.

Materials provided

- The question and answer book of 15 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided above.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination.

Question 1 (3 marks)

Solve $z^4 - (1+i)z^3 = z^2 - z - iz$, where $z \in C$.

Question 2 (4 marks)

The velocity v ms^{-1} of an object moving in a straight line in terms of its position x metres is given by the relation $(x+1)e^{v} - x^{2} - 2x - 3 = 0$.

Find the acceleration, in ms^{-2}, when $x = 1$.

Question 3 (5 marks)

The mass of the oranges grown in an orchard is normally distributed with a mean of μ g and a standard deviation of 10 g.

A sample of 100 oranges has a mean mass of 160 g and the standard deviation remains at 10 g.

a. Find an approximate 90% confidence interval for the population mean, given that $\Pr(z > 1.645) \approx 0.05$ where z has the standard normal distribution.

Give your answer correct to one decimal place.

2 marks

Another orchard produces oranges with a mean mass of 157 g and standard deviation of 6 g.

The owner of the orchard introduces a new fertiliser, which is intended to increase the mass of the oranges.

After using the new fertiliser, a sample of 36 oranges has a mean mass of 159 g and the standard deviation remains at 6 g.

b. **i.** Write down the appropriate null and alternate hypotheses to test whether the mean mass of the oranges has increased as a result of the new fertiliser.

1 mark

ii. Find an approximate p value for this test, correct to three decimal places.

1 mark

iii. Explain why the null hypothesis should be rejected at the 5% level of significance.

1 mark

Question 4 (3 marks)

Evaluate $\int_{0}^{\frac{\pi}{3}} \tan^2(x) + \tan^4(x)dx$.

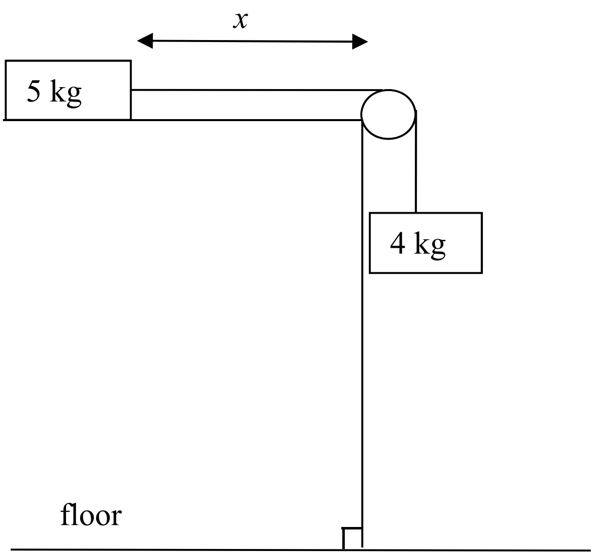
Question 5 (3 marks)

A block of mass 5 kg rests on a horizontal table. The block is connected to another block of mass 4 kg by a light inextensible string over a smooth pulley of neglible mass at the edge of the table.

The 4 kg mass is hanging vertically.

The system is released from rest when the 5 kg mass is x metres from the edge of the table.

The maximum friction force between the block and the table surface is $2g$.



If it takes one second for the 5 kg mass to reach the edge, calculate the value of x.

Question 6 (3 marks)

Solve the differential equation $\frac{dy}{dx} = 3y\sqrt{x}$, given that $y = e$ when $x = 1$. Express y as a function of x.

Question 7 (4 marks)

A curve is defined by the parametric equations:

$x(t) = 2 + \sin(2t)$

$y(t) = 2\sin^2(t)$ where $t \geq 0$.

a. Determine the cartesian equation of the curve.

2 marks

b. Find the length of the curve from $t = 0$ to $t = \frac{\pi}{3}$.

2 marks

Question 8 (5 marks)

Consider the three vectors:

$\underset{\sim}{a} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$, $\underset{\sim}{b} = 2\underset{\sim}{i} + \underset{\sim}{j} + n\underset{\sim}{k}$, where $n \in R$, and $\underset{\sim}{c} = 4\underset{\sim}{i} + 3\underset{\sim}{j} + 6\underset{\sim}{k}$.

$\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$ are unit vectors in the positive directions of the *x*, *y* and *z* axes respectively.

a. Find the values of *n* if vectors $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are perpendicular.

1 mark

b. Find the values of *n* if vector $\underset{\sim}{b}$ makes an angle of θ° with the *x*–*z* plane, where

$\cos(\theta) = \dfrac{3}{\sqrt{10}}$.

2 marks

Question 8 – continued

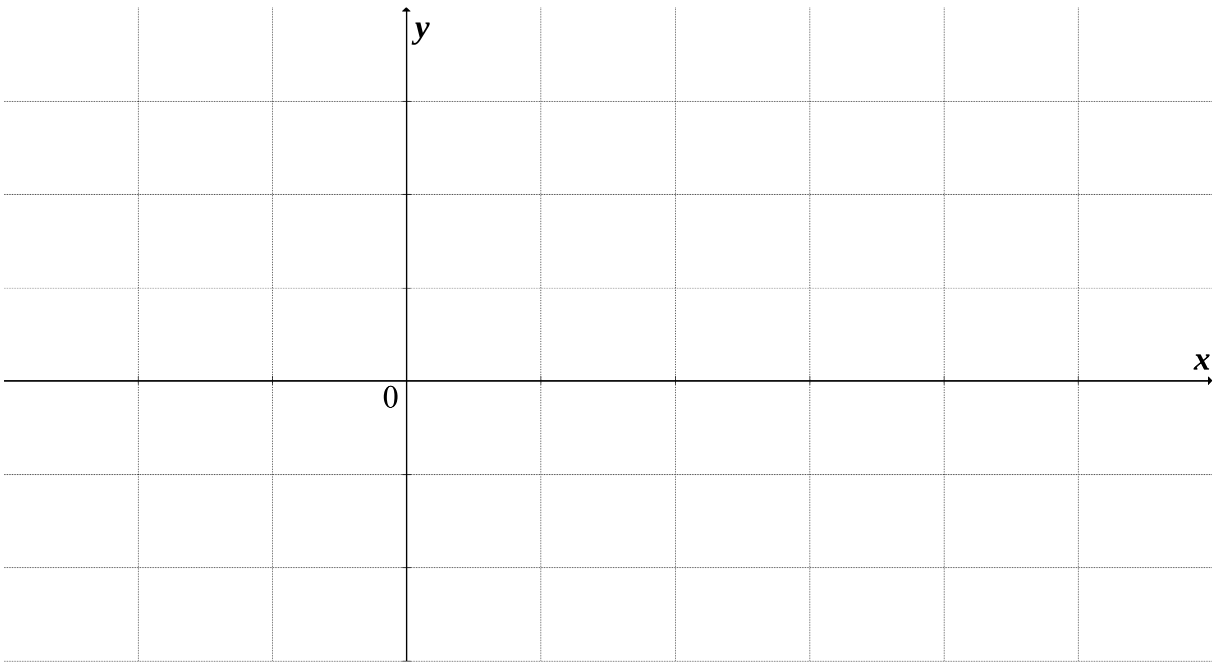
c. Find the values of n if vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly independent.

2 marks

Question 9 (4 marks)

a. Sketch the graph of $y=\left|\log_e(x)\right|$ on the axes below. Clearly label any axis intercepts with their coordinates and any asymptotes with their equations.

1 mark



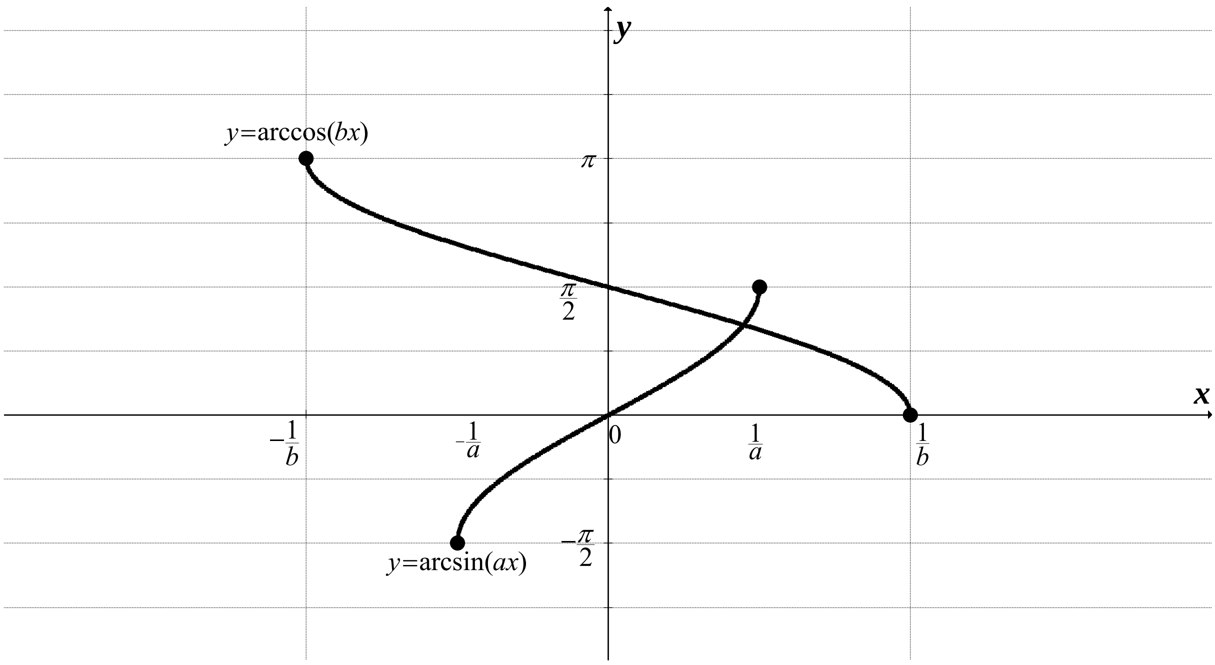
b. The area bounded by $y = \left|\log_e(x)\right|$ and the line $y = a$, where $a > 0$, is rotated about the y-axis to form a solid of revolution of volume 2π.

Show that $e^{2a} + e^{-2a} = 6$.

3 marks

Question 10 (6 marks)

The graphs of $y = \arcsin(ax)$ and $y = \arccos(bx)$, where $a, b \in R$ and $a > b > 0$, are shown below.



a. Find, in terms of a and b, the solution of the equation $\arcsin(ax) = \arccos(bx)$.

2 marks

The derivative of $x\arcsin(ax)+\dfrac{\sqrt{1-a^2x^2}}{a}=\arcsin(ax)$.

b. Show that the derivative of $x\arccos(bx)-\dfrac{\sqrt{1-b^2x^2}}{b}=\arccos(bx)$.

1 mark

c. Hence find the area bounded by $y=\arcsin(2x)$, $y=\arccos(x)$ and the y-axis.

3 marks

END OF QUESTION AND ANSWER BOOK