2016 VCE Specialist Mathematics 1 examination report

General comments

In Specialist Mathematics examination 1 students were required to answer 10 short-answer questions worth a total of 40 marks. Students were not permitted to bring technology or notes into the examination.

One area requiring improvement was the clarity of students' responses and the manner in which they set out their mathematics. Students should be reminded that if an assessor is not certain as to what an answer is conveying, that assessor cannot award marks. Students are expected to set out their work properly. If an assessor is unable to follow a student's working (or reasoning), full marks will not be awarded. Working should not appear to be a number of disjointed statements. If there are inconsistencies in the student's working, full marks cannot be awarded. For example, if an equals sign is placed between quantities that are not equal, full marks cannot be awarded.

It should also be emphasised that students should always consider the reasonableness of their answers. Several of the answers given to Questions 2 and 9 were not feasible.

Areas of strength included:

- recognising the need to use the chain rule when differentiating implicitly (Question 3)
- recognising the need to use the product rule when differentiating implicitly (Question 3).

Areas of weakness included:

- not reading the question carefully enough – this included not answering the question, proceeding further than required or not giving the answer in the specified form. These were common and particularly evident in Questions 1a., 2, 3, 5a., 8a. and 10. Many students would benefit from highlighting key words in the question. Students should be reminded that good examination technique includes re-reading the question after it has been answered to ensure that they have answered what was required and that they have given their answer in the correct form
- algebraic skills. Difficulty with algebra was evident in several questions. The inability to simplify expressions often prevented students from completing the question. Incorrect attempts to factorise, expand and simplify were common. Poor use of brackets was also common
- arithmetic skills. Difficulty with arithmetic was evident in several questions. The inability to evaluate expressions, especially those involving fractions or surds, was common
- notation, especially the omission of the *dx* or equivalent in integration, and showing the dot in the dot product (Question 5b.)
- showing a given result. This was required in Question 1b. In such questions, the onus is on students to include sufficient relevant working to demonstrate that they know how to derive the result. Students should be reminded that they can use a given value in the remaining part(s) of the question whether they were able to derive it or not
- recognising the method of integration required (Questions 7 and 10)
- knowing the exact values for circular functions (Questions 3, 4, 6 and 8b.)

- consideration of quadrants when dealing with circular functions (Question 6).

In this examination, students are expected to be able to apply techniques, routines and processes, involving rational, real and complex arithmetic, without the use of technology. Students are expected to be able to simplify simple arithmetic expressions. Many students found this difficult and missed out on marks as a consequence.

Many students made algebraic or numerical slips at the end of an answer, which meant that the final mark could not be awarded. This often occurred when students had a correct answer and there was no need for further simplification.

In the comments on specific questions in the next section, many common errors are highlighted. These should be brought to the attention of students so that they can develop strategies to avoid them. A particular concern is the need for students to read the questions carefully as responses to several questions indicated that students had not done so.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Question 1a.

Marks	0	1	Average
%	40	60	0.6

Students were expected to label a tension force acting up the rope and a weight force (20*g*) acting vertically downwards.

This question was answered reasonably well. The most common errors involved including extra forces (often a normal reaction force and sometimes a friction force) or incomplete labelling. Some students did not attempt this question. Tension was sometimes shown in the incorrect direction or in both directions. Some students labelled the resolved forces as if they were forces and not just the resolutions of the forces.

Question 1b.

Marks	0	1	Average
%	7	93	1

$\sin(\theta) = \frac{3}{5}$

This question was well answered by most students. A few wrote $\sin(\theta) = \frac{1}{5/3} = \frac{5}{3}$, despite the correct answer being given.

Question 1c.

Marks	0	1	2	Average
%	39	13	48	1.1

245

Most students handled this question reasonably well. Typical errors included $T = 20g\cos(\theta)$, $T\sin(\theta) = 20g$ and sometimes $T = 20g$, while some students substituted $\cos(\theta) = \frac{4}{3}$ or $\frac{3}{4}$. A few used $g = 10$. Some students used Lami's theorem but most were unsuccessful.

Question 2

Marks	0	1	2	3	Average
%	21	42	20	17	1.4

(103.4,106.6) or (517/5, 533/5)

This question was not answered well. A large proportion of students were unable to find the mean mass, dividing 2625 by 25 and giving 15 instead of 105. An estimation of the mean would have been helpful with this. A smaller number of students gave 150, while some used the total for the bag, 2625. Many students seemed to misunderstand the question wording, 'integer multiple of the standard deviation …', using ± 1.96 instead of ± 2. Some took the standard deviation to be four rather than $\frac{4}{5}$. Occasionally students used $\frac{16}{5}$, while a small number rounded $\frac{4}{5}$ to 1 (misinterpreting the 'integer multiple' phrase).

Question 3

Marks	0	1	2	3	4	Average
%	15	8	19	5	53	2.8

$y = -\frac{2}{\pi}x - \frac{\pi}{2}$

Students generally dealt well with the implicit differentiation, with most realising the need for the chain rule and the product rule. Several students made a sign error when substituting the given point; others found the gradient of the perpendicular line and did not continue. Others found the gradient and/or equation of the tangent, while some thought that the gradient of the normal was equal to the reciprocal rather than the negative reciprocal of the gradient of the tangent. Others used the negative of the gradient of the tangent.

Question 4

Marks	0	1	2	3	4	Average
%	23	13	20	4	40	2.3

$\frac{dA}{dt} = \frac{3\pi}{2}$

Students had mixed success with this question. Quite a few students made errors with the formula for the surface area of a cube, including $A = x^2$, $2x^2$, $4x^2$ or more commonly x^3. There was some confusion with t and x, which resulted in a denominator $(1 + x^2)$ rather than $(1 + t^2)$. Some found the correct derivative using the chain rule but then did not continue by substituting for t and/or x. Some students did not use the chain rule. A few wrote $t = \tan(x)$ and some progressed successfully from there. Other methods were seen, including some students converting completely to the variable x. Some students wrote $\arctan(t)$ as $\tan^{-1}(t)$ and then converted this to $\frac{1}{\tan(t)}$.

Question 5a.

Marks	0	1	2	Average
%	32	14	54	1.2

$-\frac{13}{14}\left(\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}\right)$

This question was well answered by students who knew what a vector resolute was and used the correct formula. Some found the scalar resolute and several had an incorrect formula for the vector resolute (sometimes not using the unit vector, occasionally finding the vector resolute in the direction of $\underset{\sim}{i}$). A number of students did not show the dot in the dot product.

Question 5b.

Marks	0	1	2	Average
%	24	18	59	1.4

$d = 1$

Most students performed well on this question. The main issues were due to algebraic errors in the solution of the simultaneous equations. Some insightful solutions were seen using the fact that $2\underset{\sim}{a} + \underset{\sim}{b}$ eliminated $\underset{\sim}{j}$. Others used the discriminant of a 3×3 matrix, with varied success.

Question 6

Marks	0	1	2	3	Average
%	8	14	26	52	2.2

$4 + 4\sqrt{3}i$

This question was well answered overall. Those students who used polar form tended to have greater success than those who tried to solve the equation in Cartesian form and often made algebraic or arithmetical errors. It was common for the incorrect argument to be used, usually due to the incorrect quadrant but sometimes due to not knowing exact values. A sketch may have been helpful. Many sign errors were seen. An elegant solution used the fact that the numerator turns out to be four times the square of the denominator.

Question 7

Marks	0	1	2	3	4	Average
%	9	15	30	2	43	2.6

$\frac{14}{3}$

Some students responded very well to this question but others had some difficulty. A number made an error in the formula, despite it being on the formula sheet. Most students found the derivative correctly, but some made errors leading to an impossible integral. A common error was to give the derivative as $\frac{x}{\sqrt{x^2+2}}$. A large proportion of those who found the correct derivative and substituted correctly into the formula were then unable to recognise the perfect square inside the square root. Some of the functions that were used in attempts to substitute could not lead to a correct answer. A small number of students took the square root of individual terms. Some students did not use *dx*.

Question 8a.

Marks	0	1	2	Average
%	9	25	66	1.6

$|\underset{\sim}{\mathrm{v}}| = \sqrt{36\cos^2(2t)+16\sin^2(2t)}$

Most students answered this question very well. There were errors seen in the derivative, usually involving sign but sometimes mixing up sin and cos. Occasionally the $\underset{\sim}{\mathrm{i}}$ and $\underset{\sim}{\mathrm{j}}$ were left out at this stage. The most frequent error was not attempting to find the modulus. A small number of students removed the $\underset{\sim}{\mathrm{i}}$ and $\underset{\sim}{\mathrm{j}}$ in an attempt to convert from velocity and speed. A few correctly found an expression for the speed but then made errors in trying to simplify it.

Question 8b.

Marks	0	1	Average
%	34	66	0.7

$\sqrt{31}$

This question was answered well, with most students who had made a reasonable attempt at part a. answering correctly. Typical errors included leaving the answer as a vector or simplifying incorrectly to obtain $3\sqrt{3}+2$ by taking the square root of individual terms.

Question 8c.

Marks	0	1	2	3	Average
%	22	31	21	26	1.5

36

A broad spread of levels of achievement was seen for this question. Most students were able to make an attempt but the majority encountered some difficulty. The most common errors involved sign errors in the derivative, using velocity rather than acceleration or making errors in differentiation when attempting to find a maximum. Several students found the correct expression for either acceleration or force but were then unable to progress. Some chose a value for *t*, while others used $\sin(2t)=1$ and $\cos(2t)=1$ simultaneously to find the maximum force. A small number of students attempted to use an ellipse to solve this question but most were unsuccessful. Some substituted $t=\frac{\pi}{12}$. A few made the incorrect assumption that to find the maximum value for acceleration, one or both of the components of acceleration must equate to zero.

Question 9

Marks	0	1	2	3	Average
%	15	28	13	44	1.9

$-\frac{1}{5}$

This question was reasonably well answered, with most students using one or more trigonometric identities correctly. A large number of students had difficulty with the algebra but many persisted and answered correctly. Some used $\tan(x - y)$, which was not always successful. Of great concern was the number of students who gave answers for sine or cosine that were either less than -1 or greater than 1.

Question 10

Marks	0	1	2	3	4	5	Average
%	20	15	12	31	7	14	2.4

$$y = 2 - \sqrt{4 + \frac{\pi}{2} - 2\arcsin\left(\frac{x}{\sqrt{2}}\right)}$$

Students found this question challenging. Most students were able to separate the variables correctly, but there were then many errors in the subsequent integration. Errors were seen in the integration of the polynomial part but far more in the integration of the term involving the reciprocal of the square root, despite the formula being on the formula sheet. It was common for logarithms to be seen. Some students were unable to proceed by completing the square or otherwise having integrated. A large number of students, when confronted with a square equals a constant, gave only the positive root. Many gave both roots but did not realise that only the negative root satisfied the initial conditions. Several students omitted the constant of integration; others made mistakes when attempting to evaluate the constant. A number of students interpreted $y(1) = 0$ as $x = 0$ when $y = 1$, while some attempted to solve for x rather than y.