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Trial Examination 2016

# **VCE Specialist Mathematics Units 3&4**

Written Examination 1

**Suggested Solutions**

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**Question 1** (2 marks)

$$\int_0^{\frac{\pi}{4}} \frac{\sec^2(x)}{1 + \tan(x)} dx = [\log_e(1 + \tan(x))]_0^{\frac{\pi}{4}} \quad \text{M1}$$

$$= \log_e(2) \quad \text{A1}$$

**Question 2** (3 marks)

a. Using  $\underline{p} = m\underline{v}$  with  $m = 0.25$  and  $\underline{v} = (4t^3 - 4t)\underline{i} + (12t^2 - 4t^3)\underline{j}$ .

$$\text{So } \underline{p} = (t^3 - t)\underline{i} + (3t^2 - t^3)\underline{j}. \quad \text{A1}$$

b. Using  $\underline{F} = m\underline{a}$  with  $m = 0.25$  and  $\underline{a} = (12t^2 - 4)\underline{i} + (24t - 12t^2)\underline{j}$ . M1

$$\underline{F} = (3t^2 - 1)\underline{i} + (6t - 3t^2)\underline{j}$$

$$3t^2 - 1 = 0 \Rightarrow t = \frac{1}{\sqrt{3}} \text{ (seconds) (since } t \geq 0) \quad \text{A1}$$

**Question 3** (3 marks)

$$\cos\left(\frac{x}{2}\right) = \sin\left(\frac{x}{4}\right)$$

$$1 - 2\sin^2\left(\frac{x}{4}\right) = \sin\left(\frac{x}{4}\right)$$

$$2\sin^2\left(\frac{x}{4}\right) + \sin\left(\frac{x}{4}\right) - 1 = 0, 0 \leq \frac{x}{4} \leq \pi \quad \text{M1}$$

$$\left(2\sin\left(\frac{x}{4}\right) - 1\right)\left(\sin\left(\frac{x}{4}\right) + 1\right) = 0$$

$$\sin\left(\frac{x}{4}\right) = -1, \frac{1}{2}$$

$$\text{As } 0 \leq \sin\left(\frac{x}{4}\right) \leq 1, \sin\left(\frac{x}{4}\right) = \frac{1}{2}. \quad \text{A1}$$

$$\frac{x}{4} = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\therefore x = \frac{2\pi}{3}, \frac{10\pi}{3} \quad \text{A1}$$

**Question 4** (6 marks)

a. By the conjugate root theorem,  $z = 1 + \sqrt{3}i$  is a root. A1

$$\text{So } (z - (1 + \sqrt{3}i))(z - (1 - \sqrt{3}i)) = z^2 - 2z + 4.$$

$$\therefore z^3 + z^2 + bz + 12 = (z^2 - 2z + 4)(z - m) \quad \text{M1}$$

By equating coefficients,  $-4m = 12 \Rightarrow m = -3$ .

So the roots are  $-3, 1 \pm \sqrt{3}i$ . A1

b.  $1 - \sqrt{3}i = 2 \operatorname{cis}\left(-\frac{\pi}{3}\right)$  M1

$$(1 - \sqrt{3}i)^k \in R \text{ for } k\left(-\frac{\pi}{3}\right) = n\pi, n \in Z$$

So  $k = -3n \Rightarrow k_{\min} = 3$ . A1

$$\begin{aligned} (1 - \sqrt{3}i)^3 &= 2^3 \operatorname{cis}(-\pi) \\ &= -8 \end{aligned} \quad \text{A1}$$

**Question 5** (4 marks)

a.  $E(\bar{X}) = E(X)$   
 $= 30$  A1

b.  $\operatorname{sd}(\bar{X}) = \frac{\sigma}{\sqrt{n}}$   
 $= \frac{7}{\sqrt{100}}$   
 $= \frac{7}{10}$  A1

c.  $E(\bar{X}) = 30$  A1

*Note: This is unchanged as it is independent of  $n$ .*

$$\begin{aligned} \operatorname{sd}(\bar{X}) &= \frac{\sigma}{\sqrt{n}} \\ &= \frac{7}{\sqrt{400}} \\ &= \frac{7}{20} \end{aligned}$$

$\operatorname{sd}(\bar{X})$  is reduced by a factor of 2. A1

**Question 6** (3 marks)

Let  $y = \cos^{-1}(u)$  and so  $u = 2x^{-1}$ .

$$\frac{dy}{du} = \frac{-1}{\sqrt{1-u^2}} \text{ and } \frac{du}{dx} = -2x^{-2} = \frac{-2}{x^2} \quad \text{M1}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= \frac{-1}{\sqrt{1-\left(\frac{2}{x}\right)^2}} \times \frac{-2}{x^2} \quad \text{A1} \end{aligned}$$

$$= \frac{2}{x^2 \sqrt{\frac{x^2-4}{x^2}}}$$

$$\frac{dy}{dx} = \frac{2}{|x|\sqrt{x^2-4}} \text{ but } \sqrt{x^2} = |x|, \text{ and since } x > 2, \sqrt{x^2} = x. \text{ So } \frac{dy}{dx} = \frac{2}{x\sqrt{x^2-4}}. \quad \text{A1}$$

*Note: The final A1 should only be awarded if the second-last line (or equivalent) is shown.*

**Question 7** (9 marks)

$$\begin{aligned} \text{a. } \underline{\underline{r}}'(t) &= \int -g\underline{\underline{j}} dt \\ &= -gt\underline{\underline{j}} + \underline{\underline{c}} \quad \text{M1} \end{aligned}$$

When  $t = 0$ ,  $\underline{\underline{r}}'(0) = V\cos(\theta)\underline{\underline{i}} + V\sin(\theta)\underline{\underline{j}}$  and so  $\underline{\underline{c}} = V\cos(\theta)\underline{\underline{i}} + V\sin(\theta)\underline{\underline{j}}$ .

$$\text{So } \underline{\underline{r}}'(t) = V\cos(\theta)\underline{\underline{i}} + (V\sin(\theta) - gt)\underline{\underline{j}}. \quad \text{A1}$$

$$\begin{aligned} \underline{\underline{r}}(t) &= \int V\cos(\theta)\underline{\underline{i}} + (V\sin(\theta) - gt)\underline{\underline{j}} dt \\ &= V\cos(\theta)t\underline{\underline{i}} + \left(V\sin(\theta)t - \frac{1}{2}gt^2\right)\underline{\underline{j}} + \underline{\underline{d}} \quad \text{M1} \end{aligned}$$

When  $t = 0$ ,  $\underline{\underline{r}}(0) = h\underline{\underline{j}}$  and so  $\underline{\underline{d}} = h\underline{\underline{j}}$ .

$$\text{So } \underline{\underline{r}}(t) = V\cos(\theta)t\underline{\underline{i}} + \left(V\sin(\theta)t - \frac{1}{2}gt^2 + h\right)\underline{\underline{j}}. \quad \text{A1}$$

*Note: The final A1 should only be awarded if the second last line is shown.*

- b. Parametric equations are  $x = V\cos(\theta)t$  and  $y = V\sin(\theta)t - \frac{1}{2}gt^2 + h$ .

Substitute  $t = \frac{x}{V\cos(\theta)}$  into  $y = V\sin(\theta)t - \frac{1}{2}gt^2 + h$ . M1

$$y = \tan(\theta)x - \frac{gx^2}{2V^2\cos^2(\theta)} + h$$

$$= h + \tan(\theta)x - \frac{g\sec^2(\theta)}{2V^2}x^2$$
A1

*Note: The final A1 should only be awarded if the second-last line is shown.*

- c. Consider  $y = h + \tan(\theta)x - \frac{gx^2}{2v^2}\sec^2(\theta)$ .

Projectile fired horizontally:

$$\theta = 0, V = U, y = 0$$

$$0 = h - \frac{gx^2}{2U^2} \Rightarrow h = \frac{gx^2}{2U^2} \quad \dots (1)$$
A1

Projectile fired at an angle of elevation of  $\tan^{-1}(3)$ :

$$0 = h + 3x - \frac{10gx^2}{2U^2} \quad \dots (2)$$

Substituting  $h = \frac{gx^2}{2U^2}$  into (2) gives  $3x - \frac{9gx^2}{2U^2} = 0$ . M1

Solving gives  $x = \frac{2U^2}{3g}$  (metres). A1

### Question 8 (3 marks)

$\underline{c} = m\underline{a} + n\underline{b}$  where  $m, n \in R \setminus \{0\}$  for linear dependence.

$$\underline{i} + 6\underline{j} + 10\underline{k} = m(\underline{i} + 2\underline{j}) + n(-\underline{i} + 5\underline{k})$$
A1

Equating  $\underline{i}$  components:  $m - n = 1$

Equating  $\underline{j}$  components:  $2m = 6 \Rightarrow m = 3$

Equating  $\underline{k}$  components:  $5n = 10 \Rightarrow n = 2$  M1

*Note: Award M1 for attempting to form three equations.*

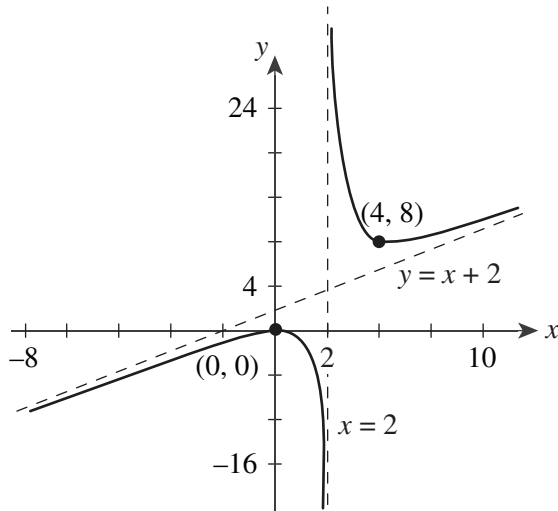
$m = 3$  and  $n = 2$  satisfy  $m - n = 1$  and so  $\underline{a}$ ,  $\underline{b}$  and  $\underline{c}$  are linearly dependent. A1

**Question 9** (7 marks)

a. Re-expressing  $y = \frac{x^2}{x-2}$  to give  $y = x + 2 + \frac{4}{x-2}$ . M1

Asymptotes are  $y = x + 2$  and  $x = 2$ . A1

Stationary points are  $(4, 8)$  and  $(0, 0)$ . A1



*correct shape* A1

b.  $x^2 = p(x^2 - 4) \Rightarrow \frac{x^2}{x-2} = p(x+2)$  M1

Due to the asymptote  $y = x + 2$ , the maximum value of  $p$  is 1,  
and we also require  $p > 0$  (gradient of the line steeper than the horizontal line joining  $(-2, 0)$  and  $(0, 0)$ ). A1

So the required values are  $0 < p \leq 1$ . A1