The Mathematical Association of Victoria

Trial Exam 2016

SPECIALIST MATHEMATICS

Written Examination 1

STUDENT NAME _____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 15 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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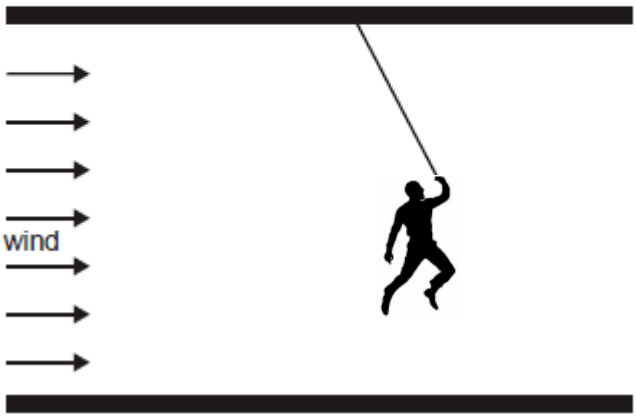
Instructions

Answer **all** questions in the space provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams are **not** drawn to scale.
Take the acceleration due to gravity to have magnitude g ms^{-2}, where g =9.8

Question 1 (3 marks)

An emergency rescue worker with a mass of 70 kg is training in a wind tunnel. He is hanging from the roof of the wind tunnel by a rope and a light wind exerts a force of magnitude $\frac{g}{3}$ newtons on him.



Calculate the magnitude in newtons of the tension in the rope. Give your answer in the form $\frac{g\sqrt{a}}{b}$ where a and b are positive integers.

Question 2 (5 marks)

The vector $\underset{\sim}{a} = -\underset{\sim}{i} - 2\sqrt{3}\,\underset{\sim}{j} - \sqrt{2}\,\underset{\sim}{k}$, where $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$ are unit vectors in the positive directions of the x, y and z axes respectively.

a. Find the vector of length $\sqrt{45}$ in the direction of $\underset{\sim}{a}$. 1 mark

b. The vector $\underset{\sim}{b} = 2\underset{\sim}{i} - m\,\underset{\sim}{j} - 3\sqrt{2}\,\underset{\sim}{k}$ where $m \in R$. Find all possible values of m if:

i. $\left|\underset{\sim}{b}\right| = 5$. 2 marks

ii. $\underset{\sim}{b}$ is perpendicular to $\underset{\sim}{a}$. 2 marks

Question 3 (3 marks)

Solve $z^3 - (2-i)z^2 = z - 2 + i$ where $z \in C$.

Question 4 (3 marks)

Solve $\frac{dy}{dx} = \frac{y-1}{x+1}$ for y, given that $y = \frac{1}{2}$ when $x = -2$.

TURN OVER

Question 5 (4 marks)

The velocity $v\,\text{ms}^{-1}$ of a body moving in a straight line in terms of its position x m is given by the relation $2v^2 - 3vx - 2x^2 = 1$ for $x \geq 0$.

Find the acceleration of the body when $x = \frac{1}{5}$ and $v > 0$.

Question 6 (5 marks)

The position of a particle at time t seconds is given by $\underset{\sim}{r} = t^2 \underset{\sim}{i} + \frac{4t^2}{5}\sqrt{t}\,\underset{\sim}{j}$, $t \geq 0$, where components are measured in metres.

a. Find the velocity of the particle when it has an acceleration of $2\underset{\sim}{i} + 2\underset{\sim}{j}$ ms^{-2}. 2 marks

b. Find the distance travelled by the particle in the first 3 seconds of its motion. 3 marks

Question 7 (5 marks)

Galaxian Highscore and Max Hardcase regularly run a distance of 5 km as part of a fitness program. The time it takes Galaxian to run 5 km is normally distributed with a mean of 31 minutes and a standard deviation of 3 minutes. The time it takes Max to run 5 km is normally distributed with a mean of 33 minutes and a standard deviation of 4 minutes. The times run by Galaxian and Max are independent.

Let the random variable X represent the time it takes Galaxian to run 5 km and let the random variable Y represent the time it takes Max to run 5 km.

a. Find the standard deviation of the difference between the times it takes Max and Galaxian to run 5 km. 2 marks

Max Hardcase decides to try and decrease his running times by also doing some high intensity interval training. Max finds that for his next 25 runs over 5 km, the mean time is 31 minutes. Assume that the standard deviation of 4 minutes is unchanged.

b. **i.** State appropriate null and alternative hypotheses for the running time T of Max in this situation. 1 mark

ii. The p-value for this test is given by the expression $\Pr(Z \leq a \mid H_0)$, where Z has the standard normal distribution.

Find the value of a and **hence** determine whether or not the null hypothesis should be rejected at the 0.05 level of significance. 2 marks

TURN OVER

Question 8 (3 marks)

The region enclosed by the graph with equation $y = \frac{\sqrt{2x-1}}{x-1}$ and the lines $y = 0$ and $x = \frac{3}{4}$ is rotated about the x-axis to form a solid of revolution.
Calculate the volume of the solid of revolution.

Question 9 (9 marks)

Consider the functions with rules $f(x) = \dfrac{x^2 + 1}{2x^2 - 1}$ and $g(x) = \pi - \cos^{-1}(x)$ where each function is defined over its maximal domain.

a. State the equations of all asymptotes of $f(x)$. 1 mark

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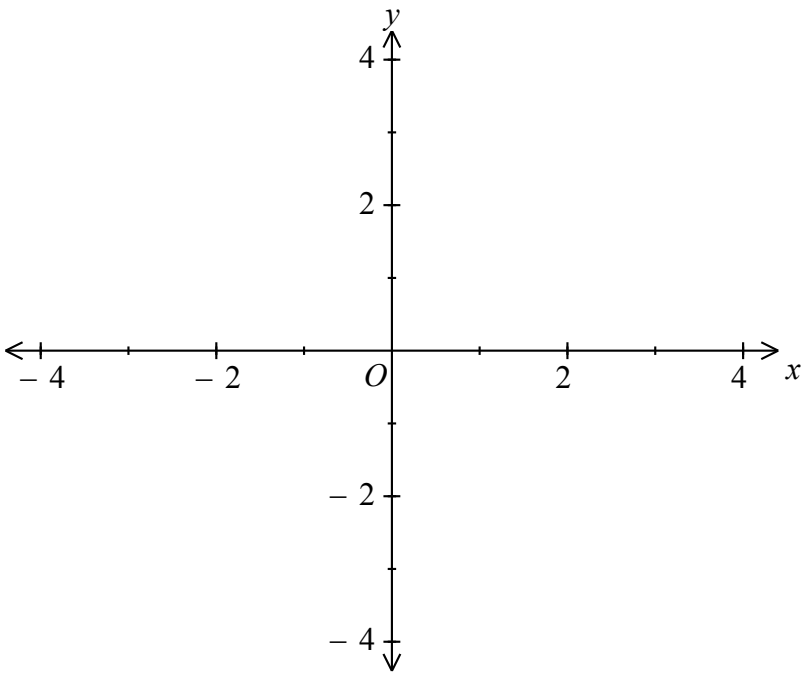
b. Sketch a graph of $y = f(x)$. Label all asymptotes with their equation and all stationary points with their coordinates. 3 marks



Working space

c. Hence find the implied domain of $g(f(x))$. 3 marks

d. Find the implied range of $g(f(x))$. 2 marks

END OF QUESTION AND ANSWER BOOK