2016 TRIAL EXAMINATION 2

UNITS 3 & 4

STUDENT NAME

First Name	
Last Name	

SPECIALIST MATHEMATICS

Written examination 2

2016

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
		Total	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 26 pages.
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION 1

Instructions for Section 1

Answer **all** questions on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where g = 9.8.

Question 1

Which of the following functions has three asymptotes over its maximal domain?

A. $f(x) = \dfrac{x^2}{x^2 - 4x + 4}$

B. $f(x) = \dfrac{x^2}{x^2 + 4x + 4}$

C. $f(x) = \dfrac{x^2}{x^2 - 4x + 3}$

D. $f(x) = \dfrac{x}{x - 4}$

E. $f(x) = \dfrac{x}{4x + 3}$

Question 2

When a suitable substitution is used, the integral $\int_2^3 x\sqrt{x-1}\,dx$ is the same as

A. $\int_2^3 u\sqrt{u}\,du$

B. $\int_2^3 (u+1)\sqrt{u}\,du$

C. $\int_1^{\sqrt{2}} 2u(u^2+1)\,du$

D. $\int_1^{\sqrt{2}} 2u^2(u^2+1)\,du$

E. $\int_2^3 2u^2(u^2+1)\,du$

SECTION 1 – continued
TURN OVER

Question 3

The region bounded by the graph of $y = x\sqrt{x}$ and the lines with equations $x = 0$ and $x = 1$ is rotated about the x – axis. The value of the volume generated by this rotation is

A. $\frac{1}{4}$

B. $\frac{p}{4}$

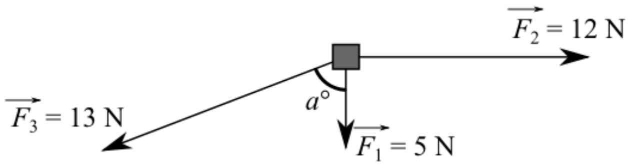
C. $\frac{2}{5}$

D. $\frac{2p}{5}$

E. $\frac{p}{2}$

Question 4

Three forces of 5 N, 12 N and 13 N respectively, act on an object as shown in the diagram below.



If the object is in equilibrium, the value of $a°$ is

A. 45.0°
B. 65.4°
C. 67.4°
D. 76.1°
E. 90.0°

Question 5

Let $y = f(x)$ be a continuous function where $\frac{dy}{dx} = \frac{x}{y}$ and when $x = 0$, $y = 0$.

Consider the following five statements:

- $y^2 = x^2$
- x can take only positive values.
- $f(x)$ has a horizontal asymptote, $y = 0$.
- $f(x)$ has a stationary point at $x = 0$.
- $\frac{d^2y}{dx^2} = \frac{y - x}{y^2}$

How many of the five statements above are **not** true?

A. 0
B. 1
C. 2
D. 3
E. 4

Question 6

Consider the two vectors $\boldsymbol{a} = m\boldsymbol{i} + n\boldsymbol{j}$ and $\boldsymbol{b} = p\boldsymbol{i} + q\boldsymbol{j}$, where m, n, p and q are non-zero real constants. Which one of the following sentences is true?

A. $\boldsymbol{a}$ and $\boldsymbol{b}$ are linearly dependent when $\frac{m}{p} = \frac{n}{q}$.
B. $\boldsymbol{a}$ and $\boldsymbol{b}$ are linearly dependent when $mp = nq$.
C. $\boldsymbol{a}$ and $\boldsymbol{b}$ have the same direction when $mp > 0$ and $nq > 0$.
D. $\boldsymbol{a}$ and $\boldsymbol{b}$ are linearly independent when $\frac{m}{p} = \frac{n}{q}$.
E. $\boldsymbol{a}$ and $\boldsymbol{b}$ are linearly independent when $mp = nq$.

Question 7

Let X and Y be two independent random variables for which $E(3Y - 2X) = 25$ and $E(4X - Y) = 18$.
Correct to one decimal place, the value of $E(-5X + Y)$ is

A. –39.5
B. –25.9
C. 7.9
D. 13.6
E. 21.5

Question 8

The maximal domain of $f(x)=\sqrt{\dfrac{x}{1-x}}+\sin^{-1}(4x)$ is

A. $\left(-\dfrac{\pi}{8},\dfrac{\pi}{8}\right)$

B. $\left[-\dfrac{\pi}{8},\dfrac{\pi}{8}\right]$

C. $(0,1)$

D. $\left[0,\dfrac{\pi}{8}\right)$

E. $\left(0,\dfrac{\pi}{8}\right)$

Question 9

The equation $\cos(x)+\sin(x)=a$, $x\in[0,2\pi]$, has at least one solution for any

A. $a\geq\sqrt{2}$

B. $a\leq-\sqrt{2}$

C. $a\in\left[-\sqrt{2},\sqrt{2}\right]$

D. $a\in(-2,2)$

E. $a\in\left[-2,2\right]$

Question 10

A biologist has collected 80 tree leaves and calculated the 95% confidence interval for the mean length of the leaves as $103.5\text{ mm}\leq\mu\leq108.3\text{ mm}$.

The standard deviation, in mm, of the lengths of the leaves from this sample, correct to one decimal place, is

A. 10.6

B. 10.7

C. 10.8

D. 10.9

E. 11.0

SECTION 1 – continued

Question 11

If $\int_0^1 f(x)\,dx = A$, then $\int_0^1 f\left(\frac{1}{5}x^2 + \frac{4}{5}x\right)(x+2)\,dx$ is equal to

A. $2A$

B. $\frac{2}{5}A$

C. $\frac{5}{2}A$

D. $\frac{A}{2}$

E. A

Question 12

Consider the functions $f(x) = \sin(ax + b)$ and $g(x) = \tan(cx + d)$, a, b, c and $d \in N$.
The period of the function $h(x) = f(x)\,g(x)$ could be

A. ac

B. $2ac$

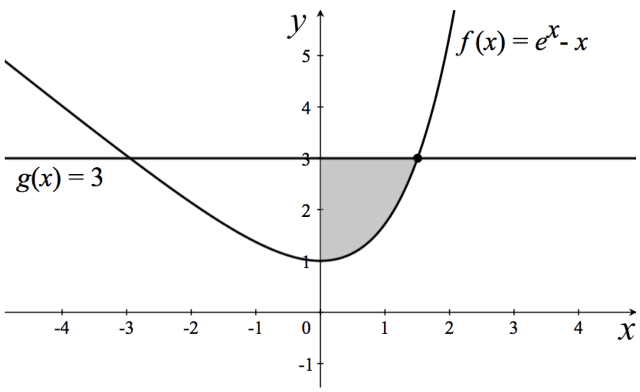
C. 2π

D. $\frac{2\pi}{ac}$

E. $\frac{\pi}{ac}$

Question 13

Part of the graphs of $f(x) = e^x - x$ and $g(x) = 3$ are shown in the diagram below.



The volume generated when the shaded region is rotated about the x–axis is closest to

A. 2.14

B. 3.55

C. 6.73

D. 10.11

E. 11.14

Question 14

Three forces are acting on a particle such that the particle is kept in equilibrium.
Two of these forces are $\boldsymbol{F_1} = 2\boldsymbol{i} + 3\boldsymbol{j}$ and $\boldsymbol{F_2} = -\boldsymbol{i} - 4\boldsymbol{j}$.
The magnitude of the third force is

A. $\sqrt{2}$
B. $\sqrt{10}$
C. $\sqrt{17}$
D. $\sqrt{50}$
E. $\sqrt{58}$

Question 15

A sample of 200 items from a population is randomly selected. For each item a variable X is measured. For the variable measured, the sample standard deviation is s and the sample mean is $\bar{x}$. A confidence interval for the population mean, μ, is calculated and given by $(\bar{x} - 0.1386s,\ \bar{x} - 0.1386s)$.
It follows that, the percentage of confidence given by this interval is

A. 90%
B. 95%
C. 96%
D. 98%
E. 99%

Question 16

The polar form of the complex number $z = (\sqrt{3} - i)(1 + i)(-2i)$ is

A. $4\sqrt{2}\text{cis}\left(\frac{17p}{12}\right)$
B. $4\sqrt{2}\text{cis}\left(-\frac{5p}{12}\right)$
C. $4\text{cis}\left(-\frac{5p}{12}\right)$
D. $4\sqrt{2}\text{cis}\left(\frac{5p}{12}\right)$
E. $4\text{cis}\left(\frac{5p}{12}\right)$

Question 17

Let $\boldsymbol{u} = 2a\boldsymbol{i} + b\boldsymbol{j}$ and $\boldsymbol{v} = b\boldsymbol{i} + a\boldsymbol{j}$, where a, b are non-zero real constants. Given that the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are collinear, which one of the following is **not** true?

A. $\boldsymbol{u} \cdot \boldsymbol{v} = 3ab$

B. $2a^2 = b^2$

C. $\frac{b}{a} = \pm\sqrt{2}$

D. $|\boldsymbol{u}| = |\boldsymbol{v}|$

E. $|\boldsymbol{v}| = \sqrt{a^2 + b^2}$

Question 18

Let $z = \text{cis}(\theta), \theta \in (-\pi, \pi]$.

Which one of the following sentences is **not** true**?**

A. $\bar{z} = \text{cis}(-q)$

B. $z^2 = \text{cis}(2q)$

C. $|z| = 1$

D. $\frac{z}{\bar{z}} = \text{cis}(2q)$

E. $z\bar{z} = \text{cis}(2q)$

Question 19

A particle moves in a straight line with a variable acceleration given by the rule $a = 2v^2 - v$ ms^{-2}, where v is the velocity of the particle, in ms^{-1}. The initial velocity of the particle is 1 ms^{-1}.

The velocity of the particle at time t seconds is given by

A. $v = \frac{2}{2 - e^t}$

B. $v = \frac{1}{2 - e^t}$

C. $v = \frac{1}{2 - e^{-t}}$

D. $v = \frac{1}{2 + e^t}$

E. $v = \frac{1}{e^t - 2}$

SECTION 1 – continued
TURN OVER

Question 20

An object of mass m kg is moved in a straight line by a force of 16 N.

After 4 seconds the object reaches a velocity of 20 ms^{-1}. The distance travelled during the first 4 seconds is 12 m.
The mass of the object is

A. 3.2 kg
B. 3.5 kg
C. 4 kg
D. 5 kg
E. 10.7 kg

Question 21

A mobile phone company offers 3 plans. Let X be the charge in dollars per month. The distribution of the random variable X is given in the table below.

x	\$6	\$10	\$20
$\Pr(X = x)$	0.65	0.1	0.25

The company decides to increase its prices by 30% and add an extra \$2 for each plan afterwards.
Which of the following expressions gives the random variable of the new charges?

A. $0.3X + 2$
B. $0.7X + 2$
C. $1.3X + 2$
D. $30X + 2$
E. $130X + 2$

Question 22

The equation $z^4 - az + b = 0$ has the solutions $z_1 = 1$ and $z_2 = 1 - i$.
The values of a and b are

A. both real numbers.
B. both imaginary numbers.
C. irrational numbers.
D. a is an imaginary number while b is a real number.
E. b is an imaginary number while a is a real number.

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
Unless otherwise specified, an exact answer is required to a question.

Question 1 (12 marks)
A 6 sided die has one of its faces showing one dot, two faces showing two dots each and three faces showing three dots each. Let *X* be the random variable that gives the number of dots shown by the top face of the die when rolled. The probability distribution for the discrete random variable *X* is shown below.

x	1	2	3
$\Pr(X = x)$	a	b	$\frac{1}{2}$

a. Determine the values of *a* and *b*. 1 mark

b. Calculate the mean value of *X*. 2 marks

SECTION 2 – Question 1 – continued
TURN OVER

c. Calculate the variance of *X*. 2 marks

An experiment is run with 2 identical dice as the one described before.
Let Y be the random variable that represents the sum of the dots shown when the two dice are rolled.

d. Show that $c = \frac{1}{36}$ and $d = \frac{5}{18}$. 2 marks

y	2	3	4	5	6
$\Pr(Y = y)$	c	$\frac{1}{9}$	d	$\frac{1}{3}$	$\frac{1}{4}$

SECTION 2 – Question 1 – continued

e. Show that $E(Y)^2 + E(Y) - \frac{E(Y^2)}{2} = 15$. 3 marks

f. Determine the median of Y. 2 marks

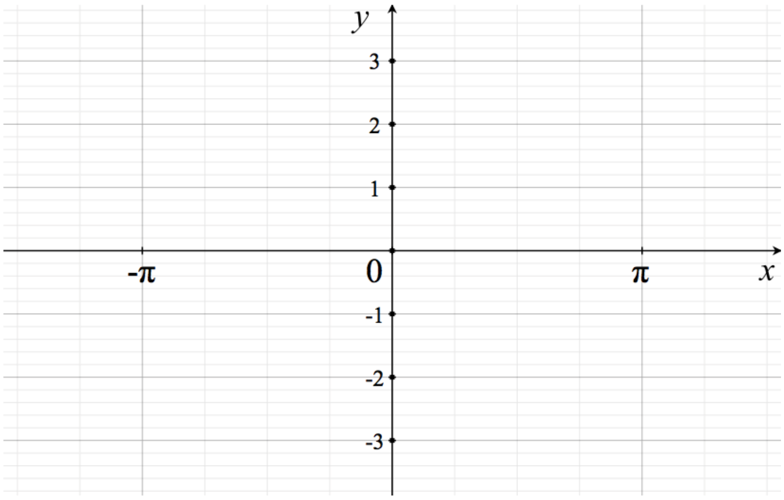
SECTION 2 – continued
TURN OVER

Question 2 (12 marks)

Consider the function $f(x) = \tan(x)e^{2x},\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

a. On the set of axes below, sketch the graph of $f(x)$ clearly showing its features such as x and y

intercepts, turning points and asymptotes, if any. 2 marks



SECTION 2 – Question 2 – continued

b. Show that $f''(x) = 2f'(x) + 2\sec^2(x)[f(x) + e^{2x}], \forall\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. 3 marks

c. Show that the function $f(x)$ has only one point of inflection. Give the coordinates of the point of inflection correct to one decimal place. 2 marks

SECTION 2 – Question 2 – continued
TURN OVER

d. Calculate the values of y and $\frac{dy}{dx}$ when $x = \frac{p}{4}$. 2 marks

e. **Hence** or otherwise determine the equation of the tangent to the curve of $f(x)$ at $x = \frac{p}{4}$.

Give your answer in the form $y = mx + c$, $m, c \in R$. 3 marks

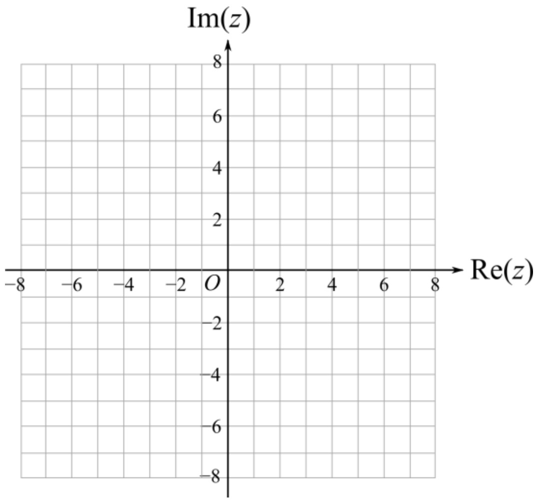
SECTION 2 – continued

Question 3 (12 marks)

Consider the set of complex numbers $P = \{z : |z + 4i - 1| = 3, z \in C\}$.

a. If $z = x + iy$, where x and y are real values and $z \in P$, determine the cartesian equation of the region represented by the set of complex numbers P. 2 marks

b. On the set of axes below, sketch the region represented by the set of complex numbers $P = \{z : |z + 4i - 1| \geq 3, z \in C\}$.
Clearly label all key features of the region (x and y – intercepts not required). 2 marks



SECTION 2 – Question 3 – continued
TURN OVER

c. Calculate all $z \in C$ such that $|z + 4i - 1| = 3$ and $|z + 2 + 4i| = |z - 4 + 4i|$. 2 marks

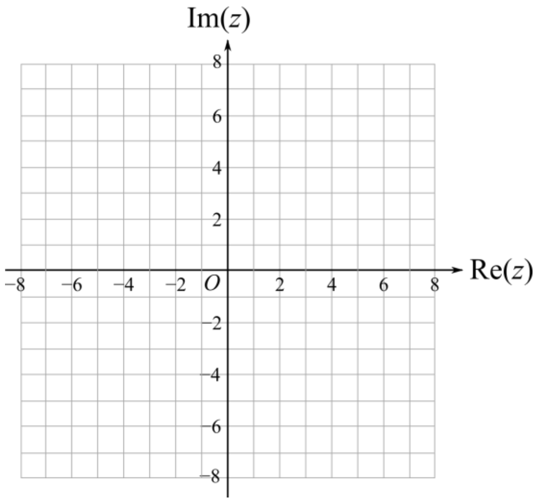
SECTION 2 – Question 3 – continued

A and *B* are two sets of complex numbers defined by

$A = \{ z = x + yi \mid x, y \in R, x^2 + (y + 4)^2 = c\}$ and

$B = \{ z = x + iy\} \mid |z + 4i - 1| \le 3, \forall z \in A\}$.

d. On the set of axes below, sketch the locus *A* when *c* = 1 and the region *B*. 1 mark



e. Let $A \subset B$ for all $z = x + iy \in A$.

i. Determine expressions for x and y in terms of c when A and B intersect. 3 mark

SECTION 2 – Question 3 – continued
TURN OVER

ii. Determine the largest value for c that satisfies the conditions given. 1 mark

iii. On the set of axes above, sketch the the set of complex numbers defined by $A = \{ z = x + yi \mid x, y \in R, x^2 + (x + 4)^2 = c\}$ with the value of c from part **e. ii**. 1 mark

Question 4 (9 marks)

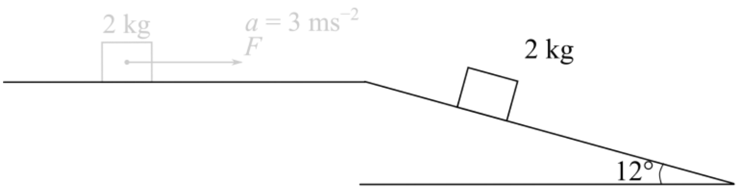
An object of 2 kg mass, initially at rest, is pulled by a horizontal force. The object is moving across a horizontal smooth surface with an acceleration of 3 ms^{-2} for 10 seconds as shown in the diagram below.

a. Calculate the speed of the object after 10 seconds. 1 mark

b. Determine the distance travelled by the object in the first 10 seconds of the motion. 1 mark

After 10 seconds the object has reached the top of an inclined plane. The inclined plane has a rough surface and makes an angle of 12° with the horizontal. The object comes to a stop after it has travelled another 25 metres.

c. On the diagram below show all forces acting on the object. 2 marks



SECTION 2 – Question 4 – continued
TURN OVER

d. Calculate the acceleration of the object travelling down the inclined plane, correct to two decimal places. 2 marks

e. Use your answer from **part d.** or otherwise, to calculate the coefficient of friction between the object and the inclined plane. Give your answer correct to 2 decimal places. 3 marks

SECTION 2 – continued

Question 5 (13 marks)

The position of a particle at any time t seconds relative to a point O, is given by the position vector

$\boldsymbol{r}(t) = \left[3\sin(t) - \sqrt{3}\cos(t) + a\right]\boldsymbol{i} + \left[-\sqrt{3}\sin(t) - 3\cos(t) + b\right]\boldsymbol{j}$, $t \in [0, 20]$, where $\boldsymbol{i}$ is the unit vector to the right and $\boldsymbol{j}$ is the unit vector in an upward direction.

All displacement components are measured in metres.

a. Calculate the values of a and b if the particle is initially at O. 2 marks

SECTION 2 – Question 5 – continued
TURN OVER

b. Determine the first two times when the particle is 6 m from the starting point O. Give your answers correct to 2 decimal places. 2 marks

SECTION 2 – Question 5 – continued

c. Show that $\left|\ddot{\boldsymbol{r}}(t)\right| = \sqrt{12}$. 2 marks

d. Determine an expression for the velocity of the particle in the form $\dot{\boldsymbol{r}}(t) = \left[m\cos(t+a)\right]\boldsymbol{i} - \left[m\cos(t+b)\right]\boldsymbol{j}$.
Give your answers correct to 2 decimal places if required. 3 marks

SECTION 2 – Question 5 – continued
TURN OVER

e. Determine the cartesian equation of the path of the particle. 4 marks

END OF QUESTION AND ANSWER BOOK