

Year 2016

VCE

Specialist Mathematics

Trial Examination 2



KILBAHA MULTIMEDIA PUBLISHING
PO BOX 2227
KEW VIC 3101
AUSTRALIA

TEL: (03) 9018 5376
FAX: (03) 9817 4334
kilbaha@gmail.com
<http://kilbaha.com.au>

IMPORTANT COPYRIGHT NOTICE

- This material is copyright. Subject to statutory exception and to the provisions of the relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Kilbaha Multimedia Publishing.
- The contents of this work are copyrighted. Unauthorised copying of any part of this work is illegal and detrimental to the interests of the author.
- For authorised copying within Australia please check that your institution has a licence from **Copyright Agency Limited**. This permits the copying of small parts of the material, in limited quantities, within the conditions set out in the licence.

Reproduction and communication for educational purposes The Australian Copyright Act 1968 (the Act) allows a maximum of one chapter or 10% of the pages of this work, to be reproduced and/or communicated by any educational institution for its educational purposes provided that educational institution (or the body that administers it) has given a remuneration notice to Copyright Agency Limited (CAL) under the Act.

For details of the CAL licence for educational institutions contact
CAL, Level 15, 233 Castlereagh Street, Sydney, NSW, 2000

Tel: (02) 9394 7600

Fax: (02) 9394 7601

Email: info@copyright.com.au

Web: <http://www.copyright.com.au>

- While every care has been taken, no guarantee is given that these questions are free from error. Please contact us if you believe you have found an error.

STUDENT NUMBER

Figures
Words

Letter

--

SPECIALIST MATHEMATICS
Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
A	20	20	20
B	5	5	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 33 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions and sign your name in the space provided.
- All written responses must be in English.

At the end of the examination

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A**Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions. Choose the response that is **correct** for the question.

A correct answer scores 1 mark, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No mark will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

The features of the graph of the function with the rule $f(x) = \frac{2x^3 + x^2 - 8x}{x^3 - 4x}$ include

- A. a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = 2$ only.
- B. vertical asymptotes at $x = -2$ and $x = 2$ only.
- C. vertical asymptotes at $x = -2$, $x = 0$ and $x = 2$ only.
- D. vertical asymptotes at $x = -2$ and $x = 2$ and the graph crosses the horizontal asymptote $y = 2$ at $x = 0$
- E. vertical asymptotes at $x = -2$ and $x = 2$ and a horizontal asymptote at $y = 2$, and a point of discontinuity at $x = 0$

Question 2

If a is a positive real constant, then the range of the with rule $f(x) = \frac{a \sin^{-1}\left(\frac{x}{a}\right)}{\cos^{-1}\left(\frac{x}{a}\right)}$ is

- A. R
- B. $\left[-\frac{a}{2}, \infty\right)$
- C. $\left(-\frac{a}{2}, \infty\right)$
- D. $\left(-\frac{a\pi}{2}, \frac{a\pi}{2}\right)$
- E. $\left[0, \frac{a\pi}{2}\right)$

Question 3

The set of points in the complex plane described by $\{ z : |z - a| = |z + ai| \}$ where $a \in \mathbb{R} \setminus \{0\}$ and $z \in \mathbb{C}$ can also be described by

- A. $\{ z : \operatorname{Re}(z) + \operatorname{Im}(z) = 0 \}$
- B. $\{ z : \operatorname{Re}(z) - \operatorname{Im}(z) = 0 \}$
- C. $\{ z : \operatorname{Re}(z) = 0 \}$
- D. $\{ z : \operatorname{Im}(z) = 0 \}$
- E. $\{ z : \operatorname{Arg}(z) = -\frac{\pi}{4} \} \cup \{ z : \operatorname{Arg}(z) = \frac{3\pi}{4} \}$

Question 4

If A, B, C, D and a are all non-zero real constants, then $\frac{x^2}{x^4 - a^4}$ expressed in partial fractions has the form

- A. $\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{(x-a)^3} + \frac{D}{(x-a)^4}$
- B. $\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x+a} + \frac{D}{(x+a)^2}$
- C. $\frac{A}{x-a} + \frac{B}{x+a} + \frac{Cx+D}{x^2+a^2}$
- D. $\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{Cx+D}{x^2+a^2}$
- E. $\frac{A}{x-a} + \frac{B}{x+a} + \frac{Cx+D}{(x^2+a^2)^2}$

Question 5

A conical tank with its axis vertical and vertex downwards has its height double the radius. It is initially filled with water to a height of h_0 metres. The tank has a hole in the vertex through which the water escapes at a rate of $c\sqrt{h}$ m³/min, where h is the height of water in the tank in metres and c is a constant. Water is poured into the tank at a rate of Q m³/min. The time in minutes taken for the tank to be empty is given by

A.
$$\int_0^{h_0} \frac{\pi h^2}{4(c\sqrt{h} - Q)} dh$$

B.
$$\int_0^{h_0} \frac{\pi h^2}{4(Q - c\sqrt{h})} dh$$

C.
$$\int_0^{h_0} \frac{4\pi h^2}{c\sqrt{h} - Q} dh$$

D.
$$\int_0^{h_0} \frac{4\pi h^2}{Q - c\sqrt{h}} dh$$

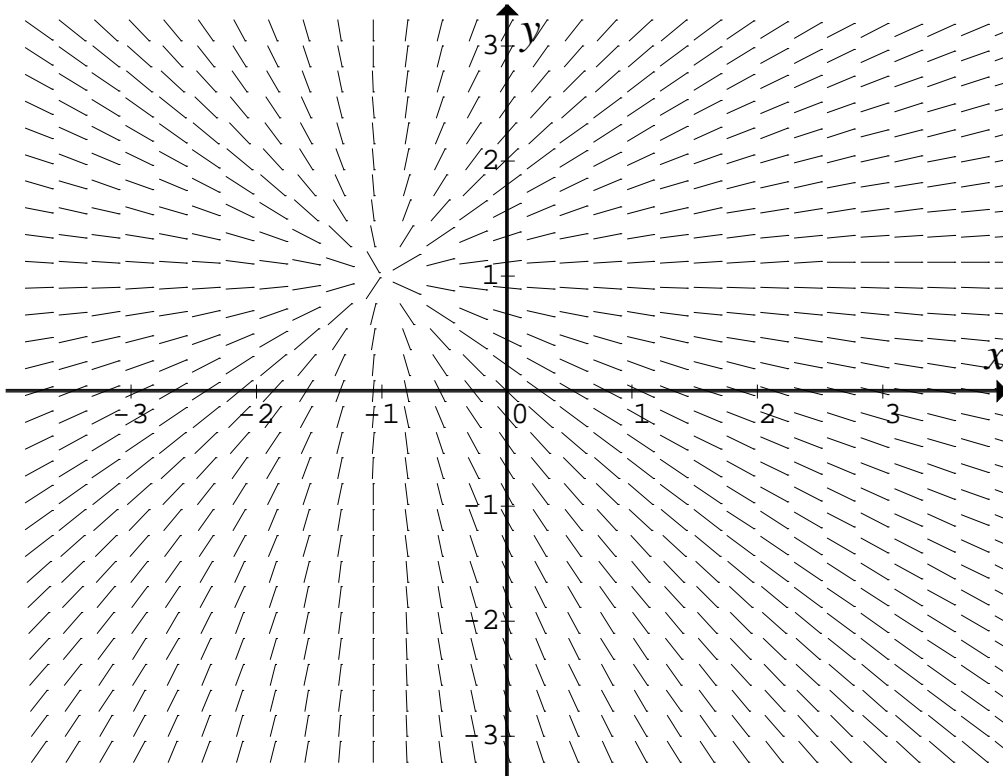
E.
$$\int_0^{h_0} \frac{4}{\pi h^2 (c\sqrt{h} - Q)} dh$$

Question 6

A particle moves so that its position vector is given by $\underline{r}(t) = \cos(t)\underline{i} + \cos(3t)\underline{j}$ where the position is measured in metres and $t \geq 0$ is the time in seconds. The particle moves along part of

- A. a straight line.
- B. a parabola.
- C. a circle.
- D. a cubic
- E. an ellipse.

Question 7



The differential equation which best represents the above direction field is

- A. $\frac{dy}{dx} = \frac{x-1}{y+1}$
- B. $\frac{dy}{dx} = \frac{y-1}{x+1}$
- C. $\frac{dy}{dx} = \frac{x+1}{y-1}$
- D. $\frac{dy}{dx} = \frac{y+1}{x-1}$
- E. $\frac{dy}{dx} = (x-1)(y+1)$

Question 8

In a chemical reaction, the velocity of the reaction is proportional to the products of the unused amounts of the substances A and B present. Initially, there is a grams of substance A and b grams of substance B. These combine in equal parts to form x grams of substance X after time t seconds. If k is a positive constant, then the differential equation which models the process is

- A. $\frac{dx}{dt} = k(a-x)(b-x)$, $x(0) = 0$
- B. $\frac{dx}{dt} = -k(a-x)(b-x)$, $x(0) = 0$
- C. $\frac{dx}{dt} = k\left(a - \frac{x}{2}\right)\left(b - \frac{x}{2}\right)$, $x(0) = 0$
- D. $\frac{dx}{dt} = -k\left(a - \frac{x}{2}\right)\left(b - \frac{x}{2}\right)$, $x(0) = 0$
- E. $\frac{dx}{dt} = k(a+b-2x)$, $x(0) = 0$

Question 9

With a suitable substitution $\int_0^1 \frac{x}{\sqrt{b-ax}} dx$ where $b > a > 0$ can be expressed as

- A. $\frac{1}{a^2} \int_b^{b-a} \frac{b-u}{\sqrt{u}} du$
- B. $\frac{1}{a^2} \int_{b-a}^b \frac{b-u}{\sqrt{u}} du$
- C. $\int_b^{b-a} \frac{b-u}{\sqrt{u}} du$
- D. $\int_{b-a}^b \frac{b-u}{\sqrt{u}} du$
- E. $\frac{1}{a^2} \int_{b-a}^b \frac{b-u^2}{u} du$

Question 10

The velocity vector in m/s of a 3 kg moving particle is given by

$\underline{v}(t) = 3 \cos(2t)\underline{i} + \sin(2t)\underline{j}$. The maximum value of the force acting on the particle in newtons is

- A. 6
- B. 9
- C. 12
- D. 18
- E. 24

Question 11

Two vectors \underline{u} and \underline{v} are such that $|\underline{u}| = 3$ and $|\underline{v}| = 4$ and $\underline{u} \cdot \underline{v} = 1$. Then

- A. the vectors \underline{u} and \underline{v} are parallel.
- B. the vectors \underline{u} and \underline{v} are perpendicular.
- C. the vectors \underline{u} and \underline{v} are linearly dependant.
- D. $|\underline{u} + \underline{v}| = 7$
- E. $|\underline{u} + \underline{v}| = 3\sqrt{3}$

Question 12

If $\frac{dy}{dx} = y \sec^2(x)$ and $y = 2$ when $x = 0$ then

- A. $y = 1 + e^{\tan(x)}$
- B. $y = 2e^{\tan(x)}$
- C. $y = 2 + \tan(x)$
- D. $y = 2e^x + \tan(x)$
- E. $y = \sqrt{2 \tan(x) + 4}$

Question 13

Let $\frac{dy}{dx} = bxy$ where $b \in \mathbb{R} \setminus \{0\}$ and $y = 2$ when $x = 1$.

Using Euler's method with a step size of 0.5, the approximation to y when $x = 2$ is

- A. $2 + b$
- B. $2 + \frac{5b}{2}$
- C. $2 + \frac{5b}{2} + \frac{3b^2}{4}$
- D. $2 + 2b + \frac{b^2}{2}$
- E. $2e^{\frac{3b}{2}}$

Question 14

Which of the following definite integrals gives the length of the curve $y = \cos(\sqrt{x})$ between $x = a$ and $x = b$, where $0 < a < b$?

- A. $\int_a^b \sqrt{1 + \sin^2(\sqrt{x})} dx$
- B. $\int_a^b \sqrt{x + \sin^2(\sqrt{x})} dx$
- C. $\int_a^b \sqrt{1 + \frac{\sin(\sqrt{x})}{2\sqrt{x}}} dx$
- D. $\int_a^b \sqrt{1 + \frac{\sin^2(\sqrt{x})}{x}} dx$
- E. $\frac{1}{2} \int_a^b \sqrt{\frac{4x + \sin^2(\sqrt{x})}{x}} dx$

Question 15

The function f is defined by $f(x) = \sqrt{x^4 + 16}$ and g is an antiderivative of f such that $g(1) = 3$, then $g(2)$ is closest to

- A. 11.72
- B. 8.72
- C. 7.69
- D. 4.69
- E. 1.69

Question 16

A ball of mass m kg is dropped and is subject to gravity and a force of air resistance equal to $k\sqrt{v}$ where k is a positive constant and $v \text{ ms}^{-1}$ is its velocity. The distance fallen vertically is x metres from the point of release. Three students were analysing the motion.

Albert stated that $\frac{dv}{dx} = \frac{g}{v} - \frac{k}{m\sqrt{v}}$.

Ben stated that $x = \int \frac{m}{mg - k\sqrt{v}} dv$

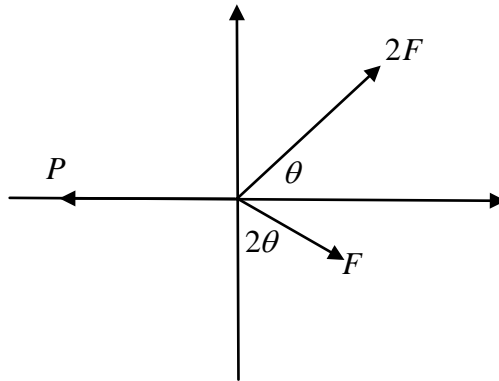
Colin stated that the terminal velocity is $\left(\frac{mg}{k}\right)^2$.

Then

- A. Only Albert is correct.
- B. Only Ben is correct.
- C. Only Colin is correct.
- D. Both Albert and Colin are correct.
- E. Albert, Ben and Colin are all correct.

Question 17

A body is on a horizontal smooth plane and acted upon by three forces, with magnitudes and directions as shown in the diagram below.



The correct statement relating the magnitude of the forces and the angle θ is

- A. $P = 3F$
- B. $P = 3F \sin(3\theta)$
- C. $P = 3F \cos(3\theta)$
- D. $P = 2F \sin(\theta) + F \cos(2\theta)$
- E. $\theta = \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$

Question 18

A type 2 error would occur in a statistical test where

- A. H_0 is accepted when H_0 is false.
- B. H_0 is accepted when H_0 is true.
- C. H_0 is rejected when H_0 is false.
- D. H_1 is accepted when H_1 is true.
- E. H_1 is rejected when H_1 is true.

Question 19

A certain brand and size of potato chips state that they contain 20 grams of potato chips. An enterprising student wishes to test this claim, thinking the manufacturers were actually under filling the packets. In performing a hypothesis test H_0 is the null hypothesis and H_1 is the alternative hypothesis, and μ is the average amount of potato chips in a packet. Then

- A. $H_0: \mu \neq 20$ and $H_1: \mu > 20$
- B. $H_0: \mu \neq 20$ and $H_1: \mu < 20$
- C. $H_0: \mu = 20$ and $H_1: \mu > 20$
- D. $H_0: \mu = 20$ and $H_1: \mu < 20$
- E. $H_0: \mu = 20$ and $H_1: \mu \neq 20$

Question 20

The heights of trees in a forest are normally distributed, with a mean of 25 metres and a standard deviation of 4 metres. A random sample of 36 trees is taken. The probability that the mean height of the trees in the sample exceeds 24 metres is closest to

- A. 0.933
- B. 0.599
- C. 0.433
- D. 0.099
- E. 0.067

END OF SECTION A

SECTION B**Instructions for Section B**

Answer **all** questions in the spaces provided.

Unless otherwise specified an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1 (9 marks)

A curve is defined by the parametric equations

$$x = 4 \sin^2(t)$$

$$y = 4 \tan(t) \sin^2(t) \text{ for } t \in [0, \pi]$$

a.i. Show that the gradient of the curve can be expressed as $\frac{\sin(t)(2\cos^2(t)+1)}{2\cos^3(t)}$.

3 marks

ii. State the coordinates on the curve where the slope of the curve is 2.

2 marks

A particle moves along the vector equation $\underline{r}(t) = 4 \sin^2(t) \underline{i} + 4 \tan(t) \sin^2(t) \underline{j}$
for $t \in [0, \pi]$.

b.i. Find the speed of the particle when $t = \frac{\pi}{4}$.

2 marks

ii. Verify that the Cartesian equation of the curve satisfies the implicit equation

$$y^2 = \frac{x^3}{4-x}.$$

2 marks

Question 2 (9 marks)

Consider the function $f(x) = \sqrt{\frac{x^3}{4-x}}$ defined on its maximal domain.

- a. If $f'(x) = \frac{\sqrt{x}(a-x)}{(4-x)^n}$. State the values of a and n .

1 mark

- b. If $f''(x) = \frac{b}{\sqrt{x}(4-x)^m}$. State the values of b and m .

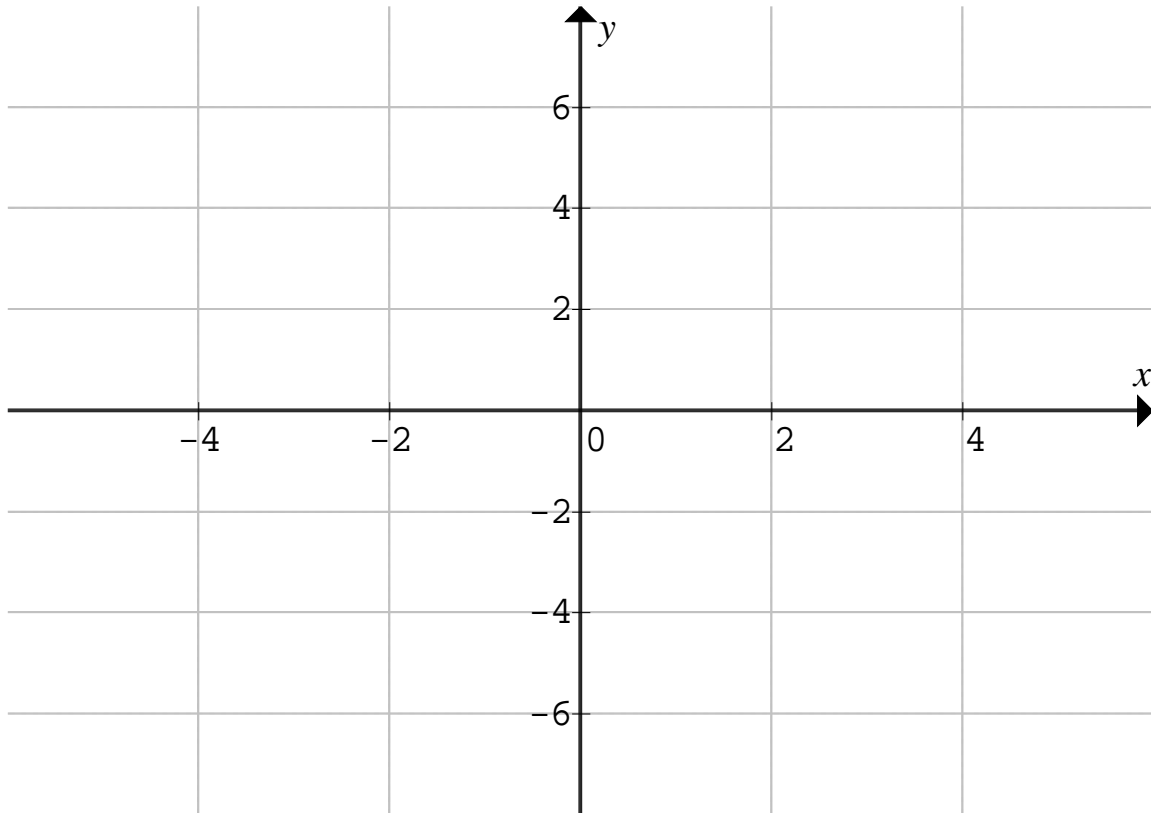
1 mark

- c. Either find or explain why the graph of $y = \sqrt{\frac{x^3}{a-x}}$ has or does not have stationary points or points of inflexion.

2 marks

- d. Sketch the graph of the relation $y^2 = \frac{x^3}{4-x}$ on the axes below, clearly stating the equations of any asymptotes.

2 marks



Question 3 (9 marks)

A, B and C are three points with coordinates $(-4, 4)$, $(-3 - \sqrt{3}, 1 + \sqrt{3})$ and $(-1 - \sqrt{3}, 3 + \sqrt{3})$ respectively

a. Find the vectors \overline{AB} and \overline{AC} .

2 marks

b. Prove that ABC is an isosceles triangle.

3 marks

c. Using vectors find the angle between \overrightarrow{AB} and \overrightarrow{AC} .

3 marks

d. Find the area of the triangle ABC .

1 mark

Question 4 (15 marks)

Consider the complex numbers $a = -4 + 4i$, $b = -(3 + \sqrt{3}) + (1 + \sqrt{3})i$ and

$$c = -(1 + \sqrt{3}) + (3 + \sqrt{3})i.$$

- a. Show that $\text{Arg}(b) = \frac{5\pi}{6}$ and hence express b in polar form.

3 marks

$$\text{Let } S = \{z : |z - a| = 2(\sqrt{3} - 1), z \in \mathbb{C}\}.$$

- b. Find and describe the cartesian equation of S .

2 marks

$$\text{Let } T = \left\{ z : \text{Arg}(z) = \frac{5\pi}{6}, z \in \mathbb{C} \right\}.$$

- c. Find and describe the cartesian equation of T .

2 marks

- d. Explain why $b \in S \cap T$

2 marks

- e. Express c in polar form.

1 mark

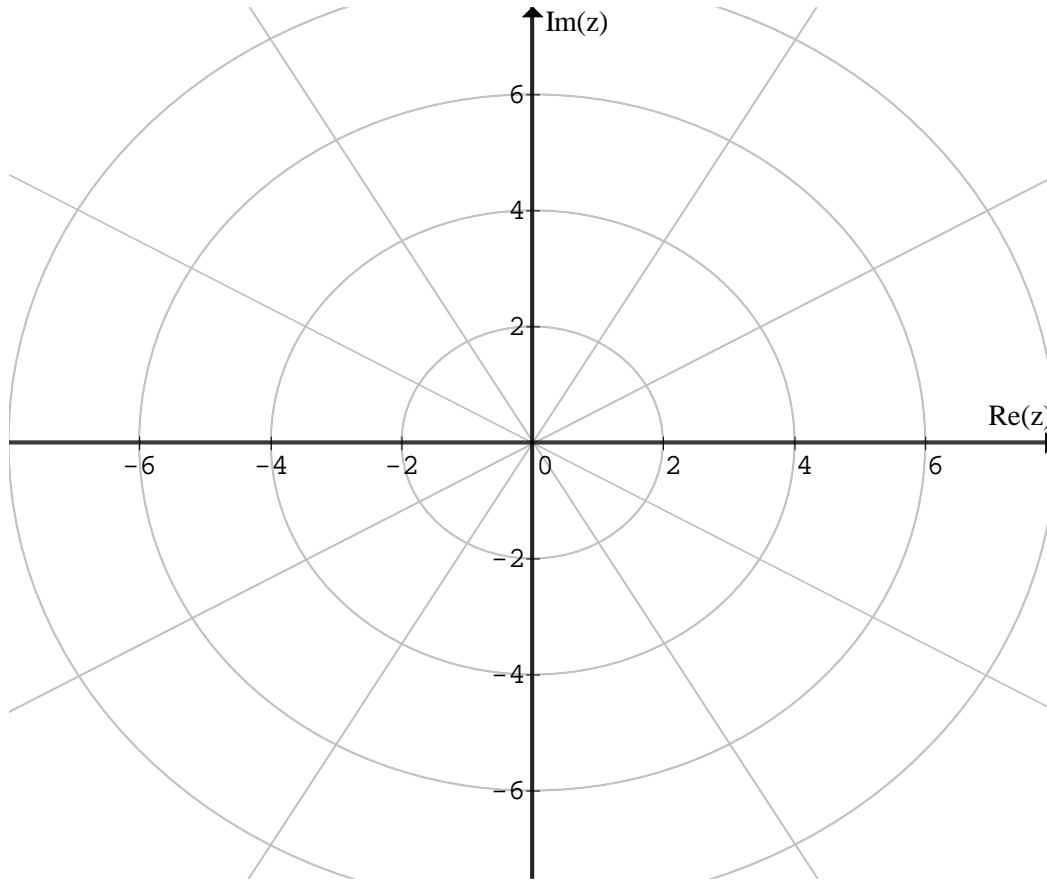
$$\text{Let } W = \{ z : \text{Arg}(z) = \theta, z \in \mathbb{C} \}$$

- f. Given that $c \in S \cap W$, find the value of θ .

1 mark

- g.** Clearly plot the points, a , b and c and the sets S , T and W on the argand diagram below.

3 marks



- h.** If $u \in S$ find the maximum value of $|u|$

1 mark

Question 5 (10 marks)

A model for the number of kangaroos N in a national park after a time t years, is modelled by the differential equation $\frac{dN}{dt} = \frac{N}{4} \left(1 - \frac{N}{500}\right)$ for $t \geq 0$. Initially there are 50 kangaroos in the park.

a.i. Set up an integral which can be used to express t in terms of N . 1 mark

ii. Use partial fractions to integrate this expression and hence show that

$$N = N(t) = \frac{500}{1 + 9e^{-\frac{t}{4}}}$$
4 marks

b. What does the model predict the eventual number of kangaroos will be?

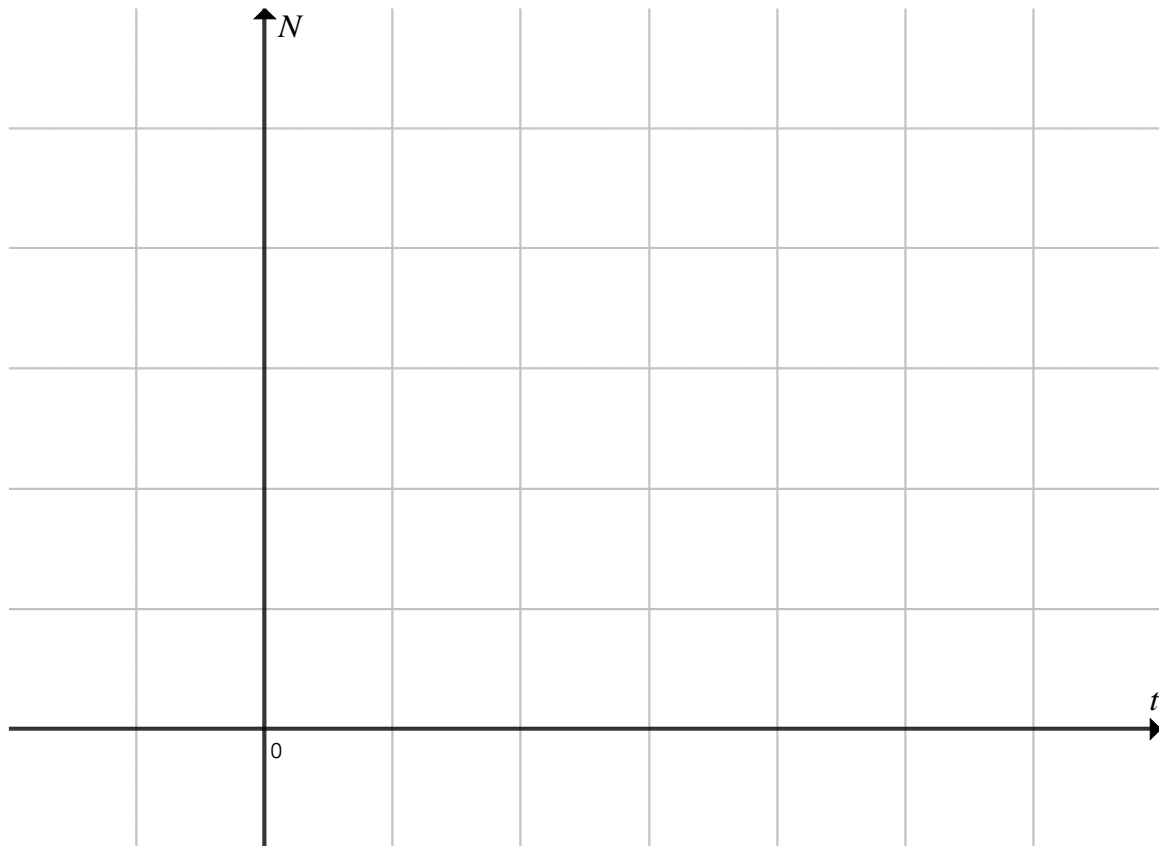
1 mark

c. Express $\frac{d^2N}{dt^2}$ in terms of N .

2 marks

d. The graph of N as a function of t has a point of inflexion. Sketch the graph of N as a function of t on the axes below, clearly labelling the scale, stating the equations of any asymptotes and the coordinates of the point of inflexion.

2 marks



Question 6 (8 marks)

- a.** Lilly stands in a lift. When the lift accelerates upwards the reaction force of the lift floor on Lilly is 650 newtons. When the lift accelerates downwards with same acceleration the reaction force of the lift floor on Lilly is 624 newtons. Find the mass of Lilly and the acceleration of the lift.

3 marks

- b.** It has been found that the weights of males are normally distributed with a mean of 85 kg and a standard deviation of 15 kg. The weights of females are normally distributed with a mean of 65 kg and a standard deviation of 20 kg. 12 males and 8 females enter a lift. The lift will be overloaded if the total weight exceeds 1500 kg. Find the probability that the lift is overloaded. Give your answer correct to four decimal places.

3 marks

- c. The lift is in service all day. A random sample of the total weight in the lift on 30 occasions was taken and was found to have an average weight of 1000 kg with a standard deviation of 250 kg. Find a 95% confidence interval for the mean weight in the lift, giving your answer correct to 2 decimal places.

2 marks

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a pyramid	$\frac{1}{3}Ah$
area of triangle	$\frac{1}{2}bc \sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab \cos(C)$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

Circular (trigonometric) functions - continued

Function	\sin^{-1} (arcsin)	\cos^{-1} (arccos)	\tan^{-1} (arctan)
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos(\theta) + i \sin(\theta)) = r \operatorname{cis}(\theta)$	
$ z = \sqrt{x^2 + y^2} = r$	$-\pi < \operatorname{Arg}(z) \leq \pi$
$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$
$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$E(aX + b) = aE(X) + b$ $E(aX + bY) = aE(X) + bE(Y)$ $\operatorname{Var}(aX + b) = a^2 \operatorname{Var}(X)$
for independent random variables X and Y	$\operatorname{Var}(aX + bY) = a^2 \operatorname{Var}(X) + b^2 \operatorname{Var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z \frac{s}{\sqrt{n}}, \bar{x} + z \frac{s}{\sqrt{n}}\right)$
distribution of sample mean \bar{X}	mean $E(\bar{X}) = \mu$ variance $\operatorname{Var}(\bar{X}) = \frac{\sigma^2}{n}$

Vectors in two and three dimensions

$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$
$ \underline{r} = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underline{r}} = \frac{dx}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}$
$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underline{p} = m\underline{v}$
equation of motion	$\underline{R} = m\underline{a}$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$	$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$
$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c, n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a} \log_e ax+b + c$
product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1+(f'(x))^2} dx$ or $\int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} dt$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

Figures
Words

Letter

--

SIGNATURE _____

SECTION A

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E