Year 2016

VCE

Specialist Mathematics

Trial Examination 1

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Victorian Certificate of Education
2016

STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 18 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (3 marks)

Resolve the vector $4\underset{\sim}{i} - 2\underset{\sim}{j} - 2\underset{\sim}{k}$ into two vector components, one which is parallel to the vector $3\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k}$ and one which is perpendicular to it.

Question 2 (3 marks)

Let $f(z) = z^4 - 6z^3 + 17z^2 - 30z + 60$, where $z \in C$.

a. Given that $z = \sqrt{5}\,\text{cis}\left(-\frac{\pi}{2}\right)$ is a solution of $f(z) = 0$, find a quadratic factor of $f(z)$. 1 mark

b. Hence find all the roots of $f(z) = 0$. 2 marks

Question 3 (3 marks)

Solve the differential equation $\frac{dy}{dx} = \frac{24y^2}{9+4x^2}$ and $y\left(\frac{3}{2}\right) = 1$, expressing y in terms of x.

Question 4 (5 marks)

A bowl is formed when the region bounded by the graph of $y = x^2 - 16$, the x-axis, the y-axis and the line $y = 20$, is rotated about the y-axis. Units are in centimetres.

a. Find the capacity of the bowl. 2 marks

b. If the bowl is being filled at a rate of 2 cm^3/sec, find the rate at which the height of the fluid in the bowl is rising at, when the height is 14 cm. 3 marks

Question 5 (5 marks)

a. Show that $\int \frac{\sin(2x)}{\cos^2(2x)}\,dx = \frac{1}{2}\sec(2x)+c$ 1 mark

A particle moves along a curve so that at time $t \geq 0$ its velocity vector is given by $\underset{\sim}{v}(t) = 4\tan(2t)\sec(2t)\,\underset{\sim}{i} + 6\sec^2(2t)\,\underset{\sim}{j}$.

b.i. Given that $\underset{\sim}{r}(0) = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ find the position vector $\underset{\sim}{r}(t)$.

2 marks

ii. The particle moves along the curve from $t = 0$ to $t = \frac{\pi}{5}$.

This distance can be expressed in terms of t as the definite integral

$\int_0^{\frac{\pi}{5}} p\sec(2t)\sqrt{q\tan^2(2t) + r}\, dt$. Determine the values of p, q and r.

Do not attempt to evaluate the definite integral. 2 marks

Question 6 (4 marks)

Find the gradient of the normal to the curve defined by $y^2(4 - x) = x^3$ at the point in the fourth quadrant where $x = 2$.

Question 7 (4 marks)

A teacher at a school wishes to test if the students in a certain year have above or below average intelligence. To do this he takes a random sample of 36 students and finds that their mean IQ score is 103. The mean population IQ is 100 with a standard deviation of 15. Perform a hypothesis test, at the 95% level of significance, stating the null and alternative hypothesis, calculating the test statistic and clearly stating your conclusions.

Question 8 (5 marks)

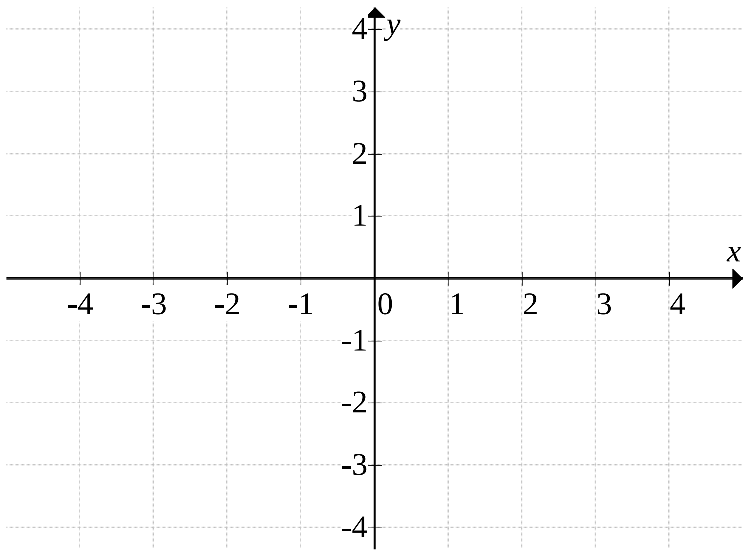
At a time t seconds, $t \geq 0$, a body of mass 3 kg is moving in a straight line. Initially the body is at rest 4 metres from the origin and is acted upon by a force of $-12x$ newtons, where x is its displacement from the origin in metres. Express x in terms of t.

Question 9 (4 marks)

Consider the function $f(x) = 2\arcsin\left(\frac{x}{3}\right)$.

a. Sketch the graph of the function on the axes below, clearly indicating the co-ordinates of the endpoints.

1 mark



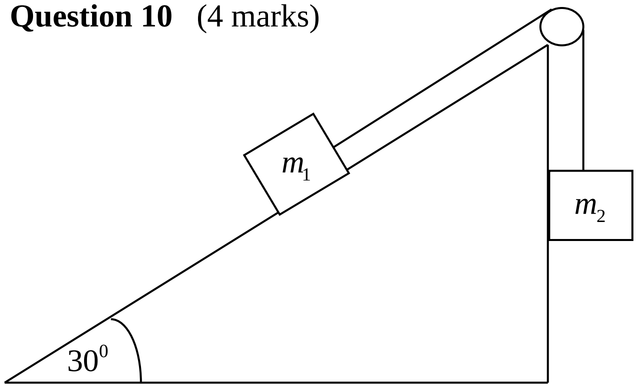
b. Find the inverse function.

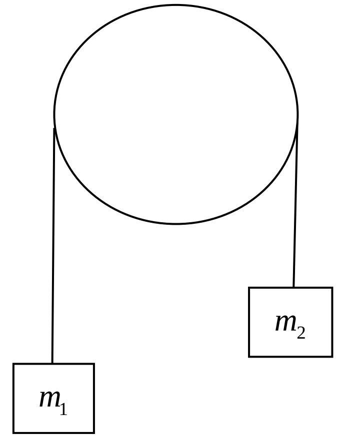
1 mark

c. Hence find the area in the first quadrant bounded by the function f and the x-axis.

2 marks

Question 10 (4 marks)





A particle of mass m_1 kg is on a smooth plane, inclined at an angle of 30^0 to the horizontal. It is connected by a light string which passes around a smooth pulley to another mass of m_2 kg hanging vertically. The mass m_2 moves downwards with an acceleration of $a\,\text{ms}^{-2}$.

In another situation both masses are hanging vertically and connected by a light string which passes around another smooth pulley. In this situation the mass m_2 moves downwards with an acceleration of $\frac{a}{2}\ \text{ms}^{-2}$. Determine the ratio $\frac{m_2}{m_1}$.

EXTRA WORKING SPACE

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a pyramid	$\frac{1}{3}Ah$
area of triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\operatorname{cosec}^2(x)$
$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$	$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$	$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

Circular (trigonometric) functions - continued

Function	$\sin^{-1}$ (arcsin)	$\cos^{-1}$ (arcos)	$\tan^{-1}$ (arctan)
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r\operatorname{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \operatorname{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$
$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$E(aX + b) = aE(X) + b$ $E(aX + bY) = aE(X) + bE(Y)$ $\operatorname{Var}(aX + b) = a^2 \operatorname{Var}(X)$
for independent random variables X and Y	$\operatorname{Var}(aX + bY) = a^2 \operatorname{Var}(X) + b^2 \operatorname{Var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $E(\bar{X}) = \mu$ variance $\operatorname{Var}(\bar{X}) = \frac{\sigma^2}{n}$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r_1} \cdot \underset{\sim}{r_2} = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e\left(x\right)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\lvert x\rvert+c$
$\frac{d}{dx}\left(\sin\left(ax\right)\right)=a\cos\left(ax\right)$	$\int \sin\left(ax\right)dx=-\frac{1}{a}\cos\left(ax\right)+c$
$\frac{d}{dx}\left(\cos\left(ax\right)\right)=-a\sin\left(ax\right)$	$\int \cos\left(ax\right)dx=\frac{1}{a}\sin\left(ax\right)+c$
$\frac{d}{dx}\left(\tan\left(ax\right)\right)=a\sec^2\left(ax\right)$	$\int \sec^2\left(ax\right)dx=\frac{1}{a}\tan\left(ax\right)+c$
$\frac{d}{dx}\left(\sin^{-1}\left(x\right)\right)=\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}\left(x\right)\right)=\frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}\left(x\right)\right)=\frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int\left(ax+b\right)^n dx=\frac{1}{a\left(n+1\right)}\left(ax+b\right)^{n+1}+c,\ n\neq -1$
	$\int\left(ax+b\right)^{-1} dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$
product rule	$\frac{d}{dx}\left(uv\right)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx}=f(x),\ x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf\left(x_n\right)$
acceleration	$a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2}\sqrt{1+\left(f'\left(x\right)\right)^2}\,dx$ or $\int_{t_1}^{t_2}\sqrt{\left(x'\left(t\right)\right)^2+\left(y'\left(t\right)\right)^2}\,dt$

END OF FORMULA SHEET