# **Year 2016**

## VCE

# Specialist Mathematics Trial Examination 1



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• While every care has been taken, no guarantee is given that these questions are free from error. Please contact us if you believe you have found an error.

# Victorian Certificate of Education 2016

#### STUDENT NUMBER

		_				Letter
Figures						
Words						

## **SPECIALIST MATHEMATICS**

## **Trial Written Examination 1**

Reading time: 15 minutes Total writing time: 1 hour

## **QUESTION AND ANSWER BOOK**

#### Structure of book

Number of	Number of questions	Number of
questions	to be answered	marks
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or white out liquid/tape.

#### Materials supplied

• Question and answer book of 18 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.

#### Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Latter

#### **Instructions**

Answer all questions in the spaces provided.

Unless otherwise specified an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the acceleration due to gravity to have magnitude g m/s<sup>2</sup>, where g = 9.8.

Question 1 (3 marks)
Resolve the vector $4\underline{i} - 2\underline{j} - 2\underline{k}$ into two vector components, one which is parallel to the
vector $3\underline{i} - 4\underline{j} + 5\underline{k}$ and one which is perpendicular to it.

**Question 2** (3 marks)

Let  $f(z) = z^4 - 6z^3 + 17z^2 - 30z + 60$ , where  $z \in C$ .

a. Given that  $z = \sqrt{5} \operatorname{cis} \left( -\frac{\pi}{2} \right)$  is a solution of f(z) = 0, find a quadratic factor of f(z).

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**b.** Hence find all the roots of f(z) = 0. 2 marks

<b>Ouestion</b>	3	(3	marks)	١
Ouesnon	J	w	marks	J

Question 5 (5 marks)	2		
Solve the differential equation $\frac{dy}{dx}$	$\frac{y}{x} = \frac{24y^2}{9+4x^2}$	and $y\left(\frac{3}{2}\right) = 1$ , expressing	y in terms of $x$ .

Oucsuon + (3 marks)	Quest	tion -	4 (	5	marks)	)
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A bowl is formed when the region bounded by the graph of  $y = x^2 - 16$ , the x-axis, the y-axis and the line y = 20, is rotated about the y-axis. Units are in centimetres.

Find the capacity of the bowl. 2 marks a. If the bowl is being filled at a rate of 2 cm<sup>3</sup>/sec, find the rate at which the height of b. the fluid in the bowl is rising at, when the height is 14 cm. 3 marks **Question 5** (5 marks)

a. Show that  $\int \frac{\sin(2x)}{\cos^2(2x)} dx = \frac{1}{2} \sec(2x) + c$  1 mark

A particle moves along a curve so that at time  $t \ge 0$  its velocity vector is given by  $y(t) = 4\tan(2t)\sec(2t)\underline{i} + 6\sec^2(2t)\underline{j}$ .

**b.i.** Given that  $\underline{r}(0) = 3\underline{i} + 2\underline{j}$  find the position vector  $\underline{r}(t)$ .

2 marks

ii.	The particle moves along the curve from $t = 0$ to $t = \frac{\pi}{5}$ .	
	This distance can be expressed in terms of $t$ as the definite integral	
	$\int_0^{\frac{\pi}{5}} p \sec(2t) \sqrt{q \tan^2(2t) + r} dt$ . Determine the values of $p, q$ and $r$ .	
	Do not attempt to evaluate the definite integral.	2 marks
Oues	tion 6 (4 marks)	
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#### **Question 7** (4 marks)

A teacher at a school wishes to test if the students in a certain year have above or below						
average intelligence. To do this he takes a random sample of 36 students and finds that						
their mean IQ score is 103. The mean population IQ is 100 with a standard deviation of 15.						
Perform a hypothesis test, at the 95% level of significance, stating the null and alternative						
hypothesis, calculating the test statistic and clearly stating your conclusions.						

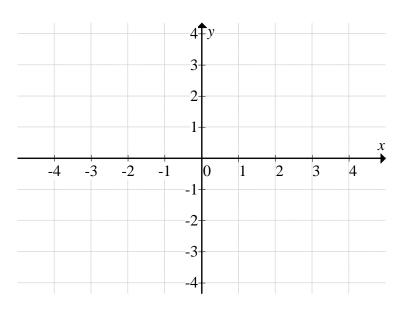
Question 8 (5 marks)						
At a time $t$ seconds, $t \ge 0$ , a body of mass 3 kg is moving in a straight line. Initially the						
body is at rest 4 metres from the origin and is acted upon by a force of $-12x$ newtons,						
where $x$ is its displacement from the origin in metres. Express $x$ in terms of $t$ .						

**Question 9** (4 marks)

Consider the function  $f(x) = 2\arcsin\left(\frac{x}{3}\right)$ .

Sketch the graph of the function on the axes below, clearly indicating the a. co-ordinates of the endpoints.

1 mark

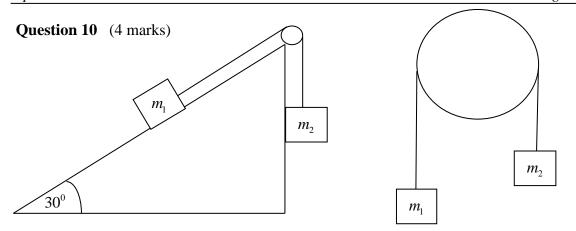


b. Find the inverse function.

1 mark

Hence find the area in the first quadrant bounded by the function f and the x-axis. c.

2 marks



A particle of mass  $m_1$  kg is on a smooth plane, inclined at an angle of  $30^{\circ}$  to the horizontal. It is connected by a light string which passes around a smooth pulley to another mass of  $m_2$  kg hanging vertically. The mass  $m_2$  moves downwards with an acceleration of  $a \, \text{ms}^{-2}$ .

In another situation both masses are hanging vertically and connected by a light string which passes around another smooth pulley. In this situation the mass  $m_2$  moves

downwards with an acceleration of	$\frac{a}{2}$ ms <sup>-2</sup> .	. Determine	the ratio	$\frac{m_2}{m_1}$ .		
			<del></del>			
					-	

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EXTRA WORKING SPACE	

#### **END OF EXAMINATION**

## **SPECIALIST MATHEMATICS**

## Written examination 1

## **FORMULA SHEET**

## **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

## **Specialist Mathematics formulas**

## Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a pyramid	$\frac{1}{3}Ah$
area of triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

## **Circular (trigonometric) functions**

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \csc^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

## Circular (trigonometric) functions - continued

Function	sin <sup>-1</sup> (arcsin)	cos <sup>-1</sup> (arcos)	tan <sup>-1</sup> (arctan)
Domain	[-1, 1]	[-1, 1]	R
Range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

## Algebra ( complex numbers )

$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r\operatorname{cis}(\theta)$	
$ z  = \sqrt{x^2 + y^2} = r$	$-\pi < \operatorname{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2 \operatorname{cis} (\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$
$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)	

## Probability and statistics

for random variables $X$ and $Y$	$E(aX+b) = aE(X)+b$ $E(aX+bY) = aE(X)+bE(Y)$ $Var(aX+b) = a^{2} Var(X)$
for independent random variables <i>X</i> and <i>Y</i>	$\operatorname{Var}(aX + bY) = a^2 \operatorname{Var}(X) + b^2 \operatorname{Var}(Y)$
approximate confidence interval for $\mu$	$\left(\overline{x} - z \frac{s}{\sqrt{n}}, \overline{x} + z \frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\overline{X}$	mean $E(\bar{X}) = \mu$ variance $Var(\bar{X}) = \frac{\sigma^2}{n}$

## Vectors in two and three dimensions

$ \tilde{\mathbf{r}} = x\tilde{\mathbf{i}} + y\tilde{\mathbf{j}} + z\tilde{\mathbf{k}} $	
$\left  \underline{r} \right  = \sqrt{x^2 + y^2 + z^2} = r$	
$\dot{r} = \frac{dr}{dt} = \frac{dx}{dt}\dot{i} + \frac{dy}{dt}\dot{j} + \frac{dz}{dt}\dot{k}$	
$rac{r}{r_1} \cdot r_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$	

## **Mechanics**

momentum	p = mv
equation of motion	R = ma

## Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c , n \neq -1$ $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_{e}(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e  x  + c$
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$ $\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$ $\int \sec^2(ax)dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a\sec^2(ax)$	$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a}\right) + c, \ a > 0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1} \left(\frac{x}{a}\right) + c, \ a > 0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1} \left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c, \ n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a} \log_e  ax+b  + c$
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$ , $x_0 = a$ and $y_0 = b$ , then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1 + (f'(x))^2}  dx  \text{or}  \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2}  dt$

## **END OF FORMULA SHEET**