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Free Exam
for 2006-15 VCE study design

Engage Education Foundation

Units 3 and 4 Specialist Maths: Exam 1

Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 1 hour writing time

Structure of book:

Number of questions	Number of questions to be answered	Number of marks
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers and rulers.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is allowed in this examination.

Materials supplied:

- This question and answer booklet of 10 pages.

Instructions:

- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Questions

Question 1

A random variable X has probability function f given by

$$f(x) = \begin{cases} \frac{3}{16}x^2 & \text{if } -a \le x \le a \\ 0 & \text{otherwise} \end{cases}$$

a. Given that $a \ge 0$, find the exact value of a.

2 marks

b. A sample of size 100 is taken from the distribution. Describe the distribution of the sample mean $\bar{X}$.

2 marks

Question 2

A normally distributed population of mice is predicted to have a mean weight of 200 g. the height variance is **known** to be 196 g. To test this prediction, a random sample of 100 mice from the population is obtained. Their sample mean is found to be 197 g.

Calculate the 95% confidence interval for the population mean, correct to the nearest gram. **Hence,** explain the confidence interval's implication and state whether the prediction of 200 g should be rejected under the significance level of 0.05.

3 marks

Question 3

Let $x^2y + \log_e y + x = 2$. Find $\frac{dy}{dx}$.

2 marks

Question 4
Solve $P(z) = z^2 + 5z + 8 = 0$ for $z \in \mathbb{C}$.

3 marks

Question 5
Find the area enclosed by the graph $\frac{7x+1}{x^2+2x-8}$ and the lines $x = -2$ and $x = 0$.

4 marks

Question 6

Let $x = sec^2(t)$ and $y = \tan(t)$ for $t \in \{t: 0 \leq t \leq \frac{\pi}{4}\}$

a. Give y as a function of x.

2 marks

b. State the domain and range of this function.

2 marks

c. Graph y in terms of x in the space provided below, giving the coordinates and t values of any intercepts and endpoints.

3 marks

Total: 7 marks

Question 7

Show that $cosec\left(\frac{5\pi}{12}\right) = \sqrt{6} - \sqrt{2}$.

3 marks

Question 8

Evaluate $\int \frac{\tan^{-1}\left(\frac{x}{5}\right)}{25+x^2}\,dx$.

3 marks

Question 9

Find the exact value of $\int_0^{\pi} cos^2(x)sin^2(x)dx$.

4 marks

Question 10

A particle's displacement is given by the vector function $\boldsymbol{r}(t) = t^3\boldsymbol{i} + \log_e t\,\boldsymbol{j}$, where $t > 0$.

a. Find a Cartesian equation for the particle's displacement.

1 mark

b. Find an equation for the speed of the particle at time *t.*

2 marks

c. At what time(s) is this speed a minimum?

4 marks

Total: 7 marks

End of Booklet

Formula sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin A$
sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

hyperbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$

$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	$\mathbb{R}$
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)

$|z| = \sqrt{x^2 + y^2} = r$

$-\pi < \text{Arg}\,z \le \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\frac{d}{dx}(e^{ax}) = ae^{ax}$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}(\log_e x) = \frac{1}{x}$

$\int \frac{1}{x} dx = \log_e|x| + c$

$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$

$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$

$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$

$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$

$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}$

$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$

$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)}{v^2}$

chain rule $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method If $\frac{dy}{dx} = f(x), x_0 = a$ and $y_0 = a$, then $y_{n+1} = y_n + hf(x_n)$

acceleration $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration $v = u + at$, $s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$, $s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$	$\boldsymbol{r} = \frac{d\boldsymbol{r}}{dt} = \frac{dx}{dt}\boldsymbol{i} + \frac{dy}{dt}\boldsymbol{j} + \frac{dz}{dt}\boldsymbol{k}$
$\lvert\boldsymbol{r}\rvert = \sqrt{x^2 + y^2 + z^2} = r$	$\boldsymbol{r}_1.\boldsymbol{r}_2 = r_1 r_2 \cos\theta = x_1x_2 + y_1y_2 + z_1z_2$

Mechanics

momentum	$\boldsymbol{p} = m\boldsymbol{v}$
equation of motion	$\boldsymbol{R} = m\boldsymbol{a}$

Probability and statistics

for random variables X and Y	$E(aX + b) = aE(X) + b$ $E(aX + bY) = aE(x) + bE(Y)$ $var(aX + b) = a^2vat(X)$
for independent random variables X and Y	$var(aX + bY) = a^2var(X) + b^2var(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	Mean $E(\bar{X}) = \mu$ Variance $var(\bar{X}) = \frac{\sigma^2}{n}$