2015 VCE Specialist Mathematics exam 1 examination report

General comments

In the 2015 Specialist Mathematics examination 1, students were required to answer nine short-answer questions worth a total of 40 marks. Students were not permitted to bring any calculators or notes into the examination.

In the comments on specific questions in the next section, many common errors are highlighted. These should be brought to the attention of students so that they can develop strategies to avoid them. A particular concern is the need for students to read the questions carefully as responses to several questions indicated that students had not done so.

Another concern was the clarity of students' answers and the manner in which they set out their mathematics. Students should be reminded that if an assessor is not certain as to what a student's answer is conveying, that assessor cannot award marks. Furthermore, if an assessor is unable to follow a student's working (or reasoning), full marks will not be awarded. Working should not appear to be a number of disjointed statements. If there are inconsistencies in the student's working, full marks will not be awarded. For example, if an equals sign is placed between quantities that are not equal, full marks will not be awarded.

Students are reminded that they should sketch graphs with care and include details such as a reasonable scale, correct domain, asymptotes and asymptotic behaviour. Smoothly drawn curves are expected.

Areas of weakness included:

- failing to read the question carefully – this included not answering the question, proceeding further than required or not giving the answer in the specified form. These were common and particularly evident in Questions 3, 4a., 4b., 8bi., 9b. and 9c. Students should be reminded that good examination technique includes re-reading the question after it has been answered to ensure that they have responded to what was required and that they have given their answer in the correct form
- algebraic skills. Difficulty with algebra was evident in several questions. The inability to simplify expressions often prevented students from completing the question. Incorrect attempts to factorise, expand and simplify were common. Poor use of brackets was also common
- arithmetic skills. Difficulty with arithmetic was evident in several questions. The inability to evaluate expressions, especially those involving fractions or surds, was common
- notation, especially the omission of the *dx* or equivalent in integration
- showing a given result. This was required in Questions 1b. and 8a. In such questions, the onus is on students to include sufficient relevant working to demonstrate that they know how to derive the result. Students should be reminded that they can use a given value in the remaining part(s) of the question whether or not they were able to derive it
- recognising the need to use the chain rule when differentiating implicitly (Question 9a.)
- recognising the need to use the product rule when differentiating implicitly (Question 9a.)

- recognising the method of integration required (Questions 5, 6 and 8a.)
- knowing the exact values for circular functions (Questions 4, 7a., 8bi. and 9c.)

In this examination, students are expected to be able to apply techniques, routines and processes, involving rational, real and complex arithmetic, without the use of technology. Students are expected to be able to simplify simple arithmetic expressions. Many students found this difficult and missed out on marks as a consequence.

For a number of questions, students who drew carefully labelled diagrams and graphs were distinctly advantaged over those who did not.

Many students made algebraic or numerical slips at the end of an answer, which meant that the final mark could not be awarded. This was especially unfortunate when they had a correct answer and there was no need for further simplification.

Students should be reminded that the final answer must be supported by relevant and correct working.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Question 1a.

Marks	0	1	Average
%	29	71	0.7

$a = \sqrt{3}$

This question was answered reasonably well by students. The most common errors involved finding the magnitude to be $a = 1$ or $a = \pm\sqrt{3}$ (even though the question stated that a was positive). Another common error was $a = 3$. A small number of students gave the answer $a = 0$.

Question 1b.

Marks	0	1	2	Average
%	25	15	59	1.4

Students needed to show the given result, using $\overrightarrow{OB}.\overrightarrow{CA} = 2 - 1 - 1 = 0$.

This question was quite well answered. The main errors were sign errors in finding the diagonal vectors and sign errors in the dot product. A few students found $\overrightarrow{OM}$ and $\overrightarrow{AM}$ where M is the point of intersection of the diagonals. Some students proved that the diagonals in a rhombus intersect at right angles using general vector methods. Brackets were often omitted and the notation used with vectors was often poor. There were some unconvincing arguments, often due to insufficient steps shown.

Question 2a.

Marks	0	1	2	Average
%	25	17	58	1.4

$N = 220$

Most students made a reasonable attempt at an equation of motion. Students who used a force diagram generally dealt well with this question. Typical mistakes involved sign errors with a conflict in the direction assigned to be positive. Many students made arithmetical errors when simplifying, and some did not attempt to simplify. In some cases, m was missing from the ma term or ma was replaced by mga. A small number of students used $g = 10$.

Question 2b.

Marks	0	1	2	Average
%	25	18	58	1.4

$a = 1.5$

Most students made a reasonable attempt at an equation of motion, but there were numerous sign errors with a conflict in the direction assigned to be positive. Many students made arithmetical errors when simplifying. The most common of these yielded $a = \frac{2}{3}$. In some cases, m was missing from the ma term or ma was replaced by mga. The most common incorrect answer was $a = -1.5$, without any indication that the acceleration was downwards.

Question 3

Marks	0	1	2	3	4	Average
%	5	5	23	14	53	3.1

13

Most students handled this question very well. Common errors included finding an incorrect antiderivative vector, forgetting to include a constant (vector) of integration, finding the incorrect constant of integration, or a sign error in consolidating the vector answer. A large proportion of these errors were caused by incorrect use (or lack) of brackets. Some students added $5t$ to 2 and got $7t$. Several students left the displacement vector as the answer, and many made arithmetic errors in calculating the modulus of the vector. Some assumed that the distance from the origin was given by the modulus of $\underset{\sim}{r}(2) - \underset{\sim}{r}(0)$. Some were unable to recognise that $\sqrt{169} = 13$.

Question 4a.

Marks	0	1	2	3	Average
%	19	23	18	40	1.8

$-2i, \sqrt{3} + i, -\sqrt{3} + i$

This question was quite well answered by students who used polar form, but not by the small number of students who tried to solve the equation in cartesian form. The most common errors included finding the incorrect polar form for $8i$ or finding the correct polar form for $8i$ but making errors in finding the other two solutions. Some students who found the correct solutions in polar form either left them in polar form or converted them to cartesian form with arithmetical errors. Many students assumed that the Conjugate Root Theorem applied. Others tried to use the formula for perfect cubes. Some gave factors rather than solutions, and a number of students gave only one solution for this cubic.

Question 4b.

Marks	0	1	Average
%	56	44	0.5

$0, \sqrt{3}+3i, -\sqrt{3}+3i$

Students were expected to recognise that the solutions to Question 4a. needed to be translated two units up, and so add 2*i*. Several students subtracted 2*i* from the answers in part a., and a small number tried to solve the equation without using their answer to part a.

Question 5

Marks	0	1	2	3	Average
%	24	30	9	37	1.6

16π

This question divided the cohort, with several students answering it very well but others having some difficulty. Typical errors included finding an area rather than a volume and rotating about the incorrect axis. Several students who rotated about the correct axis integrated from 0 to 5 rather than –3 to 5. A number of students made mistakes when trying to put x^2 in terms of y. Many students made arithmetic and transcription errors in their calculation of the integrand. Others rotated about the wrong axis. Some omitted π or included 2π instead of π. There were many arithmetical slips in the final substitution and evaluation.

Question 6

Marks	0	1	2	3	4	Average
%	15	11	8	6	59	2.8

e^5

The majority of students understood the need to use the relevant form for the acceleration a. Some used an incorrect form. Quite a few integrated $\frac{1}{4v}$ to get $\log_e(4v)$, omitting the coefficient of $\frac{1}{4}$. A relatively small number of students forgot the constant of integration, while others made arithmetical slips in finding the constant of integration or a simplification slip in working with it. Some used modulus at the integration step and then gave the final answer as $\pm e^5$. A few were unable to simplify $\sqrt{e^{10}}$. Others integrated with respect to v to obtain a cubic.

Question 7a.

Marks	0	1	2	3	Average
%	7	45	14	34	1.8

$x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

This question divided the cohort, with some students answering it very well but a number having difficulty. The most common error was to use the double-angle formula for $\sin(2x)$ but then cancel the $\sin(x)$ term from both sides, which many students did, thereby losing a set of solutions. Some tried to use a graphical approach and missed solutions. Others solved the two equations they generated but got only two solutions for $\sin(x) = 0$ (usually 0 and π but occasionally 0 and 2π). Some were not able to solve $2\cos(x) = 1$. Errors were made with exact values.

Question 7b.

Marks	0	1	2	Average
%	59	30	11	0.5

$x \in \left(0, \frac{\pi}{3}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

High-scoring students used a graphical argument for this question. Typical errors included incorrect simplification with inequalities (multiplying by a term that could be negative but not changing the inequality), choosing the incorrect interval in the first quadrant – i.e. $\left(\frac{\pi}{3}, \pi\right)$ – including endpoint(s) and giving single value answer(s) rather than intervals. A common incorrect response was $x < \frac{\pi}{3}$.

Question 8a.

Marks	0	1	2	Average
%	44	9	47	1.1

Students needed to show the given result.

This question was answered reasonably well. There were many instances of poor choices of substitution, such as $u = \sin(2x)$, $u = \tan(2x)$, $u = \sec(2x)$ or $u = \cos(x)$ (after the use of double-angle formulas) rather than $u = \cos(2x)$. These attempts led to a more complicated solution and were rarely successful. Some students who used the correct substitution then made sign or arithmetical errors or did not use a modulus sign at the integration stage (although it often appeared at the end). Some attempted to use the tan double-angle formula, but this was rarely successful. A few students used differentiation as their method, but only a small number could correctly obtain the derivative of the right-hand scale. There were some unconvincing arguments, often due to insufficient steps shown.

Question 8bi.

Marks	0	1	Average
%	28	72	0.7

$y = \pm\frac{\pi}{4}$

Most students answered this question correctly. Some gave $y=\pm\frac{\pi}{2}$ or $y=\pm\pi$. Others gave $\pm\frac{\pi}{4}$ rather than equations.

Question 8bii.

Marks	0	1	Average
%	46	54	0.6

Students were expected to sketch tan graph with $y=\tan(2x),\ x\in\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$ with asymptotes $x=\pm\frac{\pi}{4}$.

Students were expected to be able to reflect the given graph in the line $y=x$, or find the equation $y=\tan(2x)$ and sketch that directly. Typical errors included poor attempts at the shape of the inverse (sometimes graphed as $y=-\tan(2x)$), poor positioning of the vertical asymptotes, and either not labelling or incorrect labelling of the vertical asymptotes – for example, $y=\pm\frac{\pi}{4}$ – and drawing the graph beyond its domain. Asymptotic behaviour was lacking in some of the attempts, with curves sometimes moving away from the asymptotes.

Question 8c.

Marks	0	1	Average
%	14	86	0.9

$\frac{\pi}{6}$

This question was well answered. The main errors seen were $\frac{\pi}{3}$ and $\frac{\pi}{12}$, with some students not knowing the exact values.

Question 8d.

Marks	0	1	2	Average
%	26	52	23	1

$\frac{\sqrt{3}\pi}{6}-\frac{1}{2}\log_e 2$

Only a small proportion of students answered this question correctly. Some gave the correct expression for the area in terms of arctan, but no progress or poor attempts at integration (usually where the supposed antiderivative was actually the derivative) was made. Many gave an incorrect (incomplete) expression for the area in terms of the inverse (omitting $\frac{\sqrt{3}\pi}{6}$) or incorrect terminals. Some students tried to integrate $\tan(2y)$ rather than using the information contained in part a. but were usually unsuccessful. Several of those who used the correct antiderivative of $\tan(2y)$ using part a., made subsequent substitution errors. A number of students found the wrong area and obtained $\frac{1}{2}\log_e 2$. Using a diagram would have been helpful for many students.

Question 9a.

Marks	0	1	2	Average
%	14	16	70	1.6

$\frac{dy}{dx} = \frac{y - 2x}{3y - x}$

This question was answered well. Most students correctly used the product and chain rules. A number of sign errors appeared on the left-hand scale, while some left the right-hand side as 9 after differentiating the left-hand scale. Algebraic simplification errors were common.

Question 9b.

Marks	0	1	2	Average
%	18	20	62	1.5

$y = 2x - 6,\ y = \frac{x}{3} + \sqrt{6}$

This question was answered well. The most common errors were finding only one equation and arithmetic errors in converting one or both equations to the required form.

Question 9c.

Marks	0	1	2	Average
%	77	6	16	0.4

$\frac{\pi}{4}$

Most students had little idea of how to proceed with this question. Students were expected to apply $\tan(A - B)$ or use a vector method, but other methods were possible. Some correctly applied $\tan(A - B)$ but made simplification errors. There were also some poor attempts to use a vector method. In attempting to convert to vectors, sometimes equations such as $y = mx + c$ became $m\underset{\sim}{i} + c\underset{\sim}{j}$. Some found the intersection point of the tangents but were unable to progress from there.

A small number attempted to use the cosine rule, with a few of these being successful.