TSSM™
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'2016 Examination Package' -
Trial Examination 5 of 5

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STUDENT NUMBER | | | | | | | | | Letter |

SPECIALIST MATHEMATICS

Written examination 2

*(*TSSM's *2015 trial exam updated for the current study design)*

Reading time: 15 minutes
Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 24 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION 1 - Multiple Choice Questions

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where g = 9.8.

Question 1

The equations of the asymptotes of the hyperbola given by the parametric equations

$$x = 5 - 6\sec(2t)\,, y = 3 + 5\tan(2t)$$

are

A. $y = \frac{5}{6}x - \frac{7}{6}$ and $y = -\frac{5}{6}x + \frac{43}{6}$

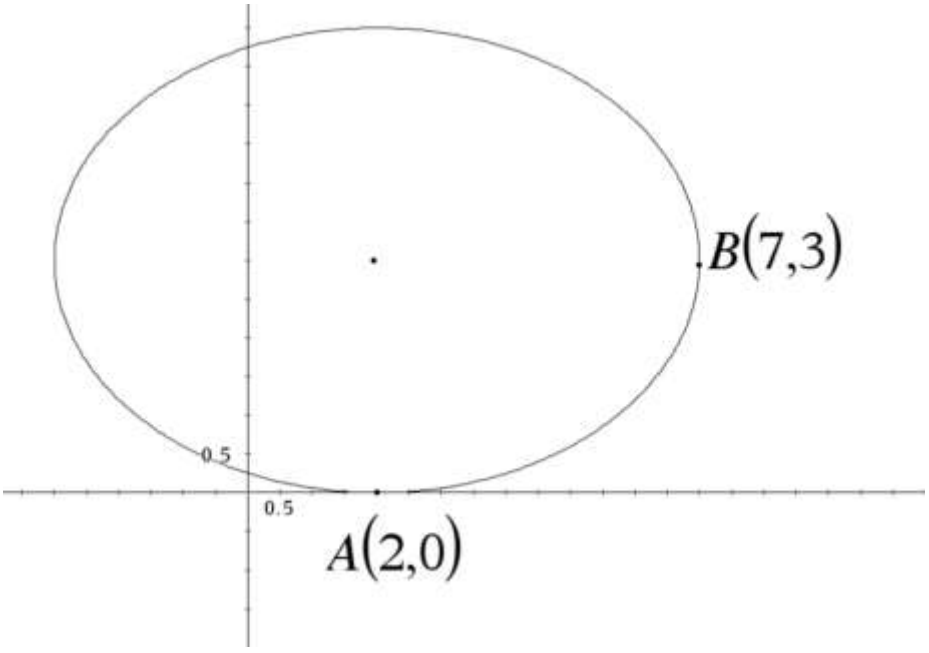
B. $y = \frac{5}{6}x$ and $y = -\frac{5}{6}x$

C. $y = \frac{5}{6}x - \frac{7}{6}$ only

D. $y = -\frac{5}{6}x - \frac{7}{6}$ and $y = \frac{5}{6}x + \frac{43}{6}$

E. $y - 3 = \pm\frac{6}{5}(x - 5)$

Question 2



SECTION 1 - continued

In the graph of an ellipse shown above, two vertices A(2, 0) and B(7, 3) are labelled. Then the equation of the ellipse is

A. $\frac{(x+2)^2}{25} - \frac{(y-3)^2}{9} = 1$

B. $\frac{(x-2)^2}{25} + \frac{(y+3)^2}{9} = 1$

C. $\frac{(x-2)^2}{25} - \frac{(y+3)^2}{9} = 1$

D. $\frac{(x+2)^2}{25} + \frac{(y-3)^2}{9} = 1$

E. $\frac{(x-2)^2}{25} + \frac{(y-3)^2}{9} = 1$

Question 3

The graph of the function $f(x) = \frac{2x^2+3x+7}{x^2+2x+2}$ has

A. a horizontal asymptote at $y = 2$ and a minimal point at $x = -1$.

B. a horizontal asymptote at $y = 2$ and a maximal point at $x = -1$.

C. a vertical asymptote at $x = -1$ and a maximal point at $x = -1$.

D. a y-intercept at $(0, \frac{7}{3})$ and a x-intercept at $x = 3$.

E. no intersection with the horizontal asymptote $y = 2$.

Question 4

The range of the function $f(x) = 4\arctan(2 - 3x) + 5$ is

A. $\left(-\frac{\pi}{2},\ \frac{\pi}{2}\right)$

B. $(-\infty,\ \infty)$

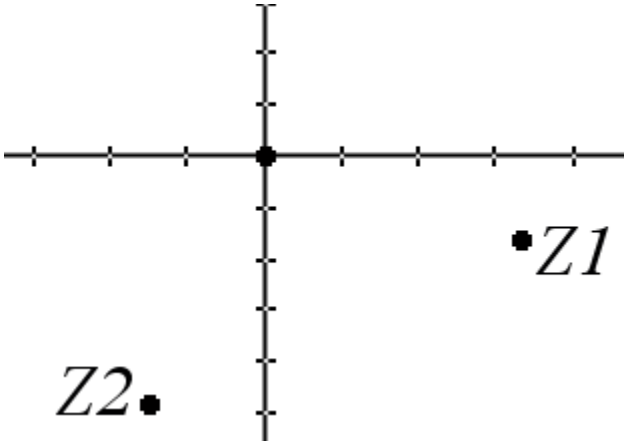
C. $\left(2 - \frac{3\pi}{2}, 2 + \frac{3\pi}{2}\right)$

D. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

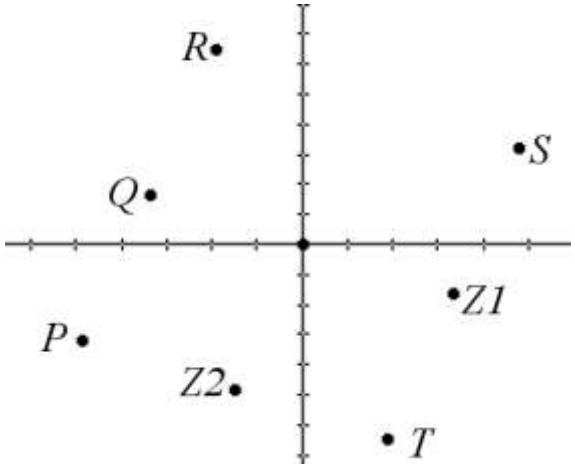
E. $(5 - 2\pi,\ 5 + 2\pi)$

Question 5

Two complex numbers z_1, z_2 are shown in the axes below.



Then the point representing the correct position of $z = z_1 - z_2$ in the diagram shown below is



A. P

B. Q

C. R

D. S

E. T

SECTION 1 - continued

Question 6

The shape of the graph of the locus represented by $|z - z_1| = |z - z_2|$ is

A. an ellipse

B. a hyperbola

C. a parabola

D. a circle

E. a straight line

Question 7

Let $z = -5\sqrt{3} - 5i$.

Then the magnitude and principal argument of z^{10} are respectively

A. 10^{10} and $\left(-\frac{5\pi}{6}\right)^{10}$

B. 10^{10} and $-\frac{\pi}{3}$

C. 10^{10} and $-\frac{25\pi}{3}$

D. 100 and $-\frac{\pi}{3}$

E. 5^{10} and $-\frac{\pi}{3}$

Question 8

The complex number, $z = 4cis(72°) \cdot z_1$, can be obtained from z_1 by

A. a rotation of 72° around the origin in clockwise.

B. a rotation of 72° around the origin in anti-clockwise.

C. a rotation of 72° around the origin in anti-clockwise, followed by a dilation of factor 4 from the origin.

D. a rotation of 72° around the origin in clockwise, followed by a dilation of factor 4 from the origin.

E. a reflection about the origin.

Question 9

u and v are two complex numbers with $u + v = \frac{3i}{5}$, $u \times v = \frac{7}{10}$. Then u and v must be the two roots of the equation

A. $10z^2 - 6iz + 7 = 0$

B. $\left(z - \frac{3i}{5}\right)\left(z - \frac{7}{10}\right) = 0$

C. $z^2 + \frac{3i}{5}z + \frac{7}{10} = 0$

D. $10z^2 + 6iz + 7 = 0$

E. $10z^2 - 6iz - 7 = 0$

Question 10

By an appropriate substitution, the definite integral

$$\int_{\log_e(\frac{\pi}{6})}^{\log_e(\frac{\pi}{2})} \frac{e^x}{1 + e^{2x}}\, dx$$

is equivalent to

A. $\int_{\log_e(\frac{\pi}{6})}^{\log_e(\frac{\pi}{2})} \frac{u}{1+u^2}\, du$

B. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{1+u^2}\, du$

C. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{u}{1+u^2}\, du$

D. $\int_{\log_e(\frac{\pi}{6})}^{\log_e(\frac{\pi}{2})} \frac{1}{1+u^2}\, du$

E. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{u}{1+u}\, du$

SECTION 1- continued

Question 11

$G(x)$ is a differentiable function over real numbers. $g(x)$ is the derivative function of $G(x)$ and $\int_1^5 g(x)\,dx = 8$. (1, 2) is a point on the graph of $G(x)$. Then the value of $G(5)$ is

A. 5

B. 2

C. 8

D. 1

E. 10

Question 12

A type of chemical solution with concentration 12g/L flows into a cylindrical tank at a rate of 2L per minute. This tank initially has 85 litres of the same type of solution. At the same time the solution flows out of the tank at a rate of 1.5 L per minute. Let x g be the amount of chemical in the tank after t minutes. Then a correct differential equation regarding x is

A. $\frac{dx}{dt} = 24 - \frac{3x}{170-t}$

B. $\frac{dx}{dt} = 24 - \frac{1.5x}{85}$

C. $\frac{dx}{dt} = 24 + \frac{1.5x}{85+0.5t}$

D. $\frac{dx}{dt} = 24 - \frac{3x}{170+4t}$

E. $\frac{dx}{dt} = 24 - \frac{3x}{170+t}$

Question 13

Let $\frac{dy}{dx} = e^{\frac{xy}{100}}$ and $y = 6$ when $x = 2$.

Using Euler's method with a step size of 0.2, the approximate value of y when $x = 2.4$ is

A. 1.1676

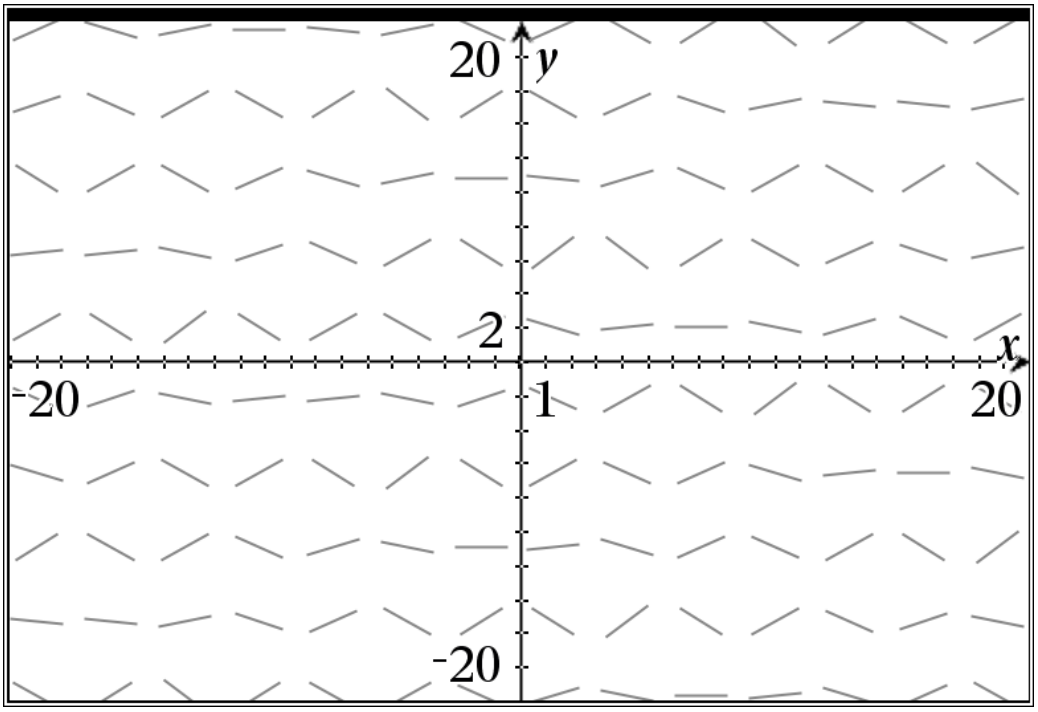
B. 1.1468

C. 6.4549

D. 6.2255

E. 6

Question 14



The differential equation that best represents the above direction field is

A. $\frac{dy}{dx} = \sin(x + y)$

B. $\frac{dy}{dx} = \sin(x - y)$

C. $\frac{dy}{dx} = \tan(x + y)$

D. $\frac{dy}{dx} = \cos(x - y)$

E. $\frac{dy}{dx} = \tan(x + y)$

Question 15

Three vectors given by $\underset{\sim}{u} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$, $\underset{\sim}{v} = 2\underset{\sim}{i} + a\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{w} = 5\underset{\sim}{i} + 5\underset{\sim}{j} + a\underset{\sim}{k}$ are linearly dependent. Then the possible values of a are

A. 1, 4

B. 2, 5

C. 3, 6

D. 0, 3

E. $-1,\ 2$

SECTION 1 - continued

Question 16

If θ is the angle between $\underset{\sim}{a} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{j} - \sqrt{11}\,\underset{\sim}{k}$ then $\tan(2\theta)$ is

A. $\frac{5}{13}$

B. $\frac{9}{40}$

C. $-\frac{120}{119}$

D. $\frac{40}{169}$

E. $\frac{120}{169}$

Question 17

The velocity of a particle is $\underset{\sim}{v} = \frac{e^t+e^{-t}}{2}\underset{\sim}{i} + \frac{e^t-e^{-t}}{2}\underset{\sim}{j} + 2t\underset{\sim}{k}$. The initial position of the particle is $\underset{\sim}{r}(0) = 2\underset{\sim}{j} + 2\underset{\sim}{k}$. Then the position of the particle at time t is

A. $\frac{e^t-e^{-t}}{2}\underset{\sim}{i} + \frac{\left(e^{\frac{t}{2}}+e^{-\frac{t}{2}}\right)^2}{2}\underset{\sim}{j} + (t^2+2)\underset{\sim}{k}$

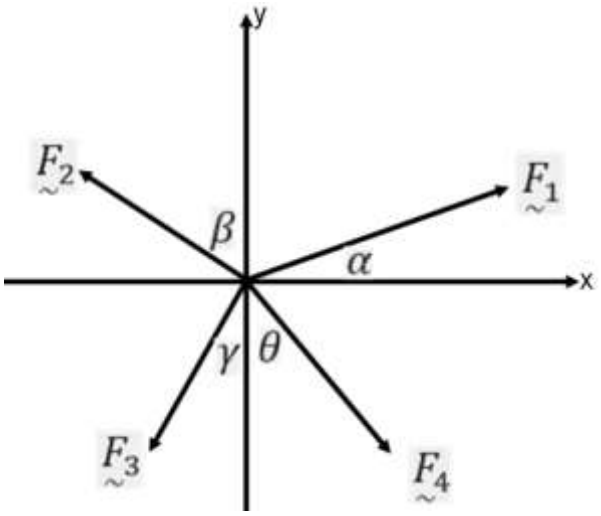
B. $\frac{e^t-e^{-t}}{2}\underset{\sim}{i} + \frac{e^t+e^{-t}}{2}\underset{\sim}{j} + t^2\underset{\sim}{k}$

C. $\frac{e^t-e^{-t}}{2}\underset{\sim}{i} + \frac{e^t+e^{-t}}{2}\underset{\sim}{j} + 2\underset{\sim}{k}$

D. $\frac{e^t-e^{-t}}{2}\underset{\sim}{i} + \frac{e^t+e^{-t}}{2}\underset{\sim}{j} + (t^2+2)\underset{\sim}{k}$

E. $\frac{e^t+e^{-t}}{2}\underset{\sim}{i} + \frac{e^t-e^{-t}}{2}\underset{\sim}{j}$

Question 18



A body in a smooth plane is acted upon by four forces $\underset{\sim}{F}_1, \underset{\sim}{F}_2, \underset{\sim}{F}_3$ and $\underset{\sim}{F}_4$ as shown above. Then the resultant force acting upon the body is

A. $\underset{\sim}{F}_1 \cos(\alpha) + \underset{\sim}{F}_4 \sin(\theta) - \underset{\sim}{F}_2 \cos(\beta) - \underset{\sim}{F}_3 \sin(\gamma)$

B. $\underset{\sim}{F}_1 \sin(\alpha) + \underset{\sim}{F}_4 \cos(\theta) - \underset{\sim}{F}_2 \sin(\beta) - \underset{\sim}{F}_3 \cos(\gamma)$

C. $\underset{\sim}{F}_1 \cos(\alpha) + \underset{\sim}{F}_4 \sin(\theta) + \underset{\sim}{F}_2 \cos(\beta) + \underset{\sim}{F}_3 \sin(\gamma)$

D. $\underset{\sim}{F}_1 + \underset{\sim}{F}_4 - \underset{\sim}{F}_2 - \underset{\sim}{F}_3$

E. $\underset{\sim}{F}_1 + \underset{\sim}{F}_4 + \underset{\sim}{F}_2 + \underset{\sim}{F}_3$

Question 19

The position vector, in metres of a particle is given by

$$\underset{\sim}{r}(t) = (8\sin(3t) + 2)\underset{\sim}{i} + (5 - 15\cos(3t)))\underset{\sim}{j},\ t \geq 0.$$

Then the minimum speed of the particle is

A. 289

B. 51

C. 24

D. 66

E. 29

Question 20

A body of 0.5kg was thrown up from a point at a height of 10 m from the ground at a speed of 2 m/s. Neglecting the effect of air resistance on the particle, then the speed of the particle when hitting the ground is

A. $8\sqrt{3}$

B. $10\sqrt{2}$

C. 196

D. 192

E. 0

Question 21

The acceleration, in m/s^2, of a particle is given by $a = ve^{\frac{x^2}{100}}$, where v and x are the velocity and displacement t seconds after $t = 0$. $v = 3\ m$/s when $x = 1$. Then the velocity when $x = 5$ is approximately

A. 5.45 m/s

B. 7.45 m/s

C. 4.45 m/s

D. 4.12 m/s

E. 14.70 m/s

Question 22

The velocity of a particle travelling on a straight line is given by

$$v(t) = t^3 - 4t^2 - 4t + 16, t \geq 0.$$

Then the displacement of the particle from the initial position when $t = 6$ is

A. $\frac{220}{3}\ m$

B. 76 m

C. 64 m

D. 60 m

E. 48 m

END OF SECTION 1
TURN OVER

SECTION 2- Extended Response questions

Instructions for Section 2

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where g = 9.8.

Question 1 (12 marks)

A curve is defined by the parametric equations

$$x = 2 + 3\tan(t), \qquad y = 4\sec(t), \qquad t \in [0, 2\pi]$$

a. Find the Cartesian equation of the curve.

2 marks

b. State the equations of the asymptotes.

2 marks

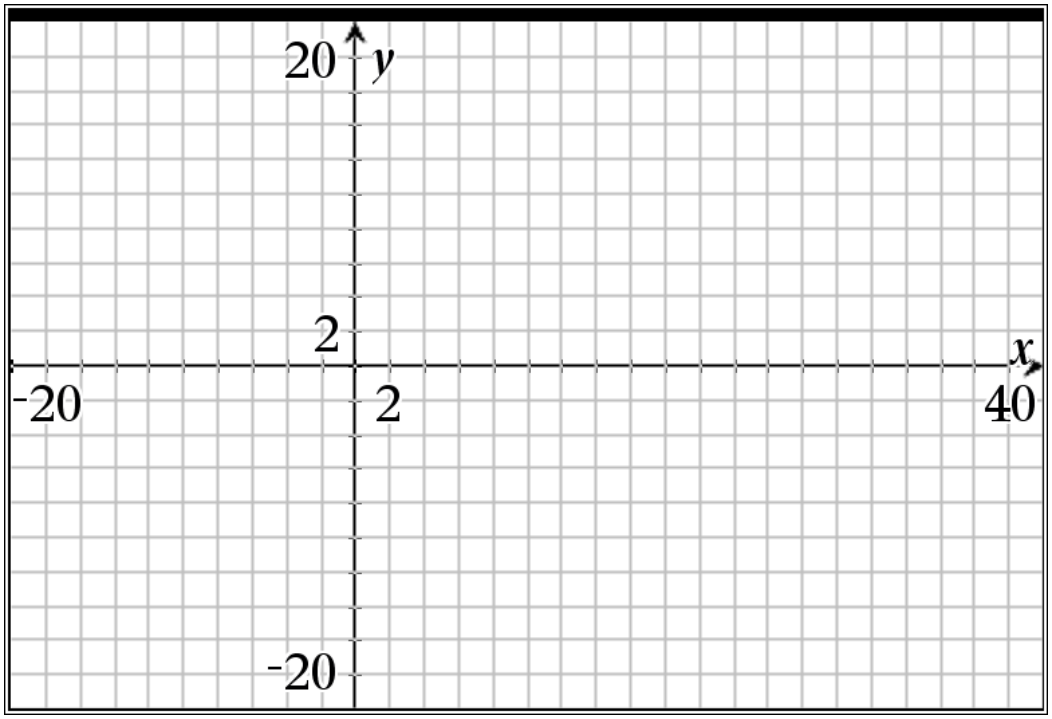
c. Find the equations of the tangent at $(6, \frac{20}{3})$

2 marks

SECTION 2- Question 1- continued

d. Sketch the graphs of the curve, showing the asymptotes, the stationary points and the tangents found in Part c.

4 marks



A solid revolution is formed by rotating the region bounded by the x-axis and the curve for $x \in [0, 4]$ about the x-axis.

e. Write down a definite integral in terms of x that gives the volume of the solid of revolution.

1 mark

f. Find the volume of this solid, correct your answer to two decimal places.

1 mark

SECTION 2- continued

TURN OVER

Question 2 (10 marks)

a. Show that $z = 2\sqrt{2} + 2\sqrt{2}i$ is a solution of the equation $z^4 = -256$.

2 marks

b. Find the other solutions of $z^4 = -256$ in the form of $z = rcis(\theta)$.

2 marks

c. Find the Cartesian equation of the locus $|z - 2\sqrt{2} - 2\sqrt{2}i| = |z + 2\sqrt{2} + 2\sqrt{2}i|$

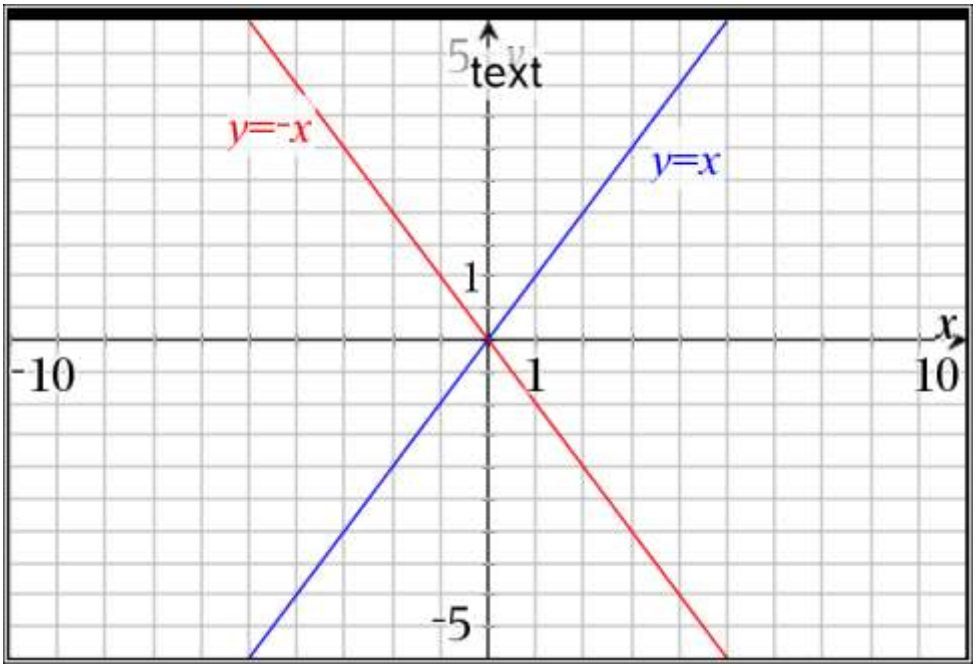
2 marks

SECTION 2- Question 2- continued

d. Sketch the graphs of the sets of points in the set of axes below, where two axes of symmetry are shown.

$$S = \{z \in C, |z| = |2\sqrt{2} + 2\sqrt{2}i|\}$$

1 mark



e.

i. Show that $z = 2\sqrt{2} + 2\sqrt{2}i$ is a point on the ellipse represented by the equation

$$\left|z - \frac{8\sqrt{42}}{7}\right| + \left|z + \frac{8\sqrt{42}}{7}\right| = 16$$

SECTION 2- Question 2- continued

TURN OVER

ii. Hence find the Cartesian form of the equation of the locus in part i.

1 + 2 = 3 marks

SECTION 2- continued

Question 3 (12 marks)

Let $\overrightarrow{OA} = \underset{\sim}{i}, \overrightarrow{OB} = \underset{\sim}{j}$. M is the middle point between O and A.

a. Find $|\overrightarrow{MB}|$.

1 mark

b. $\overrightarrow{ON}$ is in the opposite direction to $\overrightarrow{OA}$ with $\left|\overrightarrow{MN}\right| = |\overrightarrow{MB}|$. Find an expression for $\overrightarrow{ON}$ in the form $\overrightarrow{ON} = a\underset{\sim}{i}$.

2 marks

c. Find the magnitude of $\overrightarrow{BN}$.

1 mark

SECTION 2- Question 3- continued

TURN OVER

Let $\overrightarrow{OC} = -\frac{\sqrt{10+2\sqrt{5}}}{4}\underset{\sim}{i} + \frac{\sqrt{5}-1}{4}\underset{\sim}{j}$.

d. Show that the angle between $\overrightarrow{OB}$ and $\overrightarrow{OC}$ is $\frac{2\pi}{5}$, given that $\cos(\frac{\pi}{5}) = \frac{\sqrt{5}+1}{4}$

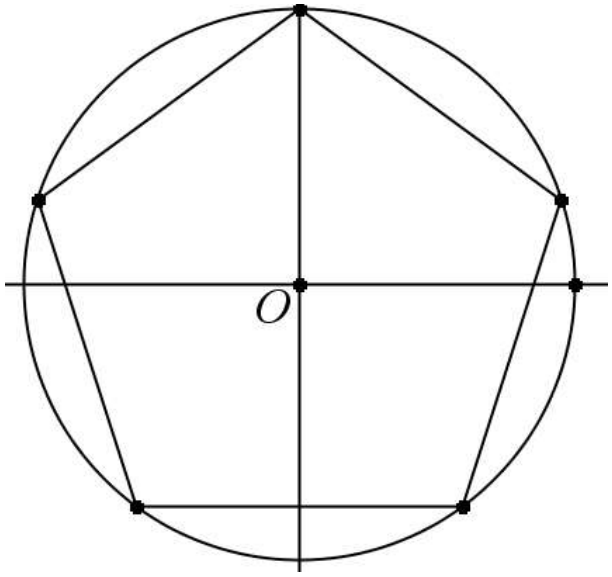
3 marks

e. Show that $\left|\overrightarrow{BC}\right| = |\overrightarrow{BN}|$.

3 marks

SECTION 2- Question 3- continued

f. A regular pentagon is inscribed in a unit circle, shown below. Find the length of the sides of the pentagon.

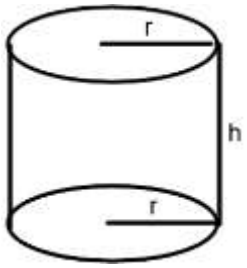


2 marks

SECTION 2- continued

TURN OVER

Question 4 (9 marks)



In a factory a cylindrical container is made by expanding a cylindrical model. The radius r of the base is increasing at a rate of $\frac{r+2}{r+1}$ cm/s and the height h is increasing at a rate of $\frac{h}{1+h}$ cm/s.

a. Find the rate of change of the height h relative to the radius r, in terms of h and r.

2 marks

b. Find the rate which the volume of the container is increasing at per second when $h = 8$ cm and $r = 5$cm.

3 marks

SECTION 2- Question 4- continued

When the container is completed, it is filled with 400L of fresh water. A salt solution with concentration of 40g/L flows into this container at a rate of 8L per minute and the solution in the container is stirred constantly. The mixed solution flows out from a hole on the bottom at the same rate. Let x be the quantity in grams of the salt in the container after t minutes.

c. Write down a differential equation representing the change of the quantity of the salt in the container.

2 marks

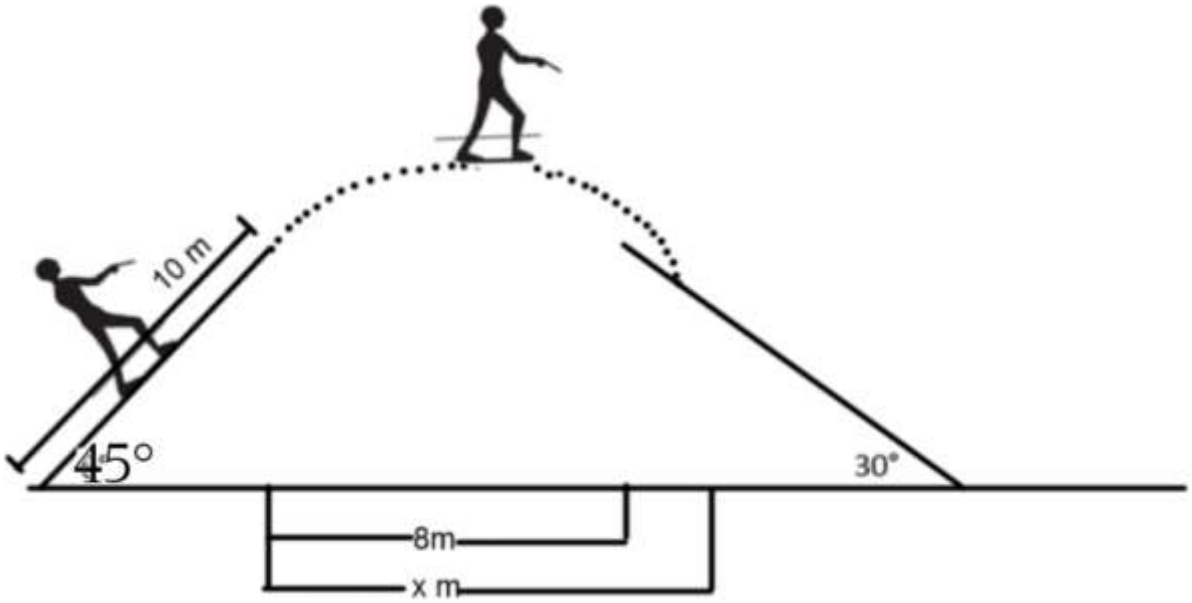
d. Find the time when there is 200g of salt in the container, rounding your answer to 2 decimal places.

2 marks

SECTION 2- continued

TURN OVER

Question 5 (15 marks)



Nick is testing a new trick at a skate park. He plans to use his skateboard to jump between two skate ramps which are 8m apart, shown in the diagram. The first ramp is 10m long and makes an angle of 45° with the ground. The second skate ramp has an angle of 30° with ground. The coefficient of friction between his skateboard and the ramps is 0.01.

Nick obtains a speed of 15m/s at the foot of the first skate ramp.

a. Label all forces acting on Nick and his board when he is moving along the first skate ramp.

2 marks

b. Find the acceleration of Nick when he is moving along the first skate ramp, rounding your answer to a whole number in m/s^2.

2 marks

SECTION 2- Question 5- continued

c. Find the speed in m/s when Nick reaches the top of the first skate ramp. Round your answer to two decimal places

2 marks

d. Show that Nick can safely land on the second skate ramp, given that it is the same height as the first ramp.

3 marks

e. Find the horizontal distance x between the top of the first skate ramp and where Nick lands on the second skate ramp. Round your answer to 2 decimal places.

3 marks

SECTION 2- Question 5- continued

TURN OVER

f. Find the minimal angle, in degrees, between the first skate ramp and the ground such that Nick can safely land on the second skate ramp. Round your answer to 1 decimal place. (Assume that Nick leaves the first ramp with the same speed of the answer found in **part c**, and both ramps reach the same height and they are always 8 meters apart.)

3 marks

END OF QUESTION AND ANSWER BOOK