SPECIALIST MATHEMATICS

Written examination 1

TSSM™
Creating VCE Success

(TSSM's 2015 trial exam updated for the current study design)

SOLUTIONS

Question 1 (6 marks)

a. $\left|\underset{\sim}{v}\right| = \sqrt{\left(8+3\sqrt{11}\right)^2 + 7^2} = \sqrt{212+48\sqrt{11}}$

Now

$\left(6+4\sqrt{11}\right)^2 = 36 + 48\sqrt{11} + 176 = 212 + 48\sqrt{11}$

Therefore

$$\left|\underset{\sim}{v}\right| = 6 + 4\sqrt{11}$$

2 marks

b. Let θ be the angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$.

Then

$$\cos(\theta) = \frac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{\left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|} = \frac{16 + 6\sqrt{11} - 7}{3(6+4\sqrt{11})} = \frac{1}{2}$$

Therefore

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

2 marks

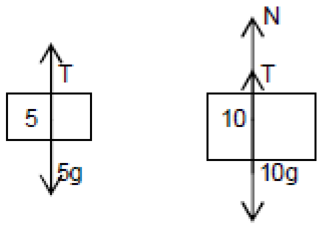
c. The resolute vector of $\underset{\sim}{v}$ in the direction of $\underset{\sim}{u}$ is

$$\left|\underset{\sim}{v}\right| \cos\left(\frac{\pi}{3}\right) \underset{\sim}{\hat{u}} = \frac{3+2\sqrt{11}}{3}\left(2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}\right)$$

2 marks

Question 2 (5 marks)

a.



$$\mathrm{T} - 5\mathrm{g} = 0, \quad \mathrm{N} + \mathrm{T} - 10\mathrm{g} = 0$$
$$T = 5 \times 9.8 = 49N$$

4 marks

b. $N = 10 \times 9.8 - 49 = 49N$

1 mark

Question 3 (7 marks)

a. $x = 4(e^t + e^{-t}) \Rightarrow x^2 = 16(e^{2t} + e^{-2t} + 2)$

$y = 3(e^t - e^{-t}) \Rightarrow y^2 = 9(e^{2t} + e^{-2t} - 2)$

$\Rightarrow \frac{x^2}{64} - \frac{y^2}{36} = \frac{16(e^{2t} + e^{-2t} + 2)}{64} - \frac{9(e^{2t} + e^{-2t} - 2)}{36} = 1$

2 marks

b. Domain: $[8, \infty)$ Range: $[0, \infty)$ 1 mark

c. The equation of the asymptote: $y = \frac{3}{4}x$ 1 mark

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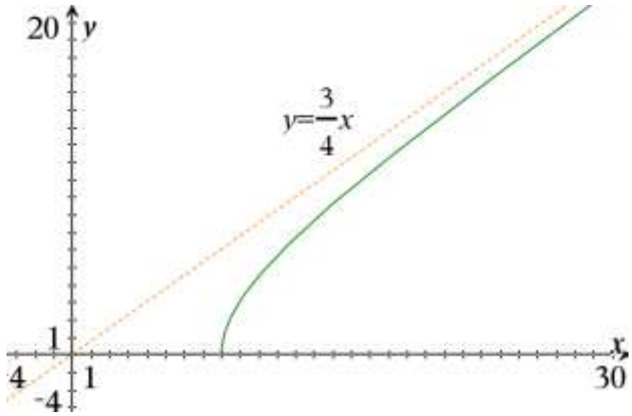

Graph	1 mark
Asymptote	1 mark
x-int	1 mark

Question 4 (3 marks)

a. $P\left(-\frac{1}{2}\right) = 2 \times \frac{-1}{8} + 5 \times \frac{1}{4} + 6 \times \frac{-1}{2} + 2 = 0 \Rightarrow z = -\frac{1}{2}$ is a root of $P(z)$. 1 mark

b. According to the result of Part a, $(2z+1)$ is a factor of $P(z)$.

$$2z^3 + 5z^2 + 6z + 2 = (2z+1)(z^2+2z+2)$$
$$= (2z+1)[(z+1)^2+1]$$
$$= (2z+1)(z+1+i)(z+1-i)$$

The other roots of $P(z)$ are

$$-1-i = \sqrt{2}cis(-\frac{3\pi}{4}) \quad \text{and} \quad -1+i = \sqrt{2}cis(\frac{3\pi}{4})$$

2 marks

Question 5 (4 marks)

a. Differentiate the equation of the curve, $2yy' = 4 \Rightarrow y' = \frac{2}{y}$.

The gradient of the tangent at P: $m = \frac{2}{2\sqrt{p}} = \frac{1}{\sqrt{p}}$. 1 mark

b. $\tan(\alpha) = m = \frac{1}{\sqrt{p}}$, $\quad \tan(\beta) = \frac{2\sqrt{p}}{p-1}$.

$$\tan(2\alpha) = \frac{2\tan(\alpha)}{1-\tan^2(\alpha)} = \frac{\frac{2}{\sqrt{p}}}{1-\frac{1}{p}} = \frac{2\sqrt{p}}{p-1} = \tan(\beta)$$

Therefore $\beta = 2\alpha$ since $\alpha,\ \beta \in (0,\ \pi)$. 3 mark

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Question 6 (5 marks)

a. $1+\cos(4x) = 1+2\cos^2(2x)-1 = 2\cos^2(2x)$. 1 mark

b. $\int \frac{\sin(2x)}{1+\cos(4x)}\,dx = \int \frac{\sin(2x)}{2\cos^2(2x)}\,dx = -\frac{1}{4}\int \frac{1}{\cos^2(2x)}\,d(\cos(2x))$

Let $u = \cos(2x)$. Then $\int \frac{\sin(2x)}{1+\cos(4x)}\,dx = -\frac{1}{4}\int \frac{1}{u^2}\,du$ 2 marks

c. When $x = \frac{\pi}{6},\ u = \cos(2x) = \frac{1}{2}$

$x = \frac{\pi}{8},\ u = \cos(2x) = \frac{1}{\sqrt{2}}$.

$\therefore \quad \int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \frac{\sin(2x)}{1+\cos(4x)}\,dx = -\frac{1}{4}\int_{\frac{1}{\sqrt{2}}}^{\frac{1}{2}} \frac{1}{u^2}\,du = -\frac{1}{4}\left[-\frac{1}{u}\right]_{\frac{1}{\sqrt{2}}}^{\frac{1}{2}} = \frac{2-\sqrt{2}}{4}$. 2 marks

Question 7 (4)

a. $\int \frac{16\arctan(x)}{1+x^2}\,dx = 16\int \arctan(x)\,d(\arctan(x)) = 8(\arctan(x))^2 + c$. 2 marks

b. Area=$2\int_0^1 f(x)\,dx = 16[(\arctan(x))^2]_0^1 = \pi^2$ 2 marks

Question 8 (6 marks)

a. $2x - x^2 = 1-(x-1)^2$. 1 mark

b. $\int \frac{1}{x(x-2)}\,dx = \int \frac{1}{2}\left(\frac{1}{x-2}-\frac{1}{x}\right)dx = \frac{1}{2}\log_e\left|\frac{x-2}{x}\right| + c$ 2 marks

c. $\pi\int_1^{\frac{3}{2}}\left(1+\frac{1}{\sqrt{2x-x^2}}\right)^2 dx = \pi\int_1^{\frac{3}{2}}\left(1+\frac{2}{\sqrt{2x-x^2}}+\frac{1}{2x-x^2}\right)dx$

$$= \pi[x]_1^{\frac{3}{2}} + \pi\int_1^{\frac{3}{2}}\frac{2}{\sqrt{1-(x-1)^2}}\,dx - \pi\int_1^{\frac{3}{2}}\frac{1}{x(x-2)}\,dx$$

$$= \frac{\pi}{2} + 2\pi[\arcsin(x-1)]_1^{\frac{3}{2}} - \frac{\pi}{2}\left[\log_e\left|\frac{x-2}{x}\right|\right]_1^{\frac{3}{2}}$$

$$= \frac{\pi}{2} + \frac{\pi^2}{3} + \frac{\pi\log_e 3}{2}$$

3 marks

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