Neap:

Trial Examination 2015

VCE Specialist Mathematics Units 3&4

Written Examination 1

Suggested Solutions

Question 1 (4 marks)

$y^2 = (2x+3)^4 \Rightarrow 2y\dfrac{dy}{dx} = 8(2x+3)^3$ M1

$\dfrac{dy}{dx} = \dfrac{4(2x+3)^3}{y}$

As $y > 0$, $y = (2x+3)^2$.

$\dfrac{dy}{dx} = \dfrac{4(2x+3)^3}{(2x+3)^2}$

$= 4(2x+3)$ (or equivalent) A1

When $x = -\dfrac{1}{2}$, $\dfrac{dy}{dx} = 8$. M1

So $y - 4 = 8\left(x - \dfrac{1}{2}\right) \Rightarrow y = 8x + 8$. A1

Question 2 (3 marks)

$\cos^2(x) = \sin^2(x) - 15\sin(x) + 8$

$(1 - \sin^2(x)) - \sin^2(x) + 15\sin(x) - 8 = 0 \Rightarrow 2\sin^2(x) - 15\sin(x) + 7 = 0$ M1

$(2\sin(x) - 1)(\sin(x) - 7) = 0 \Rightarrow \sin(x) = \dfrac{1}{2}$ or $\sin(x) = 7$ A1

$\sin(x) = \dfrac{1}{2} \Rightarrow x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ A1

Question 3 (5 marks)

a. $\bar{z}^2 = (x - yi)^2$

$= x^2 - 2xyi + i^2 y$ M1

$= x^2 - y^2 - 2xyi$ A1

b. $z^2 = (x + yi)^2$

$= x^2 + 2xyi + i^2 y$

$= x^2 - y^2 + 2xyi$ A1

$= \bar{z}^2 \Rightarrow 4xyi = 0$ M1

So $x = 0$ (purely imaginary) or $y = 0$ (purely real). A1

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Question 4 (3 marks)

$5 = 10\cos(\theta) \Rightarrow \theta = \dfrac{\pi}{3}$ A1

Using $W = 10\sin(\theta)$ with $\theta = \dfrac{\pi}{3}$ gives $W = 5\sqrt{3}$ (N). M1

(*Alternatively:* $W = \sqrt{10^2 - 5^2} = 5\sqrt{3}$)

Rearranging $W = mg$ to make m the subject with $W = 5\sqrt{3}$, we obtain $m = \dfrac{5\sqrt{3}}{g}$. A1

Question 5 (5 marks)

a. $\dfrac{dv}{dx} = -\dfrac{3x}{\sqrt{4 - x^2}}$ A1

Using $\ddot{x} = v\dfrac{dv}{dx}$, we obtain $a = 3\sqrt{4 - x^2}\left(\dfrac{-3x}{\sqrt{4 - x^2}}\right)$. M1

So $\ddot{x} = -9x$. A1

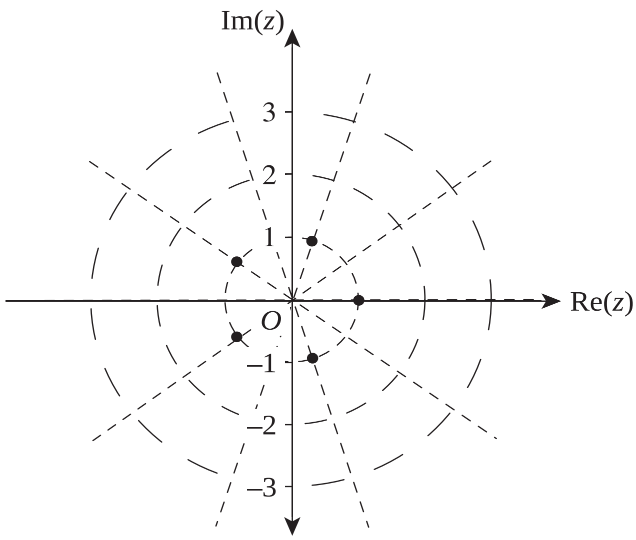
b. Differentiating $x = 2\sin(3t)$, we obtain $\dot{x} = 6\cos(3t)$ and $\ddot{x} = -18\sin(3t)$. A1

Substituting $x = 2\sin(3t)$ into $\ddot{x} = -9x$, we obtain $\ddot{x} = -18\sin(3t)$. A1

Hence it is verified.

Question 6 (3 marks)

a.



all five roots plotted correctly A1

b. $A = 5\left(\dfrac{1}{2}(1)(1)\sin\left(\dfrac{2\pi}{5}\right)\right)$ M1

So $A = \dfrac{5}{2}\sin\left(\dfrac{2\pi}{5}\right)$. $\left(k = \dfrac{5}{2}, \alpha = \dfrac{2\pi}{5}\right)$ A1

Question 7 (6 marks)

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^3(x)\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\sec^2(x)\tan(x) - \tan(x))\,dx$$ M1 A1

Note: The M1 is for using $tan^2(x) = sec^2(x) - 1$.

$$= \left[\frac{1}{2}\tan^2(x) + \log_e(\cos(x))\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$ M1 A1

Note: The M1 is for attempting integration by substitution on both terms.

$$= \frac{4}{3} - \frac{1}{2}\log_e(3)$$ M1 A1

Note: The M1 is for substituting and attempting to evaluate.

Question 8 (5 marks)

a. $\underset{\sim}{r}(t) = a\cos(2t)\underset{\sim}{i} + a\sin(2t)\underset{\sim}{j} + bt\underset{\sim}{k}$

$\underset{\sim}{r}'(t) = -2a\sin(2t)\underset{\sim}{i} + 2a\cos(2t)\underset{\sim}{j} + b\underset{\sim}{k}$ A1

b. $\underset{\sim}{r}'(t) \cdot \underset{\sim}{k} = b$ and $\underset{\sim}{r}'(t) \cdot \underset{\sim}{k} = |\underset{\sim}{r}'(t)||\underset{\sim}{k}|\cos(\theta)$ A1

$b = \sqrt{4a^2\sin^2(2t) + 4a^2\cos^2(2t) + b^2}\cos(\theta)$ M1

$= \sqrt{4a^2 + b^2}\cos(\theta) \Rightarrow \cos(\theta) = \dfrac{b}{\sqrt{4a^2 + b^2}}$ A1

$\theta = \cos^{-1}\left(\dfrac{b}{\sqrt{4a^2 + b^2}}\right)$ A1

The particle moves in such a way that it always makes a fixed angle with the $\underset{\sim}{k}$ direction.

Question 9 (6 marks)

a. Calculating the discriminant of the quadratic denominator, we obtain $\Delta = 2^2 - 4(1)(2)$

$= -4(< 0)$ A1

So the graph of f has no vertical asymptotes.

b. The axis of symmetry is $x = -1$ and so $k = -1$. A1

Let the required area be A square units, where $A = \int_{-1}^{\frac{\sqrt{3}-3}{3}} \frac{6}{x^2 + 2x + 2} dx$.

$A = \int_{-1}^{\frac{\sqrt{3}-3}{3}} \frac{6}{(x+1)^2 + 1} dx$ M1

$= 6[\tan^{-1}(x+1)]_{-1}^{\frac{\sqrt{3}-3}{3}}$ A1

$= 6\left[\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) - \tan^{-1}(0)\right]$ M1

$= 6\left(\frac{\pi}{6}\right)$

$= \pi$ A1

