



Trial Examination 2015

VCE Specialist Mathematics Units 3&4

Written Examination 1

Suggested Solutions

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Question 1 (4 marks)

$$y^2 = (2x + 3)^4 \Rightarrow 2y \frac{dy}{dx} = 8(2x + 3)^3 \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{4(2x + 3)^3}{y}$$

$$\text{As } y > 0, \quad y = (2x + 3)^2.$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{4(2x + 3)^3}{(2x + 3)^2} \\ &= 4(2x + 3) \text{ (or equivalent)} \quad \text{A1} \end{aligned}$$

$$\text{When } x = -\frac{1}{2}, \quad \frac{dy}{dx} = 8. \quad \text{M1}$$

$$\text{So } y - 4 = 8\left(x - \frac{1}{2}\right) \Rightarrow y = 8x + 8. \quad \text{A1}$$

Question 2 (3 marks)

$$\cos^2(x) = \sin^2(x) - 15 \sin(x) + 8$$

$$(1 - \sin^2(x)) - \sin^2(x) + 15 \sin(x) - 8 = 0 \Rightarrow 2\sin^2(x) - 15 \sin(x) + 7 = 0 \quad \text{M1}$$

$$(2 \sin(x) - 1)(\sin(x) - 7) = 0 \Rightarrow \sin(x) = \frac{1}{2} \text{ or } \sin(x) = 7 \quad \text{A1}$$

$$\sin(x) = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6} \quad \text{A1}$$

Question 3 (5 marks)

$$\begin{aligned} \text{a. } \bar{z}^2 &= (x - yi)^2 \\ &= x^2 - 2xyi + i^2 y^2 \quad \text{M1} \end{aligned}$$

$$= x^2 - y^2 - 2xyi \quad \text{A1}$$

$$\begin{aligned} \text{b. } z^2 &= (x + yi)^2 \\ &= x^2 + 2xyi + i^2 y^2 \\ &= x^2 - y^2 + 2xyi \quad \text{A1} \end{aligned}$$

$$= \bar{z}^2 \Rightarrow 4xyi = 0 \quad \text{M1}$$

$$\text{So } x = 0 \text{ (purely imaginary) or } y = 0 \text{ (purely real).} \quad \text{A1}$$

Question 4 (3 marks)

$$5 = 10 \cos(\theta) \Rightarrow \theta = \frac{\pi}{3} \quad \text{A1}$$

Using $W = 10 \sin(\theta)$ with $\theta = \frac{\pi}{3}$ gives $W = 5\sqrt{3}$ (N). M1

(Alternatively: $W = \sqrt{10^2 - 5^2} = 5\sqrt{3}$)

Rearranging $W = mg$ to make m the subject with $W = 5\sqrt{3}$, we obtain $m = \frac{5\sqrt{3}}{g}$. A1

Question 5 (5 marks)

a. $\frac{dv}{dx} = -\frac{3x}{\sqrt{4-x^2}}$ A1

Using $\ddot{x} = v \frac{dv}{dx}$, we obtain $a = 3\sqrt{4-x^2} \left(\frac{-3x}{\sqrt{4-x^2}} \right)$. M1

So $\ddot{x} = -9x$. A1

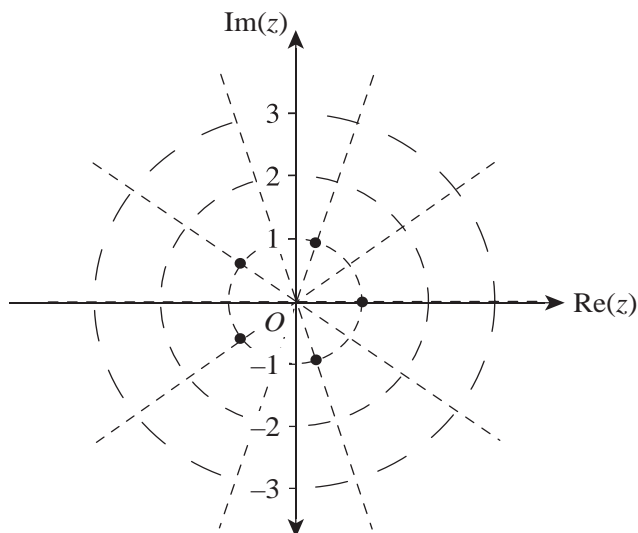
b. Differentiating $x = 2 \sin(3t)$, we obtain $\dot{x} = 6 \cos(3t)$ and $\ddot{x} = -18 \sin(3t)$. A1

Substituting $x = 2 \sin(3t)$ into $\ddot{x} = -9x$, we obtain $\ddot{x} = -18 \sin(3t)$. A1

Hence it is verified.

Question 6 (3 marks)

a.



all five roots plotted correctly A1

b. $A = 5 \left(\frac{1}{2} (1)(1) \sin\left(\frac{2\pi}{5}\right) \right)$ M1

So $A = \frac{5}{2} \sin\left(\frac{2\pi}{5}\right)$. $\left(k = \frac{5}{2}, \alpha = \frac{2\pi}{5} \right)$ A1

Question 7 (6 marks)

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^3(x) dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\sec^2(x) \tan(x) - \tan(x)) dx \quad \text{M1 A1}$$

Note: The M1 is for using $\tan^2(x) = \sec^2(x) - 1$.

$$= \left[\frac{1}{2} \tan^2(x) + \log_e(\cos(x)) \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \quad \text{M1 A1}$$

Note: The M1 is for attempting integration by substitution on both terms.

$$= \frac{4}{3} - \frac{1}{2} \log_e(3) \quad \text{M1 A1}$$

Note: The M1 is for substituting and attempting to evaluate.

Question 8 (5 marks)

a. $\underline{r}(t) = a \cos(2t) \underline{i} + a \sin(2t) \underline{j} + bt \underline{k}$

$$\underline{r}'(t) = -2a \sin(2t) \underline{i} + 2a \cos(2t) \underline{j} + b \underline{k} \quad \text{A1}$$

b. $\underline{r}'(t) \cdot \underline{k} = b$ and $\underline{r}'(t) \cdot \underline{k} = |\underline{r}'(t)| |\underline{k}| \cos(\theta)$ A1

$$b = \sqrt{4a^2 \sin^2(2t) + 4a^2 \cos^2(2t) + b^2} \cos(\theta) \quad \text{M1}$$

$$= \sqrt{4a^2 + b^2} \cos(\theta) \Rightarrow \cos(\theta) = \frac{b}{\sqrt{4a^2 + b^2}} \quad \text{A1}$$

$$\theta = \cos^{-1} \left(\frac{b}{\sqrt{4a^2 + b^2}} \right) \quad \text{A1}$$

The particle moves in such a way that it always makes a fixed angle with the \underline{k} direction.

Question 9 (6 marks)

- a. Calculating the discriminant of the quadratic denominator, we obtain $\Delta = 2^2 - 4(1)(2)$
 $= -4 (< 0)$ A1

So the graph of f has no vertical asymptotes.

- b. The axis of symmetry is $x = -1$ and so $k = -1$. A1

Let the required area be A square units, where $A = \int_{-1}^{\frac{\sqrt{3}-3}{3}} \frac{6}{x^2 + 2x + 2} dx$.

$$A = \int_{-1}^{\frac{\sqrt{3}-3}{3}} \frac{6}{(x+1)^2 + 1} dx \quad \text{M1}$$

$$= 6[\tan^{-1}(x+1)]_{-1}^{\frac{\sqrt{3}-3}{3}} \quad \text{A1}$$

$$= 6\left[\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) - \tan^{-1}(0)\right] \quad \text{M1}$$

$$= 6\left(\frac{\pi}{6}\right)$$

$$= \pi \quad \text{A1}$$