2015 Specialist Mathematics Trial Exam 1 Solutions

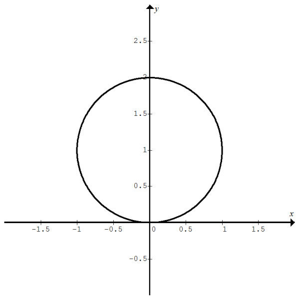
Q1a $\frac{1}{\bar{z}}-\frac{1}{z}=i$, $\frac{z-\bar{z}}{z\bar{z}}=i$, $\frac{2y\,i}{x^2+y^2}=i$, $x^2+y^2-2y=0$

$x^2+(y-1)^2=1$

Q1b $\text{Re}(z)\in[-1,1]$

Q1c $\text{Im}(z)\in[0,2]$

Q1d



Q2a

$P(z)=(z-\alpha)(z-\beta)(z-\gamma)=z^3-(\alpha+\beta+\gamma)z^2+\ldots\ldots$

$=z^3-2i\,z^2+2z-2i$

.: $\alpha+\beta+\gamma=2i$

Q2b $P(z)=(z-\alpha)(z-\beta)(z-\gamma)=z^3-2i\,z^2+2z-2i$

.: $P(i)=(i-\alpha)(i-\beta)(i-\gamma)=i^3-2i^3+2i-2i$

.: $(i-\alpha)(i-\beta)(i-\gamma)=i$

Q2c $(\alpha+\beta-\gamma)(\beta+\gamma-\alpha)(\gamma+\alpha-\beta)$

$=(2i-\gamma-\gamma)(2i-\alpha-\alpha)(2i-\beta-\beta)$

$=(2i-2\gamma)(2i-2\alpha)(2i-2\beta)$

$=8(i-\gamma)(i-\alpha)(i-\beta)=8i$

Q3 Let A be the area of the triangle. Given $A=|\tilde{a}||\tilde{b}|\sin\theta$

$A^2=|\tilde{a}|^2|\tilde{b}|^2\sin^2\theta$,

$A^2=|\tilde{a}|^2|\tilde{b}|^2(1-\cos^2\theta)=|\tilde{a}|^2|\tilde{b}|^2-|\tilde{a}|^2|\tilde{b}|^2\cos^2\theta$

$=|\tilde{a}|^2|\tilde{b}|^2-(|\tilde{a}||\tilde{b}|\cos\theta)^2=(\tilde{a}.\tilde{a})(\tilde{b}.\tilde{b})-(\tilde{a}.\tilde{b})^2$

.: $A=\sqrt{(\tilde{a}.\tilde{a})(\tilde{b}.\tilde{b})-(\tilde{a}.\tilde{b})^2}$

Q4a $|\tilde{p}|=|\tilde{q}|=|\tilde{r}|=\sqrt{\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2}=1$

Q4b $\tilde{p}.\tilde{q}=0$, $\tilde{q}.\tilde{r}=0$, $\tilde{r}.\tilde{p}=0$, .: $\tilde{p},\tilde{q},\tilde{r}$ are $\perp$ to each other.

Q4c

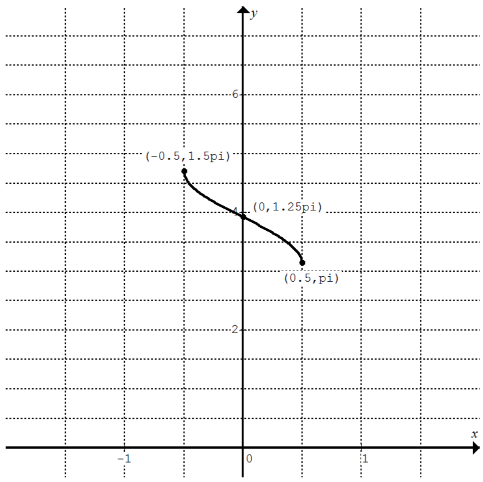
$\tilde{p}+\tilde{q}+\tilde{r}=\left(\frac{2}{3}\tilde{i}+\frac{1}{3}\tilde{j}+\frac{2}{3}\tilde{k}\right)+\left(-\frac{1}{3}\tilde{i}-\frac{2}{3}\tilde{j}+\frac{2}{3}\tilde{k}\right)+\left(\frac{2}{3}\tilde{i}-\frac{2}{3}\tilde{j}-\frac{1}{3}\tilde{k}\right)$

$=\tilde{i}-\tilde{j}+\tilde{k}=\tilde{s}$

Q4d $\tilde{s}.\tilde{t}=(\tilde{i}-\tilde{j}+\tilde{k}).\left(\frac{2}{\sqrt{3}}\tilde{p}-\sqrt{3}\tilde{q}+\frac{1}{\sqrt{3}}\tilde{r}\right)$

$=(\tilde{p}+\tilde{q}+\tilde{r}).\left(\frac{2}{\sqrt{3}}\tilde{p}-\sqrt{3}\tilde{q}+\frac{1}{\sqrt{3}}\tilde{r}\right)=\frac{2}{\sqrt{3}}-\sqrt{3}+\frac{1}{\sqrt{3}}=0$

Q5a



Q5b Given $y=f(x)=\frac{1}{2}\cos(2x)$ for $\pi\le x\le\frac{3\pi}{2}$

.: the inverse is $x=\frac{1}{2}\cos(2y-2\pi)$, $2y-2\pi=\cos^{-1}(2x)$

.: $y=\frac{1}{2}\cos^{-1}(2x)+\pi$, $f^{-1}(x)=\frac{1}{2}\cos^{-1}(2x)+\pi$

Q5c $\left(f^{-1}\right)'=\frac{1}{2}\times\frac{-2}{\sqrt{1-(2x)^2}}=\frac{-1}{\sqrt{1-4x^2}}$

At $x=0$, gradient of the tangent $=\frac{-1}{\sqrt{1-4x^2}}=-1$

.: gradient of the normal $=-\frac{1}{m_t}=1$

.: equation of the normal: $y=x+\frac{5\pi}{4}$

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Q6a $\frac{dy}{dx}=\frac{x^2}{2y}$

$x=2$, $y=1$, $\frac{dy}{dx}=\frac{2^2}{2(1)}=2$

$x=2.5$, $y\approx 1+0.5\times 2=2$, $\frac{dy}{dx}=\frac{2.5^2}{2(2)}=\frac{6.25}{4}$

$x=3$, $y\approx 2+0.5\times\frac{6.25}{4}=\frac{89}{32}$

Q6b $\frac{dy}{dx}=\frac{1}{2\sqrt{\frac{x^3-5}{3}}}\times x^2=\frac{x^2}{2y}$, .: $y=\sqrt{\frac{x^3-5}{3}}$ satisfies the differential equation $\frac{dy}{dx}=\frac{x^2}{2y}$

Q7a $f(x)=\sqrt{2x-x^2}$

Q7b $y=\pm\sqrt{2x-x^2}$, $y^2=2x-x^2$, $x^2-2x+1+y^2=1$

$(x-1)^2+y^2=1$, a circle of radius 1, .: area is π.

Q8a The particle starts from rest, .: $\tilde{v}=\frac{g}{4}\left(\sqrt{3}\,\tilde{i}-\tilde{j}\right)t$.

Displacement at time t is given by

$\int_0^t \frac{g}{4}\left(\sqrt{3}\,\tilde{i}-\tilde{j}\right)t\,dt=\frac{g}{8}\left(\sqrt{3}\,\tilde{i}-\tilde{j}\right)t^2$

.: $10\sqrt{3}\,\tilde{i}-10\,\tilde{j}=\frac{g}{8}\left(\sqrt{3}\,\tilde{i}-\tilde{j}\right)t^2$

.: $t^2=\frac{80}{g}$, $t=4\sqrt{\frac{5}{g}}$

Q8b Note: This is a 1 mark question, not 2 as indicated in the trial exam.

At $\tilde{r}=10\sqrt{3}\,\tilde{i}$, $t=4\sqrt{\frac{5}{g}}$

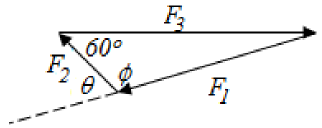
$\tilde{v}=\frac{g}{4}\left(\sqrt{3}\,\tilde{i}-\tilde{j}\right)4\sqrt{\frac{5}{g}}=\sqrt{5g}\left(\sqrt{3}\,\tilde{i}-\tilde{j}\right)$

Speed $=\left|\tilde{v}\right|=\sqrt{5g}\sqrt{3+1}=2\sqrt{5g}$

Q8c $\tilde{F}=m\tilde{a}=0.4\times\frac{g}{4}\left(\sqrt{3}\,\tilde{i}-\tilde{j}\right)=0.1g\left(\sqrt{3}\,\tilde{i}-\tilde{j}\right)$

$\left|\tilde{F}\right|=0.1g\sqrt{3+1}=\frac{g}{5}$ newtons

Q9a



$F_2^{\,2}+F_3^{\,2}-2F_2F_3\cos 60°=F_1^{\,2}$

$\left(\frac{F_3}{3}\right)^2+F_3^{\,2}-2\left(\frac{F_3}{3}\right)F_3\cos 60°=F_1^{\,2}$

$\frac{F_3^{\,2}}{9}+F_3^{\,2}-\frac{2F_3^{\,2}}{3}\times\frac{1}{2}=7$, $F_3=3$

Q9b

$\frac{\sin\phi}{3}=\frac{\sin 60°}{\sqrt{7}}$, $\sin\phi=\frac{3}{\sqrt{7}}\times\frac{\sqrt{3}}{2}=\frac{3}{2}\sqrt{\frac{3}{7}}$

.: $\sin\theta=\sin\phi=\frac{3}{2}\sqrt{\frac{3}{7}}$

.: $\alpha=\frac{3}{2}$ and $\beta=\frac{3}{7}$, or other equivalent forms.

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