Year 12 *Trial Exam Paper*

2015

SPECIALIST MATHEMATICS

Written examination 2

STUDENT NAME:

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT have to be cleared.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.

Materials provided

- The question and answer book of 27 pages, a formula sheet, and an answer sheet for the multiple-choice questions.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- You must answer the questions in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones or any other unauthorised electronic devices into the examination.

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SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where g = 9.8.

Question 1

Find the values of b and c if the graph of $y=\dfrac{9}{x^2+bx+c}$ has a local maximum at $\left(\dfrac{5}{2},-4\right)$.

A. b = –5 and c = –4

B. b = 4 and c = –5

C. b = –5 and c = 4

D. b = –4 and c = 5

E. b = –4 and c = –5

Question 2

An ellipse has a domain of $[-6, 2]$ and range of $[-4, 12]$. The equation of this ellipse is

A. $4x^2+y^2+8x-4y=44$

B. $x^2+4y^2+4x-16y+4=0$

C. $\dfrac{(x-2)^2}{16}+\dfrac{(y-4)^2}{64}=1$

D. $\dfrac{(x+2)^2}{16}+\dfrac{(y+4)^2}{64}=1$

E. $4x^2+y^2+16x-8y-32=0$

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Question 3

If $\sec(2\theta) = \frac{-5}{4}$, where $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ and $\tan(\theta) = y$, then $\tan(\theta)$ can be found by selecting an appropriate solution of the quadratic equation

A. $3y^2 - 8y + 3 = 0$

B. $3y^2 + 10y + 3 = 0$

C. $3y^2 + 8y - 3 = 0$

D. $3y^2 - 10y + 3 = 0$

E. $3y^2 - 8y - 3 = 0$

Question 4

The maximal implied domain and range of $y = 1 - 2\cos^{-1}(2 - x)$ is

A. [1, 3] and [–5.5, 1]

B. [1, 3] and [1 – 2π, 1]

C. [1, 3] and [–6, 0.9]

D. [–1, 3] and [1 – π, 1]

E. [1, 3] and [1 – π, 1]

Question 5

Find the real numbers *a* and *b* such that $(a - bi)(2 + i) = i^3$.

A. $a = \frac{-1}{5}, b = \frac{2}{5}$

B. $a = 1, b = -2$

C. $a = \frac{1}{5}, b = \frac{-2}{5}$

D. $a = -1, b = 2$

E. $a = 1, b = 2$

SECTION 1 – continued
TURN OVER

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Question 6

If $z = a - bi$, where $a \neq 0$ and $b \neq 0$, simplify $\dfrac{i\,[\text{Re}(z) - z]}{\text{Im}(z)}$

A. -1

B. 0

C. 1

D. $\dfrac{a}{b}$

E. $\dfrac{-a}{b}$

Question 7

A solution to the complex equation $z^3 = -a$, where a is a positive real number, is $a^{\frac{1}{3}}\text{cis}\left(\dfrac{\pi}{3}\right)$.

The other two solutions are

A. $-a^{\frac{1}{3}}\text{cis}(\pi),\ a^{\frac{1}{3}}\text{cis}\left(\dfrac{-\pi}{3}\right)$

B. $a^{\frac{1}{3}},\ a^{\frac{1}{3}}\text{cis}\left(\dfrac{-\pi}{3}\right)$

C. $-a^{\frac{1}{3}},\ a^{\frac{1}{3}}\text{cis}\left(\dfrac{-\pi}{3}\right)$

D. $-a^{\frac{1}{3}},\ a^{\frac{1}{3}}\text{cis}\left(\dfrac{-2\pi}{3}\right)$

E. $a^{\frac{1}{3}},\ a^{\frac{1}{3}}\text{cis}\left(\dfrac{-2\pi}{3}\right)$

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Question 8

Let $u = \text{cis}\left(\dfrac{\pi}{4}\right)$ and $v = p\,\text{cis}(\theta)$, where $p, \theta \in R$. If $u = \dfrac{2}{v}$, then

A. $p = -2,\ \theta = \dfrac{\pi}{4}$

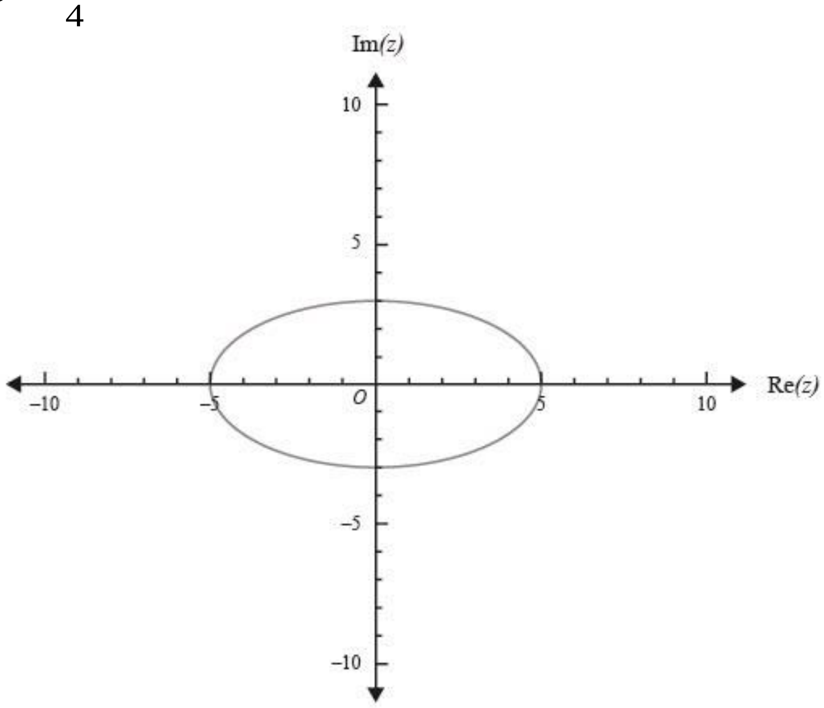
B. $p = 2,\ \theta = \dfrac{\pi}{4}$

C. $p = 1,\ \theta = \dfrac{-\pi}{4}$

D. $p = 1,\ \theta = \dfrac{\pi}{4}$

E. $p = 2,\ \theta = \dfrac{-\pi}{4}$

Question 9



The ellipse shown on the Argand diagram above is described by

A. $|z+4|-|z-4|=10$

B. $|z+4|+|z-4|=1$

C. $z\bar{z}=36$

D. $|z+4|+|z-4|=10$

E. $|z+6|+|z-6|=1$

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Question 10

Using the substitution $u = \cos(2x)$, $\int_0^{\frac{\pi}{6}} \sin(4x)\cos(2x)\,dx$ can be written as

A. $\int_0^{\frac{\pi}{6}} u^2\,du$

B. $-\int_{\frac{1}{2}}^{1} u^2\,du$

C. $-\int_0^{\frac{\pi}{6}} u^2\,du$

D. $\int_1^{\frac{1}{2}} u^2\,du$

E. $\int_{\frac{1}{2}}^{1} u^2\,du$

Question 11

The rational expression $\frac{1-x^2}{(x-1)^2}$, when expressed as a sum of partial fractions, is

A. $\frac{A}{(x-1)^2} + \frac{B}{(x-1)}$

B. $1 + \frac{A}{(x-1)^2} + \frac{B}{x-1}$

C. $\frac{A}{(x-1)^2} + \frac{B}{x-1} - 1$

D. $x + \frac{A}{(x-1)^2} + \frac{B}{x-1}$

E. $\frac{A}{(x-1)^2} + \frac{B}{x-1} - x$

SECTION 1 – continued

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Question 12

The area bounded by the curve $y = 1 + e^{\frac{1}{2}x}$, the y-axis and the line $y = 3$ can be determined by

A. $\int_2^3 2\log_e(y-1)dy$

B. $6\log_e(2) - \int_2^3 2\log_e(y-1)dy$

C. $\int_0^{2\log_e(2)} (1 + e^{\frac{1}{2}x})dx$

D. $\int_2^3 (1 + e^{\frac{1}{2}x})dx$

E. $6\log_e(2) - \int_0^3 (1 + e^{\frac{1}{2}x})dx$

Question 13

If $y = 4e^{\frac{1}{2}x} - 4e^{\frac{-1}{2}x}$, then

A. $y'' = 4y'$

B. $y' = \frac{1}{2}y$

C. $y'' = \frac{-1}{4}y$

D. $y'' = \frac{1}{4}y$

E. $y' = 2y$

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Question 14

If $\underset{\sim}{a} = 5\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = 3m\underset{\sim}{i} - 6\underset{\sim}{j} + 6n\underset{\sim}{k}$, find the values of *m* and *n* if $\underset{\sim}{a}$ is parallel to $\underset{\sim}{b}$.

A. $m = \frac{5}{3},\ n = \frac{1}{3}$

B. $m = \frac{10}{3},\ n = \frac{1}{2}$

C. $m = 2,\ n = 10$

D. $m = \frac{1}{10},\ n = \frac{1}{2}$

E. $m = 10,\ n = 2$

Question 15

In triangle *OAB*, $\overrightarrow{OB} = 4\underset{\sim}{i}$ and $\overrightarrow{OA} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$. If *M* lies on $\overrightarrow{OB}$, and $\overrightarrow{AM}$ is drawn where $\overrightarrow{AM}$ is perpendicular to $\overrightarrow{OB}$, find the angle between $\overrightarrow{AM}$ and $\overrightarrow{AB}$ to the nearest degree.

A. 27°

B. 19°

C. 90°

D. 28°

E. 26°

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Question 16

An object is projected upward from the top of a building that is 35 m tall. such that its height of *x* m above the ground after *t* seconds is given by $x = 35 + 20t - 5t^2$. Find the object's distance above the building when it reaches its maximum height.

A. 55 m

B. 195 m

C. 20 m

D. 70 m

E. 160 m

Question 17

The position of a particle at time *t* is given by $\underset{\sim}{r}(t) = 2\cos(3t)\underset{\sim}{i} - 3\sin(3t)\underset{\sim}{j}$. The initial speed of the particle is

A. 2

B. –9

C. –2

D. 9

E. 6

Question 18

A particle moves in a straight line and at time *t* its displacement from a fixed origin is *x*.

If $\dot{x} = 5 - x$, then $\ddot{x}$ in terms of *x* would be

A. –1

B. $\log_e|5 - x|$

C. $-\log_e|5 - x|$

D. $5x - \frac{1}{2}x^2$

E. $x - 5$

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Question 19

A cylindrical tank of height 5 metres and radius a metres is initially full of water. Water leaks from a hole in the base of the tank at a rate of $\sqrt{h}$ m^3/hr, where h metres is the depth of water in the tank after t hours.

Hence, $\frac{dh}{dt}$ is equal to

A. $\frac{-\sqrt{h}}{\pi a}$

B. $\frac{\sqrt{h}}{\pi a^2}$

C. $\frac{-1}{2a\pi h}$

D. $-\pi a^2\sqrt{h}$

E. $\frac{-\sqrt{h}}{\pi a^2}$

Question 20

An approximate solution to the differential equation $\frac{dy}{dx} = \frac{x}{y}$ is found using Euler's method with an increment size of 0.5. Given $y(0) = 2,$ find the solution obtained for $x = 1$.

A. 2.125

B. 2

C. 6

D. 1

E. 4.1

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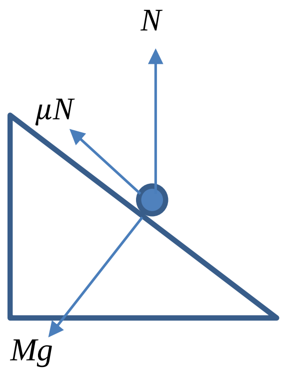
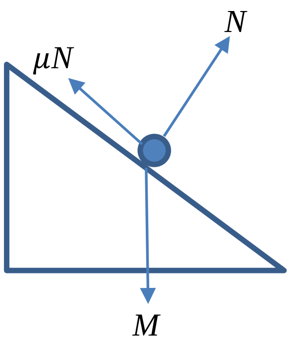
Question 21

An object of mass M kg is at rest on a rough inclined plane, and μ is the coefficient of friction between the surface of the plane and the object. If N newton is the normal reaction force, which one of the following diagrams correctly shows all forces acting on the object?

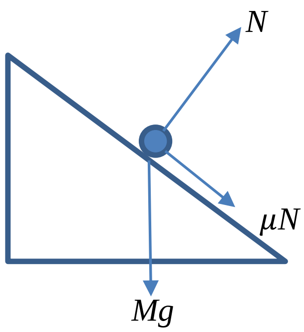
A.



B.



C.



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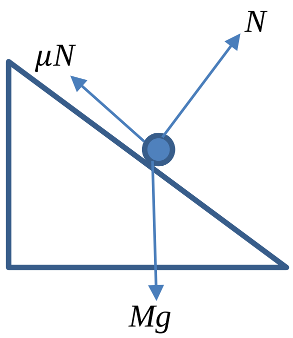
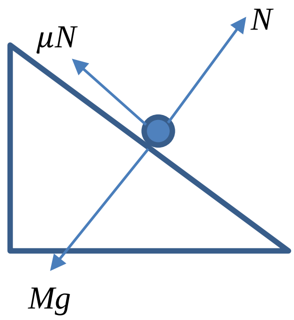


E.



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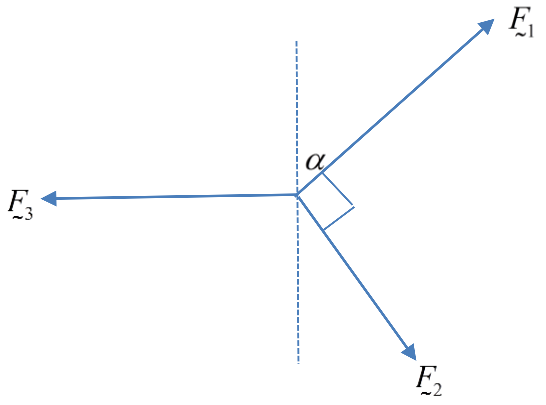
Question 22

Three coplanar forces, $\underset{\sim}{F}_1$, $\underset{\sim}{F}_2$ and $\underset{\sim}{F}_3$, act on a particle that is in equilibrium, as shown in the diagram below.



If the forces $\underset{\sim}{F}_1$, $\underset{\sim}{F}_2$ and $\underset{\sim}{F}_3$ have magnitudes F_1, F_2 and F_3, respectively, which one of the following expressions is incorrect?

A. $F_1\cos(\alpha) = F_2\sin(\alpha)$

B. $\underset{\sim}{F}_1 + \underset{\sim}{F}_2 + \underset{\sim}{F}_3 = \underset{\sim}{0}$

C. $\cot(\alpha) = \dfrac{F_1}{F_2}$

D. $F_3 = F_1\sin(\alpha) + F_2\cos(\alpha)$

E. $F_3^{\,2} = F_1^{\,2} + F_2^{\,2}$

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SECTION 2

Instructions for Section 2
Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (11 marks)

A hyperbola is described by the parametric equations $x = 2\sec(\theta)$ and $y = \tan(\theta)$, where $\theta \in \left[0, \frac{\pi}{3}\right]$.

a. Find the equation of the tangent to the hyperbola when $\theta = \frac{\pi}{6}$.

4 marks

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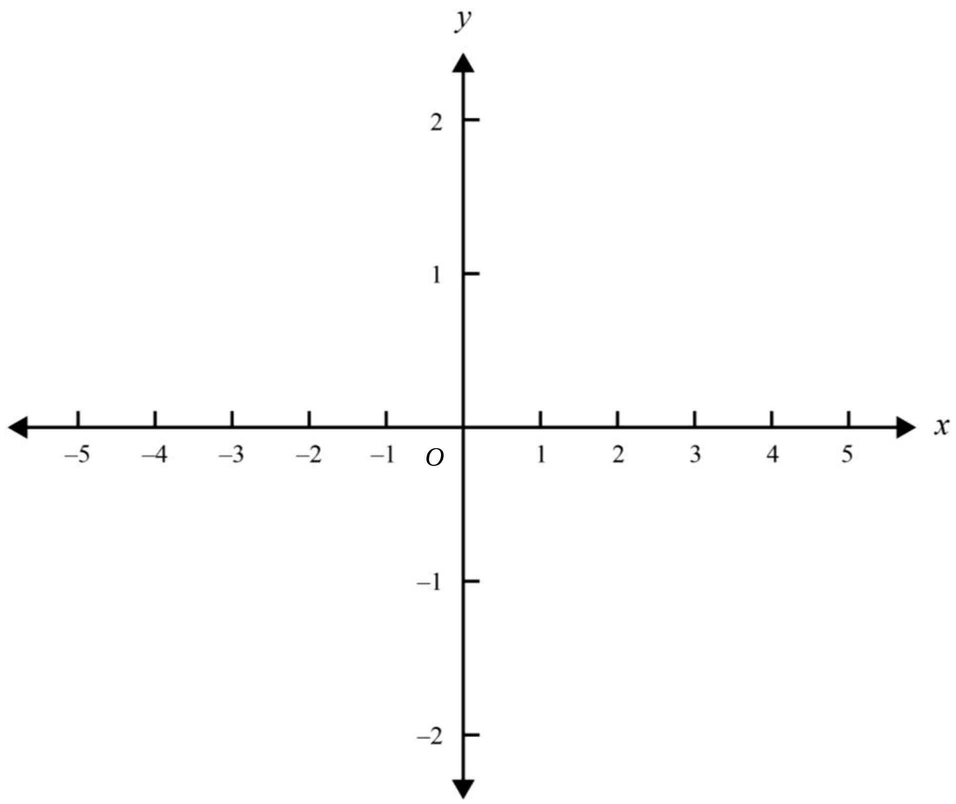
b. Show that the Cartesian equation of the hyperbola is $\frac{x^2}{4} - y^2 = 1$ for $x \in [a,\ b]$ and find the values of a and b.

3 marks

c. Sketch the hyperbola described by the parametric equations $x = 2\sec(\theta)$ and $y = \tan(\theta)$, where $\theta \in [0, \frac{\pi}{3}]$, clearly labelling the coordinates of the endpoints.

On the same graph, sketch the tangent to the hyperbola when $\theta = \frac{\pi}{6}$ and clearly label the coordinates of the point of intersection of the tangent and the hyperbola and the x-intercept of the tangent.

2 marks



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d. The area between the tangent, hyperbola and x-axis is rotated about the x-axis to form a solid of revolution. Find the exact volume of this solid.

2 marks

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Question 2 (8 marks)

An island in the Pacific Ocean has a population of 1500 people. An individual who has recently returned from a neighbouring island is diagnosed with a contagious virus. The rate at which the virus spreads through the population is proportional to the product of the number of people who have contracted the virus and the number of people who have yet to contract the virus. This can be expressed as $\frac{dN}{dt} = kN(1500 - N)$, where N is the number of individuals who have contracted the virus and t is the number of days since the virus was first detected.

a. If $N(0) = 1$, use calculus to solve the differential equation $\frac{dN}{dt} = kN(1500 - N)$ to obtain $N = \frac{Ae^{1500kt}}{B + e^{1500kt}}$ and show that A = 1500 and B = 1499.

5 marks

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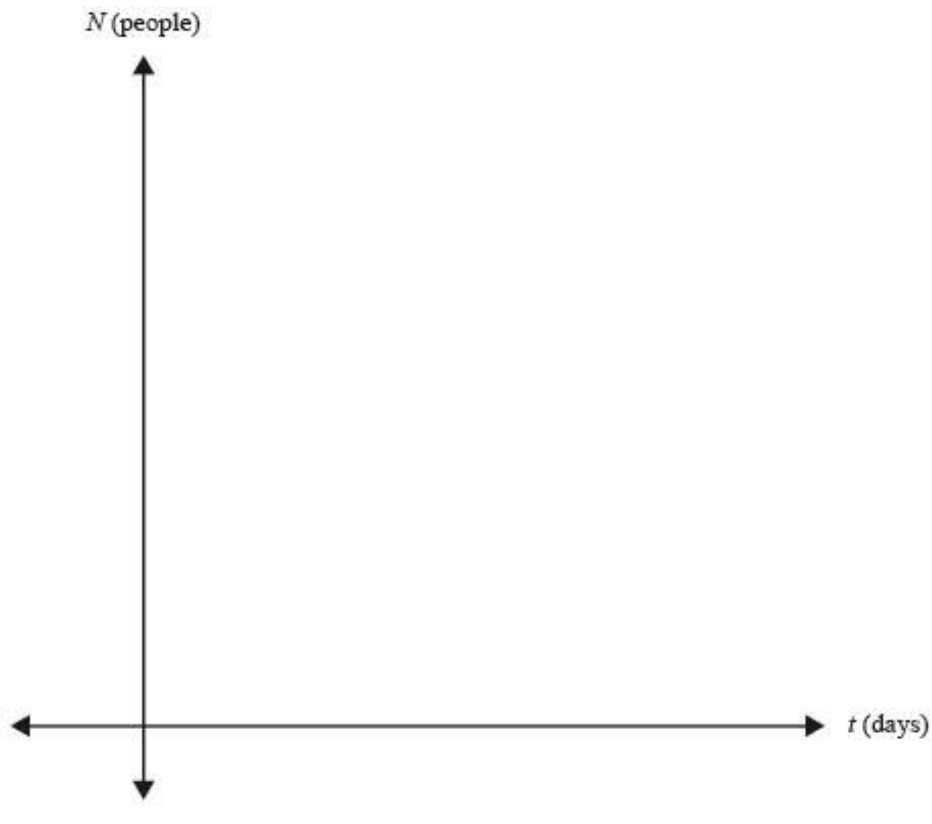
b. If $k = \dfrac{1}{3000}$, find the number of people who have contracted the virus after 10 days.

1 mark

c. Sketch a graph of N versus t for the first 30 days and clearly label the coordinates of the endpoints, to the nearest whole number.

2 marks



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Question 3 (12 marks)

Let $S = \left\{ z : (z - i)(\overline{z} + i) = 4,\ z \in C \right\}$

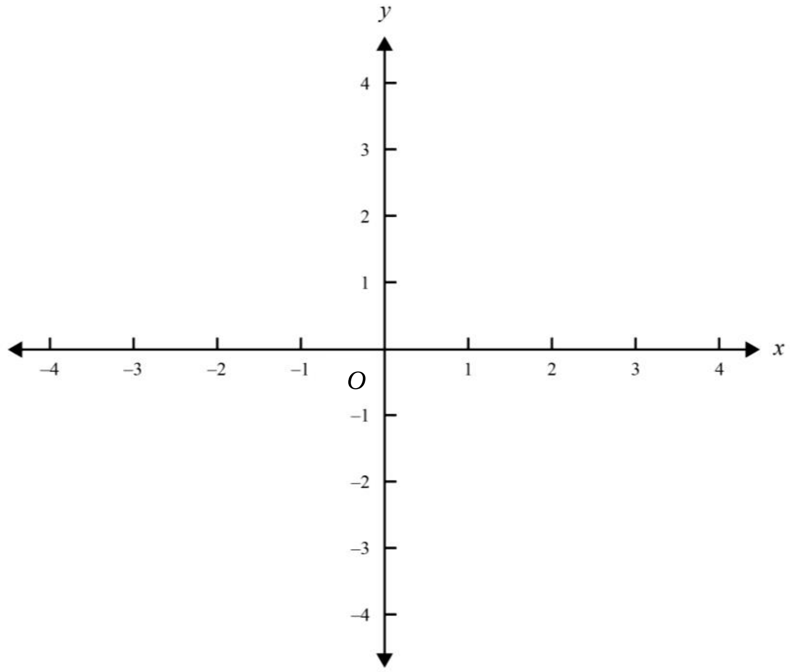
a. Show that S describes the circle with Cartesian equation $x^2 + (y-1)^2 = 4$.

2 marks

b. Let $f(x)$ be the upper half of the circle $x^2 + (y-1)^2 = 4$ as defined over its maximal domain and let $g(x) = \dfrac{1}{f(x)}$.

Sketch the graph of $g(x)$ and label the intercepts and endpoints. Clearly state the domain and range of the graph.

3 marks



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c. Let $T = \{z : z + 3 = k(iz + i),\ z \in C\}$ and $k \in R$.

Show that T represents a circle with Cartesian equation $(x+2)^2 + y^2 = 1$ and sketch T over its maximal domain on the same set of axes as the function g.

4 marks

d. Find the area enclosed by both graphs. Give your answer correct to two decimal places.

3 marks

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Question 4 (16 marks)

An ant (A) and a beetle (B) move in a Cartesian plane so that at any time $t \geq 0$ their position vectors are

$\underset{\sim}{r}_A = 4t\underset{\sim}{i} + t\underset{\sim}{j}$

$\underset{\sim}{r}_B = (8 - 8\sin(\alpha t))\underset{\sim}{i} + 8\cos(\alpha t)\underset{\sim}{j}$, where α is a positive constant, $\underset{\sim}{i}$ is a unit vector in the positive *x* direction and $\underset{\sim}{j}$ is a unit vector in the positive *y* direction.

a. Find the speeds of both insects, A and B.

3 marks

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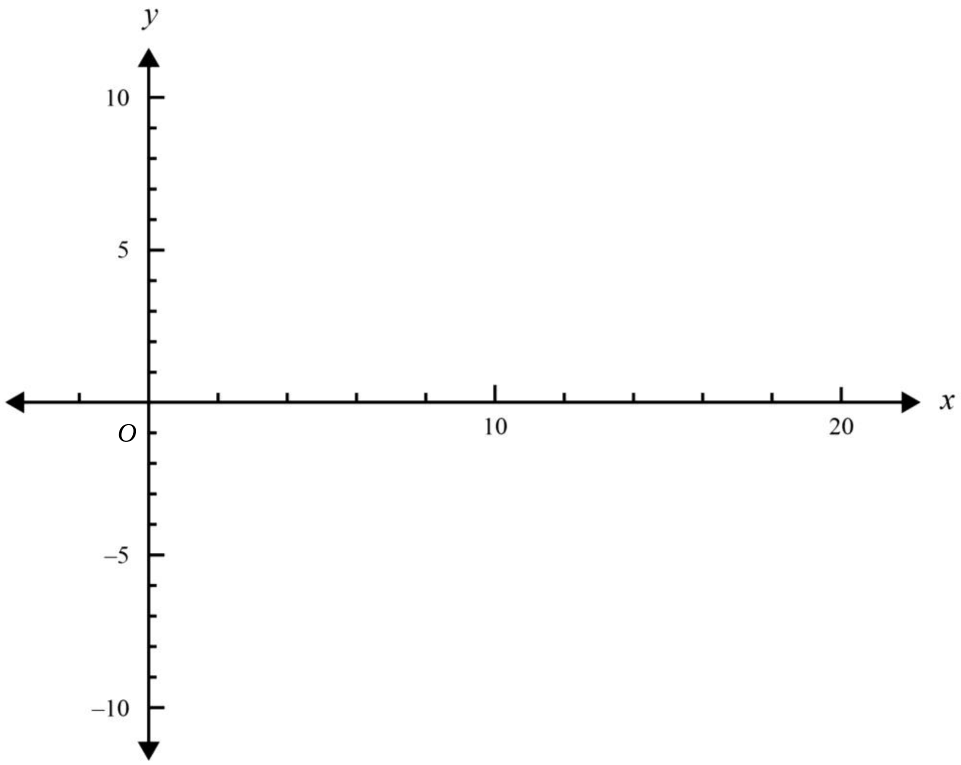
b. Find the Cartesian equations of the paths of both insects and sketch the paths on the same set of axes, clearly indicating their starting positions and direction of motion.

5 marks



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c. Find the coordinates of the point(s) where the paths of the insects intersect.

2 marks

d. Find the smallest positive value of α for which the insects collide. Give your answer correct to two decimal places.

4 marks

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e. Given that the insects collide, find the distance travelled by each insect until the collision occurs. Give your answer correct to one decimal place.

2 marks

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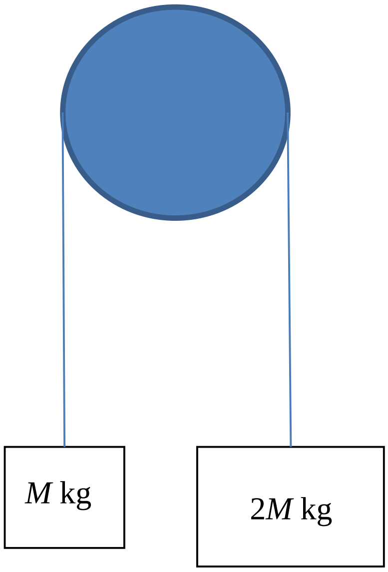
Question 5 (11 marks)

Two objects of mass *M* kg and 2*M* kg are connected by a light inelastic string that passes over a smooth pulley.



a. Let a ms^{-2} be the downward acceleration of the 2*M* kg object. Show that $a = \frac{g}{3}$ ms^{-2}.

2 marks

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b. The system starts from rest and for the first three seconds, the smaller object is yet to reach the base of the pulley. At this time, the string breaks. How long does it take the smaller object to return to its original position?

3 marks

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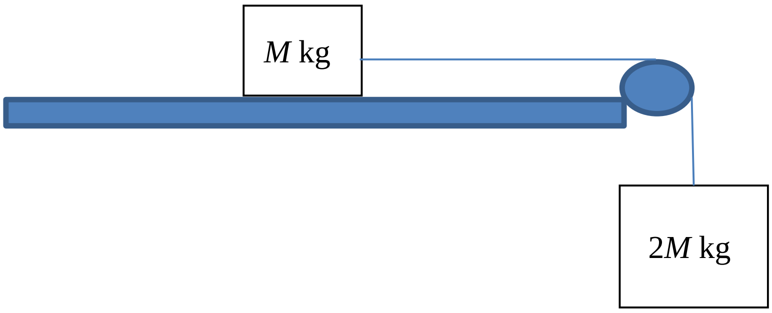
The pulley is now moved to the edge of a rough horizontal table with the smaller object (M kg) placed on the table and the larger object ($2M$ kg) hanging freely. The light inelastic string is attached between the objects parallel to the table and vertically downwards to the larger object.



c. If the coefficient of friction between the smaller object and the table is μ, find the acceleration (a) of the system and the tension (T) of the string in terms of M and μ.

4 marks

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d. The system of connected masses will not move if $\mu \geq k$. Find the value of *k*.

2 marks

END OF QUESTION AND ANSWER BOOK

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