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Year 12 *Trial Exam Paper*

2015

SPECIALIST MATHEMATICS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.
- Calculators are not permitted in this examination.

Materials provided

- The question and answer book of 15 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination.

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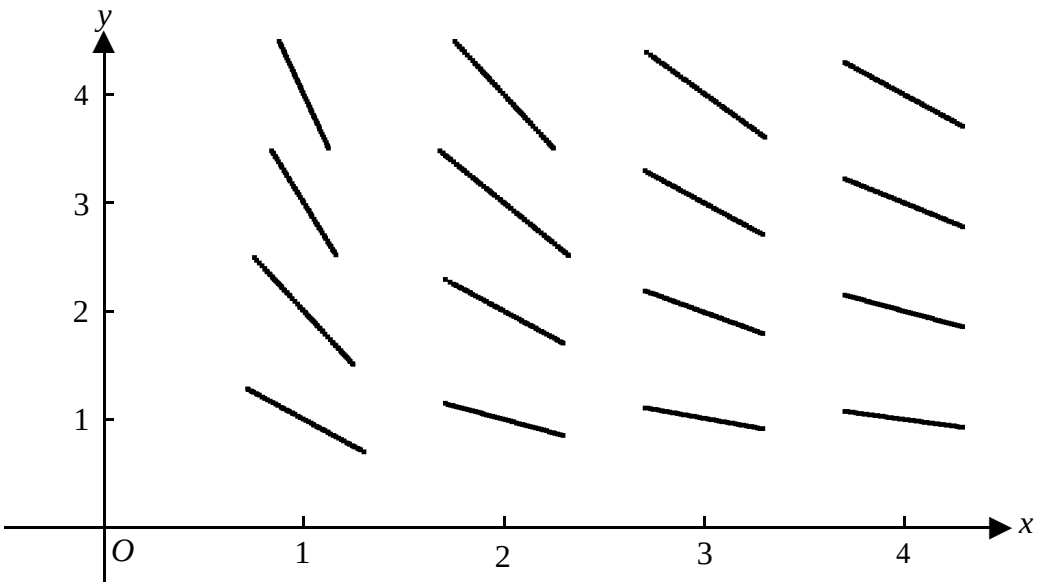
Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (2 marks)

Consider the differential equation $\frac{dy}{dx} = \frac{-y}{x}$.

A slope field using $x \in \{1, 2, 3, 4\}$ and $y \in \{1, 2, 3, 4\}$ is shown on the axes below.



a. On the slope field above, sketch the tangents for $x \in \{1, 2, 3, 4\}$ and $y = 0$.

1 mark

b. Sketch the solution curve to $\frac{dy}{dx} = \frac{-y}{x}$ that passes through (1, 1).

1 mark

TURN OVER

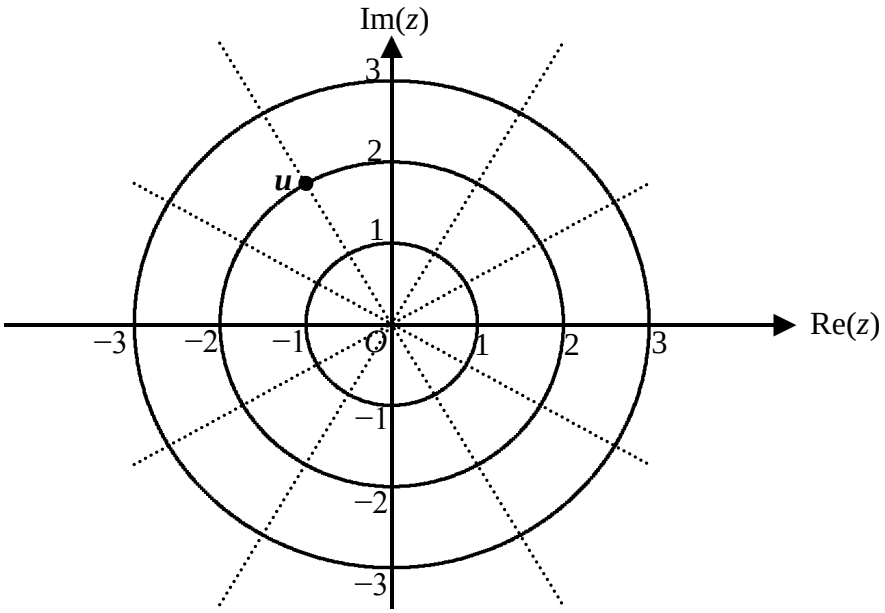
Question 2 (6 marks)

a. Find all solutions to the equation $z^4 + 3z^2 - 4 = 0,\ z \in C$.

3 marks

b. The point $u = \overline{z}$ is shown on the complex plane below. On the same graph, clearly show the point $v = iz$.

1 mark



QUESTION 2 – continued

c. Find $\sqrt{u}$ in cartesian form.

2 marks

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TURN OVER

Question 3 (3 marks)

Find the gradient of the normal to the curve $x^2y^2 - 4y - \log_e(x-1)^3 = 8$ at the point (2, 2).

Question 4 (3 marks)

Find all real solutions of the equation $\tan(x) = \sin(2x)$.

Question 5 (3 marks)

Given that $f(x) = \arcsin\left(\frac{x}{2}\right)$, find $f''\left(\sqrt{3}\right)$.

Question 6 (5 marks)

Relative to an origin *O*, points *A* and *B* have cartesian coordinates (1, 0, –1) and (2, –1, 1) respectively.

a. Find an expression for the vector $\overrightarrow{AB}$ in the form $a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k}$.

1 mark

b. If θ is the angle between the vectors $\overrightarrow{OB}$ and $\overrightarrow{AB}$, show that $\cos\theta = \frac{5}{6}$.

2 marks

c. Find the exact area of the triangle *OAB*.

2 marks

TURN OVER

Question 7 (4 marks)

a. Evaluate $\int_{e^{\frac{-\pi}{2}}}^{e^{\frac{\pi}{2}}} \frac{1}{x\sec(\log_e x)}\,dx$.

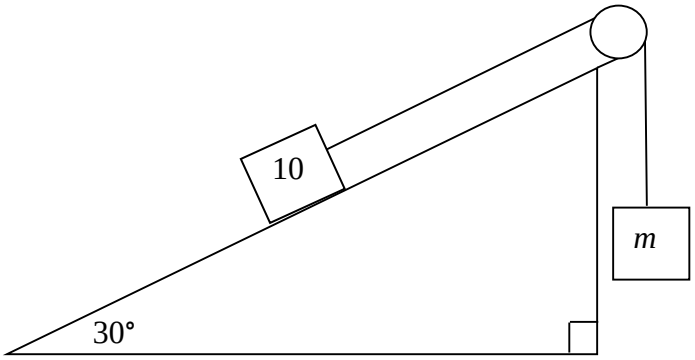
3 marks

b. Given $\frac{dy}{dx} = \frac{x-2}{(y+1)^2}$ and $y_0 = y(0) = 1$, use Euler's method to find y_1 using a step size of 0.2.

1 mark

Question 8 (4 marks)

A mass of 10 kg sits on an incline that makes an angle of 30° with the horizontal. The 10 kg mass is connected to another mass of *m* kg by a light inextensible string passing over a smooth pulley at the top of the incline. The mass of *m* kg is hanging vertically, as shown in the diagram below.



The coefficient of friction between the 10 kg mass and the incline is $\frac{1}{\sqrt{3}}$.

a. Show all forces acting on each mass on the diagram above.

1 mark

b. Calculate the **maximum** value of *m* for which the 10 kg mass will not move up the incline.

3 marks

TURN OVER

Question 9 (6 marks)

a. Show that $f(x) = \frac{1}{2(1-x)} + \frac{1}{2(1+x)} = \frac{1}{1-x^2}$.

1 mark

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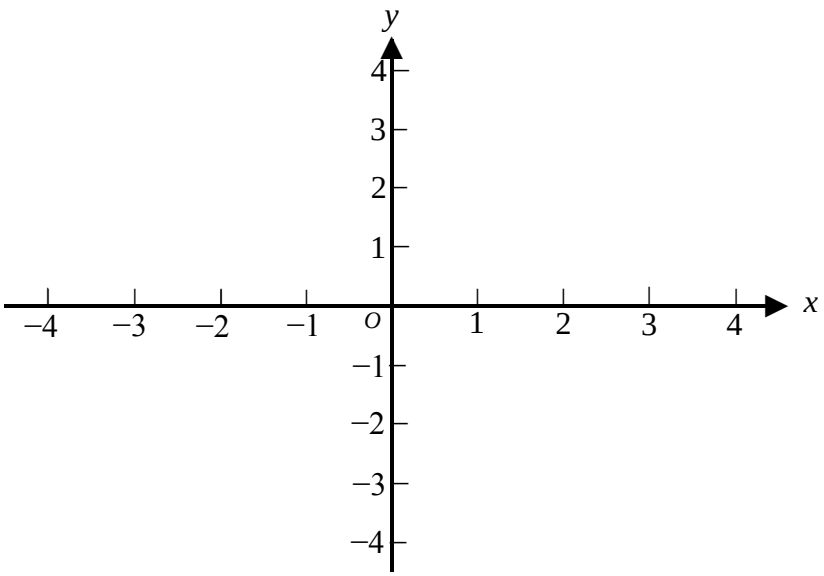
b. Sketch the graph of the function $f(x) = \frac{1}{2(1-x)} + \frac{1}{2(1+x)}$ on the axes below.

Label any asymptotes with their equations and label any axis intercepts with their co-ordinates.

2 marks



QUESTION 9 – continued

c. Calculate the volume generated when the area bounded by the curve $y = f(x)$, the y-axis, and the lines $y = -1$ and $y = -3$ is rotated about the y-axis.

3 marks

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Question 10 (4 marks)

The path of a particle is given by $\underset{\sim}{r}(t) = e^t \underset{\sim}{i} + \log_e(t+1)\underset{\sim}{j},\ t \geq 0$.

a. Calculate the initial speed of the particle.

2 marks

b. Show that the initial acceleration of the particle is perpendicular to its initial velocity.

2 marks

END OF QUESTION AND ANSWER BOOK