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Students Name:……………………………………

# SPECIALIST MATHEMATICS

# TRIAL EXAMINATION 1

# 2015

Reading Time: 15 minutes
Writing time: 1 hour

**Instructions to students**

This exam consists of 9 questions.
All questions should be answered.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
**Students may not bring any notes or calculators into the exam.**
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact answer is required to a question, a decimal approximation will not be accepted.
Unless otherwise indicated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude $g$ m/s$^2$ where $g = 9.8$
Formula sheets can be found on pages 11-13 of this exam.

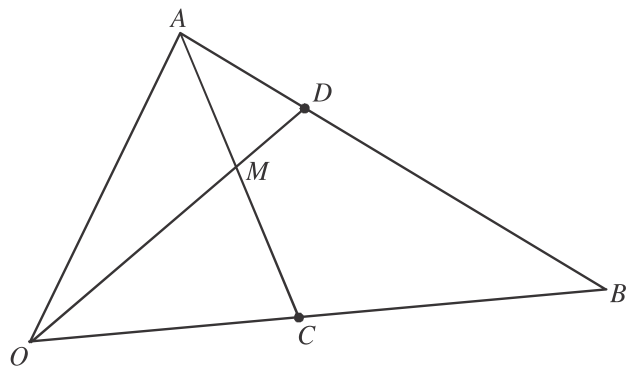
**Question 1** (3 marks)

Evaluate $\int_{0}^{\sqrt{6}} \frac{x-5}{x^2+2}dx$

**Question 2** (5 marks)

$OAB$ is a triangle where $C$ is the midpoint of $\overline{OB}$ and $3\overline{AD} = \overline{AB}$.

$\overline{OD}$ and $\overline{AC}$ intersect at point $M$.




Let $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$, $\overrightarrow{CM} = \alpha\overrightarrow{CA}$ and $\overrightarrow{OM} = \beta\overrightarrow{OD}$.

**a.** Express $\overrightarrow{CM}$ in terms of

**i.** $\underset{\sim}{a}, \underset{\sim}{b}$ and $\alpha$. 1 mark

**ii.** $\underset{\sim}{a}, \underset{\sim}{b}$ and $\beta$ 2 marks

**b.** Hence find the values of $\alpha$ and $\beta$. 2 marks

**Question 3** (3 marks)

Let $\underset{\sim}{a} = \underset{\sim}{i} - 4\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ .

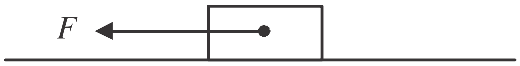
Resolve $\underset{\sim}{a}$ into two vector resolutes, one parallel to $\underset{\sim}{b}$ and the other perpendicular to $\underset{\sim}{b}$.

**Question 4** (4 marks)

Let $f(z) = z^4 - 2z^3 + 8z^2 - 8z + 16,\ z \in C$ . Given that $z = 2i$ is a solution of $f(z) = 0$, find all the other solutions in Cartesian form.

**Question 5** (5 marks)

A box of mass 2kg rests on a rough horizontal floor. It is given a one-off push which causes it to travel across the floor with an initial velocity of 10m/s. The frictional force $F$ shown in the diagram below, exerted by the floor on the box, causes the box to come to rest 4 seconds later.




**a.** Find the acceleration of the box across the floor. 2 marks

**b.** How far does the box travel before coming to rest? 1 mark

**c.** The coefficient of friction between the box and the floor equals $\frac{r}{sg}$, where $r$ and $s$ are positive integers and $\frac{r}{s}$ is a fraction expressed in simplest form.
Find the values of $r$ and $s$. 2 marks

**Question 6** (4 marks)

Find the gradient of the tangent to the curve defined by $2xy - \arctan\left(\frac{x}{2}\right) + y^2 = 5 - \frac{\pi}{4}$ at the point $(2,1)$.

**Question 7** (4 marks)

Calculate the area of the region enclosed by the graphs of $y=\dfrac{1}{x^2-2x-3}$, $x=2$ and the $x$ and $y$ axes.

**Question 8** (5 marks)

A particle moves in a straight line with acceleration $a$ m/s$^2$, velocity $v$ m/s and position $x$ m, relative to a fixed point on the line. The relationship between the position and the velocity of the particle at time $t$ seconds, $t \geq 0$, is given by $v = \sqrt{2x+4}$. The particle is initially at the fixed point.

**a.** Show that the acceleration of the particle is constant. 2 marks

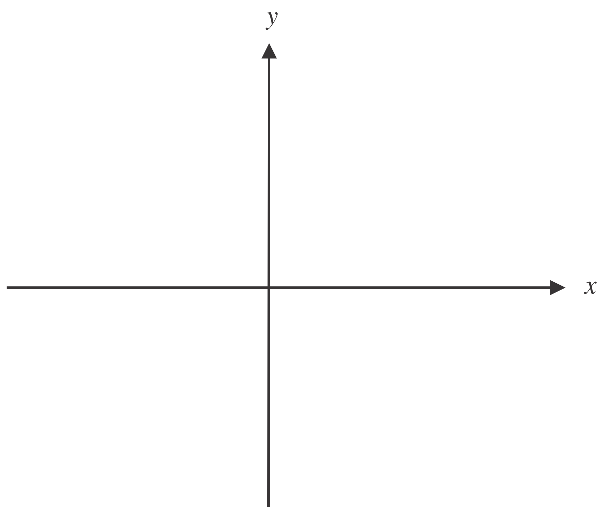
**b.** Find the value of $x$ when $t = 3$. 3 marks

**Question 9** (7 marks)

Consider the function $f(x) = 6\arccos(2x)$.

**a.** Write down the maximal domain and range of $f$. 2 marks

**b.** Sketch the graph of $y = f(x)$ on the set of axes below. Label any endpoints and axes intercepts with their coordinates. 2 marks



The region enclosed by the $x$ and $y$ axes and the graph of $y = f(x)$ is rotated about the $y$-axis.

**c.** Find the volume of the resulting solid of revolution. 3 marks

# Specialist Mathematics Formulas

## Mensuration

| | |
|---|---|
| area of a trapezium: | $\frac{1}{2}(a+b)h$ |
| curved surface area of a cylinder: | $2\pi rh$ |
| volume of a cylinder: | $\pi r^2 h$ |
| volume of a cone: | $\frac{1}{3}\pi r^2 h$ |
| volume of a pyramid: | $\frac{1}{3}Ah$ |
| volume of a sphere: | $\frac{4}{3}\pi r^3$ |
| area of a triangle: | $\frac{1}{2}bc\sin A$ |
| sine rule: | $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ |
| cosine rule: | $c^2=a^2+b^2-2ab\cos C$ |

## Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

## Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\cot^2(x)+1=\text{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

| function | $\sin^{-1}$ | $\cos^{-1}$ | $\tan^{-1}$ |
|---|---|---|---|
| domain | $[-1, 1]$ | $[-1, 1]$ | $R$ |
| range | $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ | $[0,\pi]$ | $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ |

## Algebra (Complex numbers)

$z=x+yi=r(\cos\theta+i\sin\theta)=r\text{cis}\theta$

$|z|=\sqrt{x^2+y^2}=r$ $-\pi<\text{Arg}\,z\le\pi$

$z_1z_2=r_1r_2\text{cis}(\theta_1+\theta_2)$ $\frac{z_1}{z_2}=\frac{r_1}{r_2}\text{cis}(\theta_1-\theta_2)$

$z^n=r^n\text{cis}(n\theta)$ (de Moivre's theorem)

**Calculus**

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x}\,dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax) \qquad \int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax) \qquad \int \cos\,(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax) \qquad \int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}} \qquad \int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2} \qquad \int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x),\ x_0 = a$ and $y_0 = b$,
then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$

**Vectors in two and three dimensions**

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r_1} . \underset{\sim}{r_2} = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

**Mechanics**

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$