Neap:

Trial Examination 2014

VCE Specialist Mathematics Units 3&4

Written Examination 1

Suggested Solutions

Question 1 (2 marks)

$P(-i) = (-i)^3 - (-i)^2 - 5$ M1

$= i + 1 - 5$

$= -4 + i$ A1

Question 2 (3 marks)

$x_1 = \frac{1}{2} \times 30 \times 5$

$= 75$ (m) A1

$x_2 = \frac{1}{2} \times 20 \times 4$

$= 40$ (m) A1

So the particle's displacement is 35 metres to the right of O. A1

Question 3 (4 marks)

a. $\Sigma \underset{\sim}{F} = (-\underset{\sim}{i} - 2\underset{\sim}{j}) + (4\underset{\sim}{i} - \underset{\sim}{j}) + (3\underset{\sim}{i} + 11\underset{\sim}{j})$ M1

$= 6\underset{\sim}{i} + 8\underset{\sim}{j}$

$|\Sigma \underset{\sim}{F}| = \sqrt{6^2 + 8^2}$

$= 10$ (N) A1

b. $|\underset{\sim}{a}| = \sqrt{(1.5)^2 + (2)^2}$ M1

$= 2.5$ (m/s^2)

$10 = 2.5m$

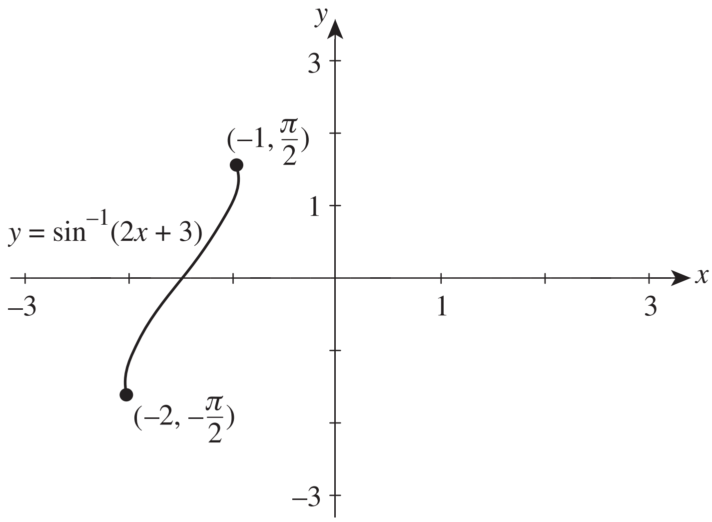
$m = 4$ (kg) A1

Question 4 (7 marks)

a. Solving $-1 \le 2x + 3 \le 1$ for x we obtain $-2 \le x \le -1$. A1

The range is $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. A1

b.



correct shape — A1

endpoints labelled with correct coordinates — A1

c. attempting the chain rule — M1

Let $u = 2x + 3$ and so $\frac{du}{dx} = 2$.

$y = \arcsin(u)$ and so $\frac{dy}{du} = \frac{1}{\sqrt{1 - u^2}}$

$\frac{dy}{dx} = \frac{2}{\sqrt{1 - (2x+3)^2}}$ (or equivalent) — A1

When $x = -\frac{5}{4}$, $\frac{dy}{dx} = \frac{4\sqrt{3}}{3}$ (or equivalent). — A1

Question 5 (4 marks)

$\cos(3x) = \cos(2x + x)$ — A1

$= \cos(2x)\cos(x) - \sin(2x)\sin(x)$

$= (2\cos^2(x) - 1)\cos(x) - (2\sin(x)\cos(x))\sin(x)$ — M1

$= 2\cos^3(x) - \cos(x) - 2\sin^2(x)\cos(x)$

$= 2\cos^3(x) - \cos(x) - 2\cos(x)(1 - \cos^2(x))$ — M1

$= 2\cos^3(x) - \cos(x) - 2\cos(x) + 2\cos^3(x)$ — A1

$= 4\cos^3(x) - 3\cos(x)$

Question 6 (4 marks)

Let $u = 4x - 1$ and so $du = 4dx$ (or equivalent).

$$\int \frac{x^2}{4x-1}dx = \int \frac{(u+1)^2}{16u}\frac{du}{4}$$ M1

$$= \frac{1}{64}\int\left(\frac{u^2+2u+1}{u}\right)du$$ A1

$$= \frac{1}{64}\int\left(u+2+\frac{1}{u}\right)du$$

$$= \frac{1}{64}\left(\frac{u^2}{2}+2u+\log_e|u|\right)$$ M1

$$= \frac{1}{64}\left(\frac{(4x-1)^2}{2}+2(4x-1)+\log_e|4x-1|\right)+c$$ A1

Alternatively:

$$\int \frac{x^2}{4x-1}dx = \int \frac{x}{4}+\frac{1}{16}+\frac{1}{16(4x-1)}dx$$ M1 A1

$$= \frac{x^2}{8}+\frac{x}{16}+\frac{1}{64}\log_e|4x-1|+C$$ M1 A1

Question 7 (4 marks)

$$-2\sin(2x) - 3\sin(3y)\frac{dy}{dx} = 0$$ M1

$$\frac{dy}{dx} = -\frac{2\sin(2x)}{3\sin(3y)}$$ A1

At $P\left(\frac{\pi}{6}, \frac{\pi}{9}\right)$, $\frac{dy}{dx} = -\frac{2}{3}$, and so the gradient of the normal is $\frac{3}{2}$. M1

$$y - \frac{\pi}{9} = \frac{3}{2}\left(x - \frac{\pi}{6}\right)$$

$$y = \frac{3}{2}x - \frac{5\pi}{36}$$ A1

Question 8 (5 marks)

$-\int_{25}^{T} \frac{dT}{T - T_0} = k\int_{0}^{t} dt$ M1

$[\log_e|T - T_0|]_T^{25} = kt$ (or equivalent)

$\log_e\left(\frac{25 - T_0}{T - T_0}\right) = kt$ A1

When $t = 5$, $\log_e\left(\frac{25 - T_0}{15 - T_0}\right) = 5k$ (1)

When $t = 10$, $\log_e\left(\frac{25 - T_0}{10 - T_0}\right) = 10k$ (2) A1

Note: Award A1 for these two equations.

(1) × 2 gives $2\log_e\left(\frac{25 - T_0}{15 - T_0}\right) = 10k$

$\log_e\left(\frac{25 - T_0}{15 - T_0}\right)^2 = \log_e\left(\frac{25 - T_0}{10 - T_0}\right)$ and so $\left(\frac{25 - T_0}{15 - T_0}\right)^2 = \frac{25 - T_0}{10 - T_0}$ M1

$(25 - T_0)(10 - T_0) = (15 - T_0)^2$

$250 - 35T_0 + T_0^2 = 225 - 30T_0 + T_0^2$

$5T_0 = 25$

$T_0 = 5$ and so the outside temperature is 5°C. A1

Question 9 (7 marks)

a. $(x + yi)(x - yi) + (a - bi)(x + yi) + (a + bi)(x - yi) + 1 = 0$ M1

$(x^2 + y^2) + (ax + by) + (ay - bx)i + (ax + by) + (bx - ay)i + 1 = 0$ A1

$(x^2 + y^2) + 2ax + 2by + 1 = 0$ A1

$(x + a)^2 + (y + b)^2 = a^2 + b^2 - 1$ or $(x + a)^2 + (y + b)^2 = |u|^2 - 1$ M1 A1

b. centre is $(-a, -b)$ A1

radius is $\sqrt{|u|^2 - 1}$ (or equivalent) A1