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# Specialist Mathematics

# 2014

# Trial Examination 2

## SECTION 1 Multiple-choice questions

### Instructions for Section 1

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
**No** marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude $g$ ms$^{-2}$, where $g = 9.8$.

**Question 1** $\frac{(x-k)^2}{4-k} + \frac{(k-y)^2}{k-6} = \frac{1}{12}$ is a hyperbola when

A. $3 \le k \le 7$

B. $4 < k < 6$

C. $k \le 3$ only

D. $k \ge 7$ only

E. $k < 4$ or $k > 6$

**Question 2** Given that the domain of $\sin x$ and $\cos x$ are restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$ respectively, the maximal domain of $\sin\left(-\cos^{-1} x\right)$ is

A. $[-1, 0]$

B. $[0, 1]$

C. $[-1, 1]$

D. $(-1, 1)$

E. $[-1, 0)$

**Question 3** Given $z = a\left[i + cis\left(-\frac{2\pi}{3}\right)\right]$ and $a \in R^+$, $Arg(z) =$

A. $\frac{7\pi}{6}$

B. $\frac{5\pi}{6}$

C. $-\frac{5\pi}{6}$

D. $\frac{11\pi}{12}$

E. $-\frac{11\pi}{12}$

**Question 4** Consider the graph of $y = \dfrac{x^3 - c}{x^2 - c}$ where $c \in R$. Which one of the following statements is true?

A. The graph does not have any straight line asymptote.

B. The graph has at least 1 straight line asymptote.

C. The graph has at least 2 straight line asymptotes.

D. The graph has at least 3 straight line asymptotes.

E. The graph has either 0, 1, 2 or 3 straight line asymptotes.

**Question 5** $\dfrac{1-\sin 2x}{1+\sin 2x}$ can be expressed as

A. $1 - 2\cot^2 4x$

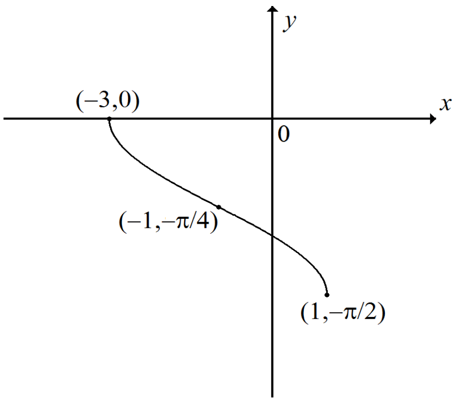
B. $1 - 2\tan^2 4x$

C. $\sec^2 2x + \tan^2 2x$

D. $(\sec 2x - \tan 2x)^2$

E. $(\operatorname{cosec} x + \cot x)^2$

**Question 6**




The diagram above is the graph of the inverse of a circular function. A possible equation of the graph is

A. $y = -\dfrac{\pi}{2} + \sin^{-1}\left(\dfrac{x-1}{2}\right)$

B. $y = -\dfrac{\pi}{2} - \cos^{-1}\left(\dfrac{x+1}{2}\right)$

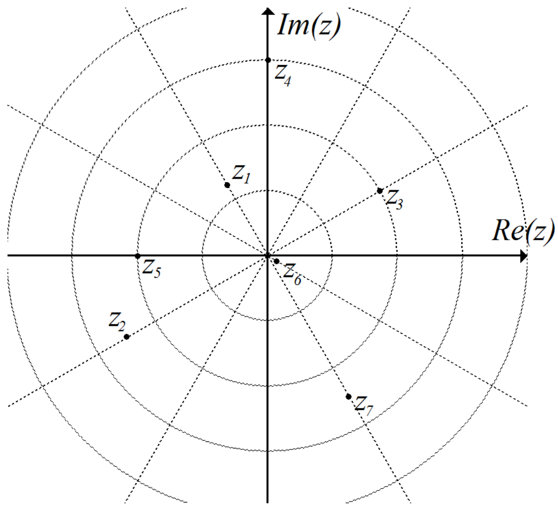
C. $y = -\dfrac{\pi}{4} + \dfrac{1}{2}\sin^{-1}\left(\dfrac{x+1}{2}\right)$

D. $y = -\dfrac{\pi}{2} + \dfrac{1}{2}\cos^{-1}\left(\dfrac{x+1}{2}\right)$

E. $y = -\dfrac{\pi}{2} - \dfrac{1}{2}\sin^{-1}\left(\dfrac{x+1}{2}\right)$

**Question 7**




Seven complex numbers are shown in the Argand diagram above. The product $z_1 z_2$ is most likely to be

A. $z_3$

B. $z_4$

C. $z_5$

D. $z_6$

E. $z_7$

**Question 8** Given $z^2 = n - ni$, $(\mathrm{Im}(z))^2$ equals

A. $\dfrac{1-\sqrt{2}}{2n}$

B. $\dfrac{\sqrt{2}-1}{2n}$

C. $-n$

D. $\dfrac{n}{2\left(\sqrt{2}+1\right)}$

E. $\dfrac{n\left(\sqrt{2}+1\right)}{2}$

**Question 9** Which one of the following statements is true according to the fundamental theorem of algebra?

A. Any polynomial (of degree one or higher) of a single variable has *at least one real root*.

B. A polynomial (of degree five) of a single variable can have *exactly two real roots*.

C. A cubic polynomial of a single variable has *exactly three complex roots*.

D. A quartic polynomial of a single variable *always has four unique roots*.

E. The roots of any polynomial of a single variable can always be paired as complex conjugates.

**Question 10** The line $y = mx + \frac{\pi}{4}$ is a tangent to the curve $y = \tan^{-1} x$ at $x = a$.
The closest approximation to $a$ is

A. $a = 2.24102$

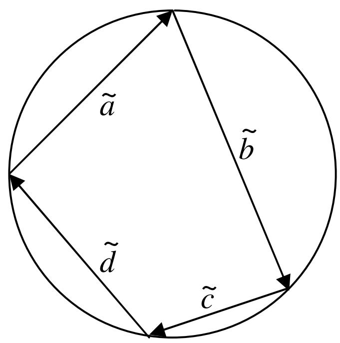
B. $a = 2.2503$

C. $a = 2.264$

D. $a = 2.28$

E. $a = 2.3$

**Question 11** Vectors $\tilde{a}$ , $\tilde{b}$ , $\tilde{c}$ and $\tilde{d}$ form a *cyclic quadrilateral*.




The value of $\tilde{a}.\tilde{b}\,|\tilde{c}\,||\,\tilde{d}\,| + |\,\tilde{a}\,||\,\tilde{b}\,|\,\tilde{c}.\tilde{d}$ is

A. $\frac{1}{2}$

B. 1

C. $\frac{\pi}{2}$

D. 0

E. $\frac{\pi}{4}$

**Question 12** If $\tilde{c}$ is a unit vector parallel to $\tilde{a} = \tilde{i} + \sqrt{2}\,\tilde{j} - \tilde{k}$ and perpendicular to $\tilde{b} = -\sqrt{2}\,\tilde{i} + 2\,\tilde{j} + \sqrt{2}\,\tilde{k}$, then $\tilde{c}$ is

A. $\frac{1}{\sqrt{2}}\tilde{i} + \frac{1}{2}\tilde{j} - \frac{1}{\sqrt{2}}\tilde{k}$

B. $\frac{1}{2}\tilde{i} - \frac{1}{\sqrt{2}}\tilde{j} + \frac{1}{2}\tilde{k}$

C. $\frac{1}{2}\tilde{i} + \frac{1}{\sqrt{2}}\tilde{j} - \frac{1}{2}\tilde{k}$

D. $-\frac{1}{2}\tilde{i} + \frac{1}{\sqrt{2}}\tilde{j} + \frac{1}{2}\tilde{k}$

E. $-\frac{1}{\sqrt{2}}\tilde{i} - \frac{1}{2}\tilde{j} + \frac{1}{\sqrt{2}}\tilde{k}$

**Question 13** If $\tilde{p} = \tilde{i} + 2a\,\tilde{j}$, $\tilde{q} = a\,\tilde{i} + \tilde{k}$, $\tilde{r} = \tilde{j} + 3a\,\tilde{k}$ and $\tilde{s} = \tilde{i} + 2\,\tilde{j} + 3\tilde{k}$, where $a \in R$ then

A. $\tilde{p}$, $\tilde{q}$, $\tilde{r}$ and $\tilde{s}$ are linearly dependent

B. $\tilde{p}$, $\tilde{r}$ and $\tilde{s}$ are linearly dependent

C. $\tilde{p}$, $\tilde{q}$ and $\tilde{r}$ are linearly dependent

D. $\tilde{p}$, $\tilde{q}$ and $\tilde{s}$ are linearly dependent

E. $\tilde{q}$, $\tilde{r}$ and $\tilde{s}$ are linearly dependent

**Question 14** Let $A$ be the area of the region enclosed by the $y$-axis, the line $y = \frac{\pi}{4}$ and the curve $y = \tan^{-1}(2x)$.

The area of the region(s) enclosed by the curve $y = \tan^{-1}(2x)$, the $x$-axis and the lines $x = \pm\frac{1}{2}$ is given by

A. 0

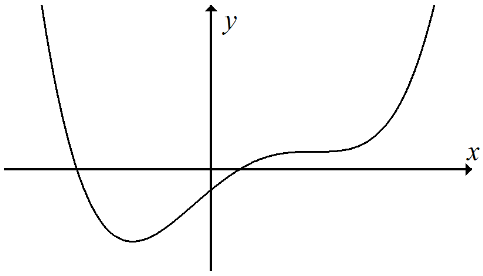
B. $2A$

C. $\frac{\pi}{4} - A$

D. $A - \frac{\pi}{8}$

E. $\frac{\pi}{4} - 2A$

**Question 15** The graph of an anti-derivative of $f(x)$ is shown below. It has a local minimum and a stationary point of inflection.




Which one of the following statements about the graph of $f(x)$ is true?

A. The graph of $f(x)$ has three $x$-intercepts and two stationary points.

B. The graph of $f(x)$ has three $x$-intercepts, one local minimum and one local maximum.

C. The graph of $f(x)$ has two $x$-intercepts and one stationary point.

D. The graph of $f(x)$ has two $x$-intercepts, one local minimum and one local maximum.

E. The graph of $f(x)$ has one $x$-intercepts, one stationary point and two points of inflection.

**Question 16** For $a, b \in R$ the function $a\sin^{-1}x + 2b\cos^{-1}x$ can be expressed as

A. $(a-2b)\sin^{-1}x - b\pi$

B. $(2b-a)\sin^{-1}x + b\pi$

C. $b\pi - (2b-a)\sin^{-1}x$

D. $(2b-a)\pi - b\cos^{-1}x$

E. $(a-2b)\pi + b\cos^{-1}x$

**Question 17** Given $\dfrac{dy}{dx} = \cos\sqrt{x^2+1}$ and $y = 5.24$ when $x = 1.7$.
When $x = 4.6$ the value of $y$ is closest to

A. 7.28

B. 3.20

C. 2.04

D. $-2.83$

E. $-3.71$

**Question 18** A particle moves in a straight line.

The velocity of the particle at time $t \geq 0$ is given by $v = \dfrac{1}{1+e^t}$, and it is at position $x = 1$ initially.

The displacement of the particle from $t = 1$ to $t = 2$ is given by

A. $\left[\log_e \dfrac{e^t}{1+e^t}\right]_1^2$

B. $\left[\log_e \dfrac{e^t}{1+e^t}\right]_1^2 + 1$

C. $\left[\log_e \dfrac{1}{1+e^t}\right]_1^2$

D. $\left[\log_e \dfrac{1}{1+e^t}\right]_1^2 + 1$

E. $\left[\log_e \dfrac{e^t - 1}{e^t + 1}\right]_1^2$

**Question 19** The position of a particle is given by $\tilde{r} = \left(\dfrac{t^3}{3} - t^2\right)\tilde{i}$ where $t \geq 0$. The acceleration of the particle at the moment when it reverses its direction of motion is

A. $\tilde{0}$

B. $\tilde{i}$

C. $2\tilde{i}$

D. $-2\tilde{i}$

E. $-\tilde{i}$

**Question 20** A 10 kg parcel rests on an accurate digital bathroom scale inside a moving lift. The reading on the bathroom scale is 9.8 kg. The lift

A. is moving upwards at constant speed

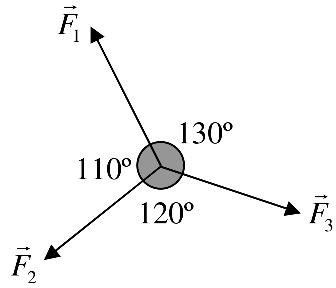
B. is moving downwards at constant speed

C. can only be moving upwards with decreasing speed

D. can only be moving downwards with decreasing speed

E. can be moving upwards or downwards

**Question 21** A particle is in equilibrium under the action of three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ (see diagram below).




Which one of the following statements is true?

A. $|\vec{F}_3| > |\vec{F}_1| > |\vec{F}_2|$

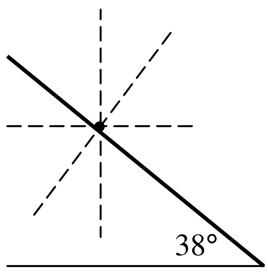
B. $|\vec{F}_3| > |\vec{F}_2| > |\vec{F}_1|$

C. $|\vec{F}_1| > |\vec{F}_2| > |\vec{F}_3|$

D. $|\vec{F}_1| > |\vec{F}_3| > |\vec{F}_2|$

E. $|\vec{F}_2| > |\vec{F}_1| > |\vec{F}_3|$

**Question 22** A particle slides *at constant speed* down a rough plane inclined at $38°$ to the horizontal. The reaction force of the plane on the particle makes an angle of $\theta°$ with the plane.




The value of $\theta$ is

A. 0

B. between 0 and 38

C. 38

D. 52

E. 90

## SECTION 2 Extended-answer questions

### Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude $g$ m s$^{-2}$, where $g = 9.8$.

**Question 1** Consider the equation $4(a-x)^2 - 9(b-y)^2 - 36 = 0$ where $a, b \in R$.

a. Show that $\frac{dy}{dx} = \pm \frac{2\,(x-a)}{3\sqrt{(x-a)^2 - 9}}$. 2 marks

Let $a = b = 0$ in the following questions.

b. Show that the equations of the tangents to the graph of $4(a-x)^2 - 9(b-y)^2 - 36 = 0$ at $x = k$, $k > 3$, are $y = \pm \frac{2k}{3\sqrt{k^2 - 9}} x \pm \frac{6}{\sqrt{k^2 - 9}}$. 3 marks



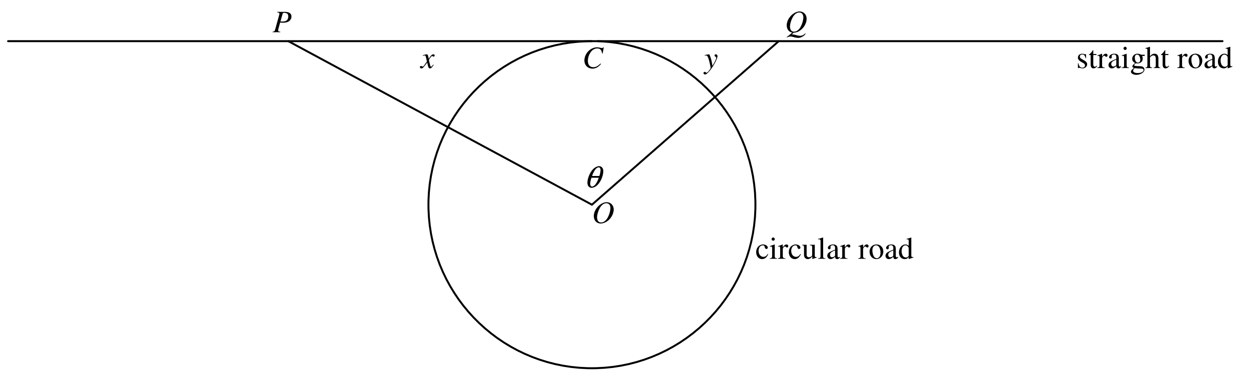
 

c. Find the $y$-coordinates of the points where the two tangents at $x = k$ cut the $y$-axis.
Hence show that the points become the same point as $k \to \infty$. 2 marks

d. Show that the two tangents at $x = k$ approach the asymptotes of $4(a-x)^2 - 9(b-y)^2 - 36 = 0$ as $k \to \infty$.
3 marks

**Question 2** Two runners marked as $P$ and $Q$ run along a straight road in the same direction. The straight road touches a circular road of radius 1 km at point $C$. Point $O$ is the centre of the circular road.
Let **distance** $PC = x$ km, **distance** $CQ = y$ km, and $\angle POQ = \theta$. ***P* and *Q* are NOT on the same side of *C***.




a. Show that $\tan\theta = \dfrac{x+y}{1-xy}$. 2 marks

b. Find the speed of runner $Q$ if $\theta$ remains constant at $\dfrac{\pi}{4}$, and when runner $P$ is 1 km from $C$ and runs towards $C$ at 12 km h$^{-1}$. 3 marks c i. If $\theta$ remains constant at $\frac{\pi}{4}$, and when $P$ is $x$ km from $C$ and runs towards $C$ at 12 km h$^{-1}$, find the time rate of change in the distance between $P$ and $Q$ in terms of $x$. 2 marks

c ii. Hence describe the change in the distance between $P$ and $Q$ if $\theta$ remains constant at $\frac{\pi}{4}$, and when $P$ is $x$ km from $C$ and runs towards $C$ at 12 km h$^{-1}$. 2 marks

Now the distance between the two runners is kept constant at $\frac{2\sqrt{3}}{3}$ km.

d. Show that the maximum value of $\theta$ is $\frac{\pi}{3}$. 3 marks

e. Find $\frac{d\theta}{dt}$ when $\theta = \frac{\pi}{3}$. 1 mark

**Question 3** The velocity of a particle at time $t \geq 0$ is given by $\underset{\sim}{v} = \frac{1}{1+t}\underset{\sim}{i} + \frac{1}{1+t^2}\underset{\sim}{j}$, and the initial position of the particle is $\underset{\sim}{r}(0) = \underset{\sim}{0}$.

a. Find the position of the particle at time $t$. 2 marks

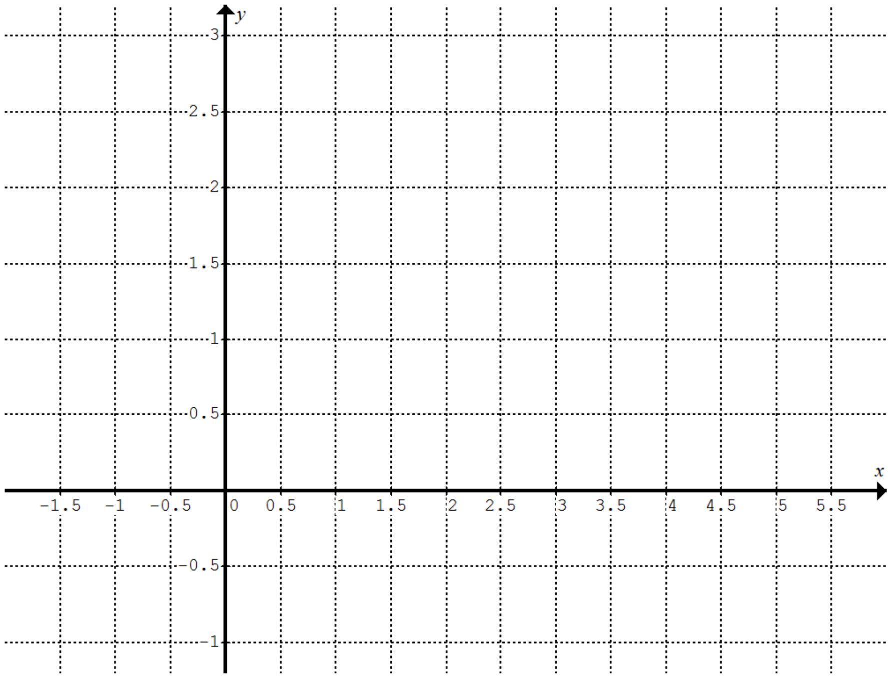
b. Show that the equation of the path of the particle is $\tan y = e^x - 1$. 1 mark

c i. Find the coordinates (correct to 2 decimal places) of the point of inflection of the path. 4 marks c ii. Find the time of arrival (correct to 2 decimal places) at the point of inflection of the path. 1 mark

c iii. Find the speed (correct to 2 decimal places) of the particle at the point of inflection of its path. 1 mark

d. Sketch the graph of the path of the particle for $t \geq 0$. Show and label the important features of the path.

3 marks




**Question 4** Consider polynomial function $P(x) = (x-5)(x-3)(x-1)(x+1) - c$ where $c \in R$.

a. Find the values of $c$ such that $P(x)$ has non-real roots. 3 marks

b. Use algebraic methods or other non-CAS means to find the roots of $P(x)$ when $c = 105$. 3 marks

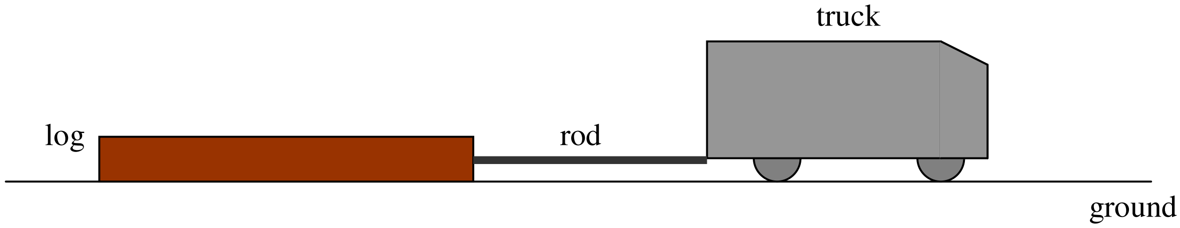
c i. Find the roots of $P(x)$ when $c = -17$ without CAS.
Write your answers as pairs of conjugates (correct to 4 decimal places). 4 marks

c ii. The roots of $P(x)$ when $c = -17$ lie on a circle in the Argand plane. Find the **exact** centre and radius of the circle. 2 marks

**Question 5** A 3000 kg truck drags a 1500 kg log with a 150 kg rigid rod along the horizontal ground in a straight line.

The coefficient of friction between the log and the ground is 0.30. The total of air resistance and other resistive forces on the truck is 200 N. The truck accelerates at 0.20 m $s^{-2}$.




a. Show that the friction between the log and the ground is 4410 N. 1 mark

b. Find the total friction between the tyres of the truck and the ground. 2 marks

c. Find the acceleration of the log. 1 mark

d. Find the maximum and minimum tensions in the rod when the truck accelerates at 0.20 m $s^{-2}$. 3 marks

Now the truck moves at constant speed of $10.0 \text{ m s}^{-1}$.

e. Find the total friction between the tyres of the truck and the ground. 1 mark

f. Find the tension in the rod. 1 mark

Now the truck slows down at $0.10 \text{ m s}^{-2}$.

g. Discuss the maximum and minimum tensions in the rod. 2 marks

# End of Exam 2