TSSM
Creating VCE Success

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'2016 Examination Package' -
Trial Examination 3 of 5

STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 1

*(*TSSM's *2013 trial exam updated for the current study design)*

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 11 pages.
- Working space is provided throughout the book.

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

Find $\int \frac{4x-1}{x^2+9}\,dx$.

3 marks

Question 2

If $2x^2 - x\sin y + y = 10$ find $\frac{dy}{dx}$ at the point $\left(1, \frac{\pi}{4}\right)$. Express your answer in the form $a\sqrt{2}+b$, where a and b are real constants.

3 marks

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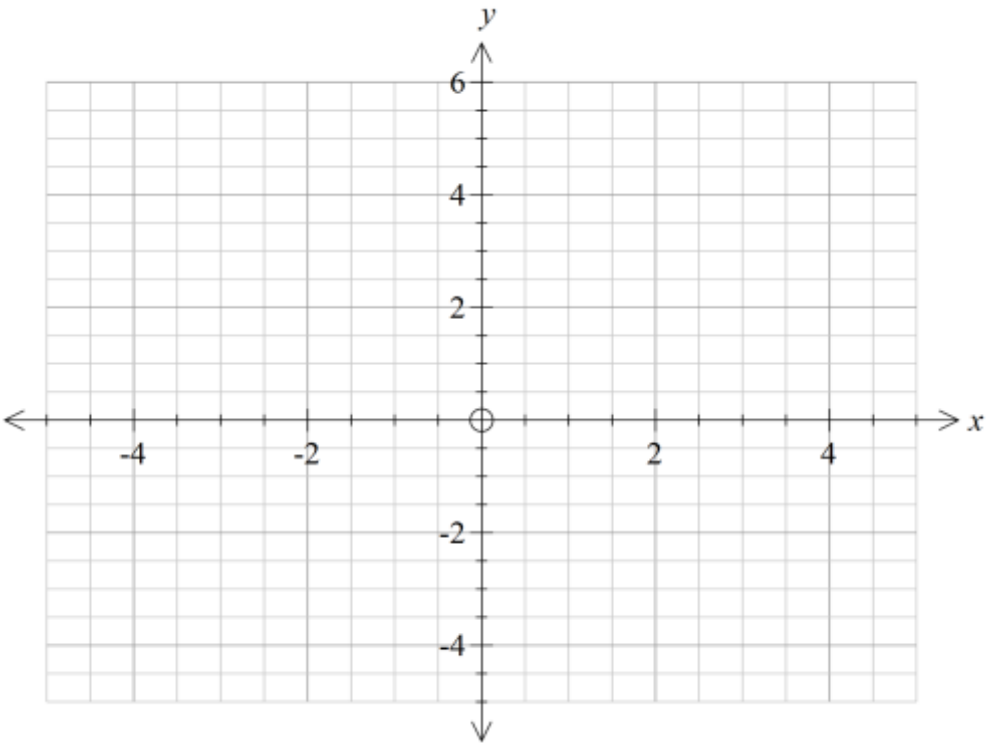
Question 3

a. Express $f(x)=\dfrac{2x^2-2}{x^2-4}$ in the form $f(x)=a+\dfrac{b}{x^2-4}$, where a and b are real constants.

1 mark

b. Sketch the graph of the relation $y=\dfrac{2x^2-2}{x^2-4}$ on the axes below.

Label any asymptotes with their equations and label any intercepts with the axes, writing them as coordinates.



3 marks

c. Find the area enclosed by the graph of the relation $y = \dfrac{2x^2 - 2}{x^2 - 4}$ and the x-axis.

3 marks

Total 7 marks

Question 4

Express $z = \left(\dfrac{-2 - 2i}{\sqrt{3} - i} \right)^3$, $z \in C$, in Cartesian form.

3 marks

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Question 5

An empty inverted right circular cone is being filled with water at the rate of $2\ \text{m}^3/\text{min}$.
If the diameter and height of the cone are equal in length, show that the rate at which the water level is rising after 1 minute is $\sqrt[3]{\frac{8}{9\pi}}\ \text{m}/\text{min}$.

3 marks

Question 6

a. Find $\frac{d}{dx}\left(x\,\text{Tan}^{-1}x\right)$.

1 mark

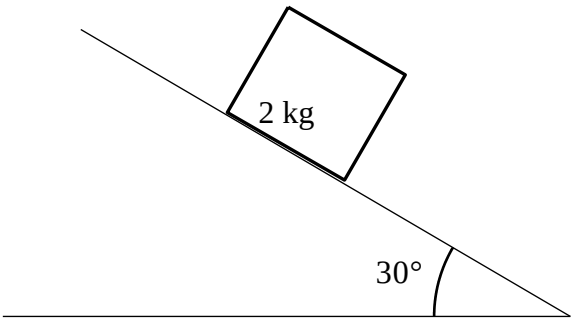
b. Hence find $\int_0^1 \text{Tan}^{-1}x\,dx$.

3 marks

Total 4 marks

Question 7

A carton of mass 2 kg is held at rest on a frictionless plane inclined at $30°$ to the horizontal as shown. After the carton is released from rest, it begins to slide down the plane.
Find the speed of the carton after it has travelled a distance of two metres, .



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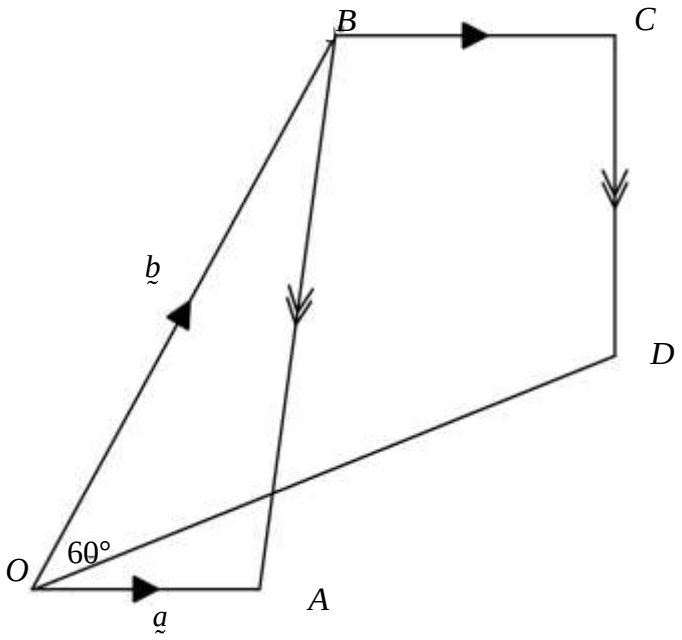
3 marks

Question 8

In the figure below, $\left|\overrightarrow{OB}\right| = 3\left|\overrightarrow{OA}\right|$, $\overrightarrow{BC} = \overrightarrow{OA}$, $\overrightarrow{CD}$ is parallel to $\overrightarrow{BA}$ and angle AOB is $60°$. The position vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are defined as $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively, and $\overrightarrow{CD} = m\left(\overrightarrow{BA}\right)$, where m is a scalar.



a. Show that the scalar product, $\underset{\sim}{a}.\underset{\sim}{b} = \frac{3}{2}\left|\underset{\sim}{a}\right|^2$.

1 mark

b. Express vectors $\overrightarrow{BA}$ and $\overrightarrow{OD}$ in terms of $\underset{\sim}{a}$, $\underset{\sim}{b}$ and the scalar m.

2 marks

c. If $\overrightarrow{OD}$ and $\overrightarrow{BA}$ are **perpendicular** vectors, show that $m = \frac{8}{7}$.

2 marks

Total 5 marks

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Question 9

The region in the first quadrant enclosed by the curve $y = \text{Cos}^{-1}(2x)$, the line $y = \frac{\pi}{4}$ and the x- and y- axes is rotated about the y-axis.
Find the volume of the resulting solid of revolution.

3 marks

Question 10

The vector equation of a particle at time, t is defined as

$$\underset{\sim}{r}(t) = (2\sec t + 1)\underset{\sim}{i} + (3\tan t - 2)\underset{\sim}{j}, \text{ where } 0 \le t < \frac{\pi}{2}$$

a. Find the Cartesian equation which describes the path of the particle.
State the domain and range.

3 marks

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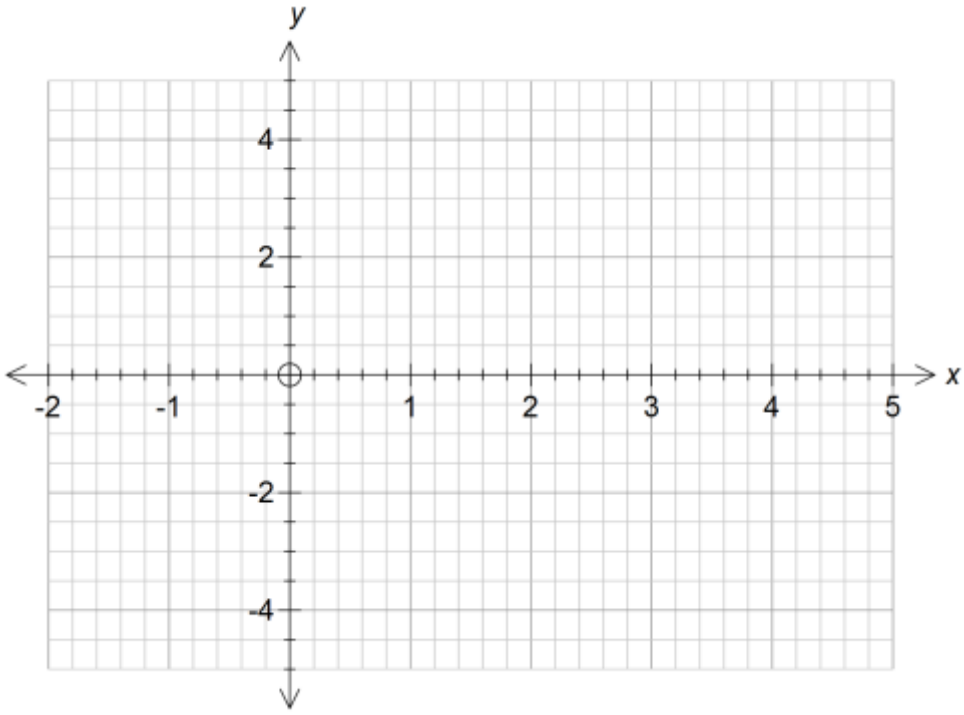
b. Find the equations of any asymptotes.

1 mark

c. Hence sketch the path of the particle on the axes below. Indicate with an arrow, the direction of motion.



2 marks
Total 6 marks

END OF QUESTION AND ANSWER BOOK

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