Neap:

Trial Examination 2013

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
Students are not permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 11 pages. Formula sheet of miscellaneous formulas.
Working space is provided throughout the booklet.

Instructions

Write **your name** and your **teacher's name** in the space provided above on this page.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2013 VCE Specialist Mathematics Units 3 & 4 Written Examination 1.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

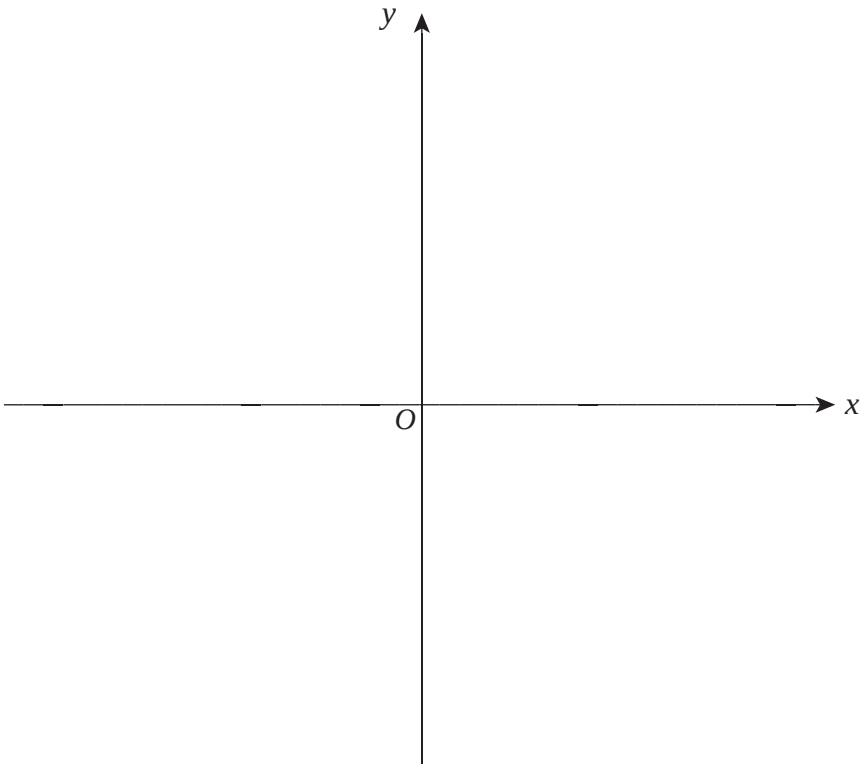
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

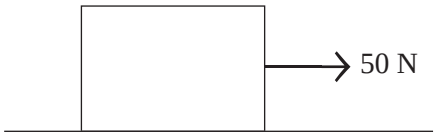
Question 1 (2 marks)

Sketch the curve with equation $y = \dfrac{1}{4x^2 + 1}$. State the coordinates of the points where the curve intersects the coordinate axes and the equations of any asymptotes.



Question 2 (4 marks)

A block of mass 4 kg is made to move in a straight line on a rough horizontal surface by a horizontal force of 50 newtons, as shown in the diagram.



a. On the diagram above, mark in the forces acting on the block. 1 mark

b. Find the magnitude of the normal reaction force acting on the block. 1 mark

The acceleration of the block is 2 m/s^2.

c. Find the magnitude of the friction force acting on the block. 2 marks

Question 3 (4 marks)

Consider the function $g(x) = x^2 e^{-x}, x \geq 0$.

Find the *x*-coordinates of the points of inflection on the graph of *g*.

Question 4 (4 marks)

Given that $\int_{\frac{\pi}{2}}^{k} \frac{\sin(x)}{1-\cos(x)}dx = \frac{1}{2}$ and $\frac{\pi}{2} < k < \pi$, find the value of k.

Question 5 (4 marks)

Solve the equation $3\cot^2(x) + 5\operatorname{cosec}(x) + 1 = 0, \ \pi < x < \frac{3\pi}{2}$.

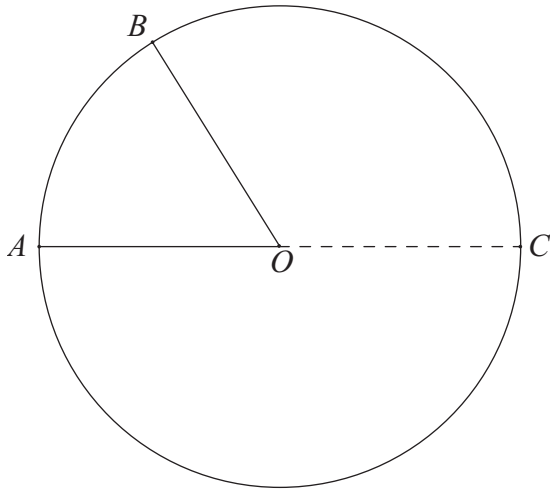
Question 6 (4 marks)

Solve $z^2 = 5 - 12i,\ z \in C$.

Express your answers in the form $z = x + yi, x, y \in R$.

Question 7 (3 marks)

A circle with centre O is shown below. The points A, B and C lie on the circumference of the circle and AC is a diameter.



Let $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.

Prove that $\angle ABC$ is a right angle.

Question 8 (6 marks)

A curve C is defined by $x^3y - \sin\left(\frac{\pi y}{2}\right) = 7$.

The point $P(k, 1)$ lies on C.

a. Show that $k = 2$. 2 marks

b. For this value of k, find the gradient at P. 4 marks

Question 9 (4 marks)

The acceleration, a m/s^2, of a body at a displacement x metres from a fixed origin O is given by $a = \dfrac{2x}{x^2 + 1}$.

Find the velocity, v m/s, of the body given that $v = 1$ when $x = 0$.

Question 10 (5 marks)

Consider the equation $\dfrac{(z + i)^n}{z^n} = 1$ where n is a positive integer.

a. Use De Moivre's theorem to show that $z = \dfrac{i}{\text{cis}\left(\dfrac{2k\pi}{n}\right) - 1}$, $k = 1, 2, ..., n - 1$. 2 marks

b. Hence show that $z = \frac{1}{2}\left(\cot\left(\frac{k\pi}{n}\right) - i\right)$. 3 marks

END OF QUESTION AND ANSWER BOOKLET